Comment on “Does the weak trace show the past of a quantum particle?”

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In the paper “Does the weak trace show the past of a quantum particle?”[arXiv:2109.14060v2], it is argued that null weak values of the spatial projectors are inadequate to infer the presence of a quantum particle at an intermediate time between preparation and detection. This conclusion relies on two arguments – (i) the role of the disturbance induced by a weak measurement, and (ii) classical-like features like continuous paths that must purportedly be associated with a quantum particle presence. Here we first show that (i) arises from a misunderstanding of null weak values by putting forward a simple counter-example that highlights that the relevant quantities to examine are the vanishing amplitudes, not the wavefunction. Then we briefly argue that enforcing classical pre-conditions in order to account for quantum properties during unitary evolution is unlikely to lead to a consistent understanding of quantum phenomena.

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In order to learn something about a physical system's properties, a measurement of the system is necessary. For a quantum system, a standard measurement radically changes the system's evolution as the premeasurement state is projected to one of the eigenstates of the measured observable. It is therefore difficult, even in principle, to imagine a procedure that would enable one to measure the properties of a system at an intermediate time, without affecting the system's evolution.

With weak measurements [1] it is possible to achieve minimally perturbing non-destructive measurements. An obervable $\hat{O}$ of a system initially prepared (“preselected”) in state $|\psi_i\rangle$ is weakly measured by coupling the system to a quantum pointer. “Weakly” means here that the system's evolution after the coupling is only affected to first order, so that when a different observable, say $\hat{N}$, is subsequently measured (through a projective measurement), the probability to obtain the final state $|\psi_f\rangle$ (an eigenstate of $\hat{N}$) is not modified to first order (relative to the same evolution without the coupling). Once $|\psi_f\rangle$ is obtained (“post-selected”), the quantum pointer coupled to $\hat{O}$ is shifted by $\mathrm{Re}\,(O^w)$ where the weak value O^w is given by

$$O^w = \frac{\langle\psi_f|\,\hat{O}\,|\psi_i\rangle}{\langle\psi_f|\,\psi_i\rangle}. \quad (1)$$

Let us now examine a quantum particle propagating inside the interferometer depicted in Fig. 1 in a pre and post-selected situation. Weak couplings can be implemented jointly on the different arms. Typically, the corresponding weak values are generally non-zero and all the coupled quantum pointers will shift. Vaidman [2] proposed that the weak values of the spatial projector, $\hat{O} = \Pi_x \equiv |x\rangle\langle x|$ could be used as a “weak trace criterion”, telling us where the quantum particle has been during its evolution inside the interferometer. In the typical case just mentioned, this criterion implies that the particle has been in all the arms in the interferometer,

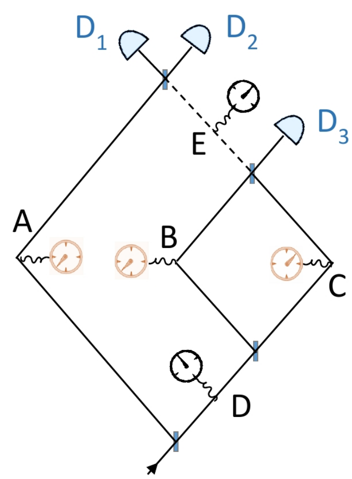

FIG. 1: The original 3-paths interferometer with a nested Mach-Zehnder, introduced in [2] and discussed in [4]. We have pictured here the weakly coupled pointers (the pointers in orange shift after post-selection, while those pictured in black remain unshifted as the weak value vanishes). The interferometer is balanced so that destructive interference is obtained along arm E in the absence of weak interactions.

i.e. a statement of the paths superposition. In some situations, the weak value for a coupling at position x_0 will vanish, $\Pi^w_{x_0} = 0$, indicating that the quantum particle was not there (in the sense that its spatial degree of freedom was not detected at x_0).

An apparent paradox pointed out by Vaidman [2, 3] happens when the interferometer is balanced such that the wavefunction along arm E vanishes. Then, taking the initial state $|\psi_i\rangle = (|\psi_D\rangle + i\,|\psi_A\rangle)/\sqrt{2}$, if post-selection is chosen when detector D_2 clicks, that is $|\psi_f\rangle = (|\psi_A\rangle + i\,|\psi_E\rangle)/\sqrt{2}$, the following weak values are ob-

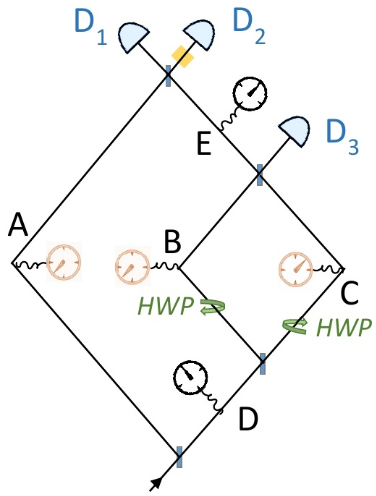

FIG. 2: Similar to Fig. 1, with the addition of polarization rotations inside the nested Mach-Zehnder and a polarizer at post-selection. Now the wavefunction on arm E does not vanish, although the weak values are the same as those of Fig. 1.

tained:

$$\Pi_A^w = 1 \qquad \Pi_D^w = 0 \tag{2}$$

$$\Pi_B^w = \frac{1}{2} \qquad \Pi_C^w = -\frac{1}{2} \tag{3}$$

$$\Pi_E^w = 0. \tag{4}$$

This means that the quantum particle is detected inside the nested interferometer, but is not detected in the ingoing (D) or outgoing (E) arms. The weak trace is therefore discontinuous.

In the paper "Does the weak trace show the past of a quantum particle?" [4] Hance *et al.* claim that the weak trace approach is inconsistent because weak measurements disturb the system. They argue that if weak interactions are made inside the nested interferometer, then the perfect destructive interference on arm E that exists when no weak couplings are implemented is disturbed, so that the wavefunction $\psi_E(x)$ is not zero anymore. This is indeed the case, but this observation is irrelevant to having a vanishing weak value. Put differently Eq. (4) may hold irrespective of whether $\psi_E(x)$ vanishes or not[1]. A vanishing weak value requires the numerator of Eq. (1), a transition amplitude, to vanish, not the wavefunctions.

This can be seen by slightly modifying the interferometer pictured in Fig. 1 in the following way (see Fig. 2). We now include the polarization of the photon and choose $|\psi_i\rangle = (|\psi_D\rangle + i\,|\psi_A\rangle)\,|H\rangle\,/\sqrt{2}$ as the initial state (where $|H\rangle\,,|V\rangle$ stand for horizontal and vertical polarization, and $|\nearrow\rangle = (|H\rangle + |V\rangle)\,/\sqrt{2}$, $|\searrow\rangle = (|H\rangle - |V\rangle)\,/\sqrt{2}$ label the polarization in the diagonal basis). Inside the nested interferometer, we add wave-plates in order to rotate the polarization on each arm such that the state of the photon inside the interferometer becomes $i\,|\psi_B\rangle\,|\nearrow\rangle + |\psi_C\rangle\,|\searrow\rangle$. Now on arm E the wavefunction becomes $|\psi_E\rangle\,(-\,|\nearrow\rangle + |\searrow\rangle)$ which does not vanish. However, it is an easy exercise to check that the structure of Eqs. (2)-(4) is left untouched – Eqs. (2) and (4) remain identical while Π_B^w and Π_C^w pick up a factor depending on the polarization rotation. Π_E^w vanishes because the state on arm E is orthogonal to the post-selected one.

This counter-example, an adaptation to the present discussion of a 3-paths atomic interferometer introduced previously [5] (see also [6]), disproves the argument given in [4] since we still have $\Pi_E^w = 0$, but now implementing the weak interactions does not change the state on arm E from an undisturbed vacuum to a "perturbed state with light present" [4] (as for any weak measurement, we have a slight perturbation introduced by the weak couplings). And the trace of the spatial degree of freedom of the quantum particle remains discontinuous. Note that in this counter-example the post-selected state evolved backward in time does not vanish in arm D, although $\Pi_D^w = 0$.

The second aim of [4] is to show that "the weak trace does not reveal the path of a quantum particle". The authors assert that a quantum particle must have a continuous path (like a classical particle), as per their condition ii). As it is given, this condition appears somewhat arbitrary and not particularly meaningful: it depends on how a path is defined for a quantum particle, and has no relation with an observational warrant that could confirm this claim. Indeed, the wavefunction is continuous and can be understood as propagating along Feynman paths. But these paths can interfere, and the destructive interference between amplitudes carried by different Feynman paths is precisely what makes a weak value vanish [7]. However the wavefunction or the Feynman paths are usually taken to be mere computational tools (at least according to standard quantum mechanics), so while it is possible to explain discontinuous weak value traces in terms of continuous but destructively interfering paths, this hardly changes the experimental fact that position weak values are discontinuous.

The underlying issue here is not to decree that quantum particles *must* have continuous paths, but (a) to put forward a cogent framework in order to define properties of a quantum system at an intermediate time; and (b) suggest an experimental scheme in order to observe such properties. Obviously, one can refuse to ascribe properties to quantum systems at intermediate times, but then the path of a quantum particle cannot even be defined (and the issue of whether these paths are continuous or not becomes moot). It turns out that weak measurements already constitute a framework in which weak val-

[1] Note this is the case here for arm D, since $\Pi_D^w = 0$ but $\psi_D(x)$ is not zero. The post-selected state evolved backward in time does vanish on arm D, but this backward evolved state does not seem to be taken as physically real in Ref. [4].

ues can be related to quantum properties, provided one is willing to relax the eigenstate-eigenvalue link (see [8] and Refs. therein). And the resulting weak values, even if they predict a non-classical behavior, can be experimentally observed.

To conclude, we have shown that the arguments given in [4] aiming to show that weak values of the spatial projector give an inconsistent account of the past position of a quantum particle are incorrect. The interpretation of weak values is controversial [9], and it is interesting to confront different viewpoints in order to understand the elusive nature and properties of quantum systems. Nevertheless, it is preferable to base the discussion on technically relevant arguments.

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