A FUZZY INTERACTIVE APPROACH FOR DECENTRALIZED BILEVEL PROGRAMMING PROBLEM WITH A COMMON DECISION VARIABLE

Guang-min Wang\(^1\), Zhong-ping Wan\(^2\) and Xian-jia Wang\(^1\)

\(^1\)Institute of System Engineering, Wuhan University, Wuhan 430072, P. R. China

\(^2\)School of Mathematics and Statistics, Wuhan University, Wuhan 430072, P. R. China

Abstract. This paper studies a decentralized bilevel programming problem with a common decision variable, in which there is a decision maker at the upper level (the leader) and multiple ones at the lower level (the followers). We propose a fuzzy interactive decision making (DM) approach to derive a satisfactory solution for decision makers by introducing fuzzy goals for objective functions of all decision makers and consulting the ratios of satisfaction between the leader and the followers. Finally, a numerical example illustrates the feasibility and efficiency of the proposed algorithm.

Keywords. bilevel programming, fuzzy interactive approach, decision making.

1 Introduction

In this paper, we consider a decentralized bilevel programming problem with a common decision variable, in which there is a decision maker at the upper level (the leader) and multiple ones at the lower level (the followers). The bilevel programming is a nested optimization problem with two levels in a hierarchy, namely the upper and lower level decision maker (namely the leader and the follower, respectively), who have their own objective functions and constraint functions. So it is a practical and useful tool for solving hierarchical decision making problems. The bilevel programming is used so extensively that many researchers devote themselves into this field \([3,5,9,11,12,18]\). Some of them survey the bilevel programming with respecting to the theory, solution approaches and applications \([6,20,23]\). And various methods, which have been proposed to solve the bilevel and multilevel programming problems, can be roughly classified into five categories \([18]\): extreme-point search; transformation approach; descent and heuristic; intelligent computation and interior point.

In a bilevel programming, the leader optimizes his/her objective function independently and is affected by the reaction of the follower who makes
his/her decision after the former. And their objective functions are generally conflict each other. However, most problems encountered in practice fall into the situation in which they depend partly on the degree of interactive or cooperation between them, although the information between them is incomplete and vague. So decision makers behave cooperatively rather than non-cooperatively. For instance, the bilevel programming problem with a common decision variable is presented by considering optimal bidding strategies between the power Sellers (the Power Companies) and the Buyer (the Distribution Center) for contract arrangements of the middle-term contracts and the spot market transactions under uncertain electricity spot market [22].

Recently, Shih, Lai and Lee [16] developed a fuzzy approach, namely interactive fuzzy decision making method, for solving the bilevel programming problems by using the concept of tolerance membership functions and multiple objective decision making. For adjusting the decision making process between the different levels and between the decision makers of the same level, Shih and Lee [17] introduced compensatory operators. By using these compensatory operators, the solution procedures were formulated for the various types of multiple level decision problems. So the interactive fuzzy decision making methods have been proposed to solve the decentralized bilevel programming problem [1,14,24] as they have extensively been applied to bilevel and multilevel programming problems [8,9,10,19]. In this paper, we also present a fuzzy interactive approach on basis of the interactive fuzzy decision making methods to solve the decentralized bilevel programming problem with a common decision variable in the upper level and lower level programming. Our algorithm is different from fuzzy programming approaches existed for it not only considers the leader is the dominant action but also treats the satisfactory degree between the leader and the followers at decision making process.

2 Problem Formulation

In this paper, we deal with the following bilevel programming problem with a common decision variable in the upper level and lower level programming:

\[
(BLP) \quad \min_{x,z} F_0(x,y,z) \tag{1}
\]

\[\text{s.t. } g(x,z) \leq 0 \tag{2}\]

where \( y = (y_1^T, \cdots, y_M^T)^T \), \( z \) solve

\[\min_{y_1,z} F_1(x,y,z) \tag{3}\]

\[\vdots\]

\[\min_{y_M,z} F_M(x,y,z) \tag{4}\]

\[\text{s.t. } h(x,y,z) \leq 0. \tag{5}\]
where $F_0(x, y, z)$ is the objective function of the leader and $F_j(x, y, z) (j = 1, \cdots, M)$ is the objective function of the $j^{th}$ follower corresponding the relate space, $g : \mathbb{R}^{n_1} \times \mathbb{R}^{n_3} \rightarrow \mathbb{R}^p$, $h : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \rightarrow \mathbb{R}^q$, where $n_2 = l_1 + l_2 + \cdots + l_M$, $y_j \in R^{l_j} (j = 1, \cdots, M)$. Here $z$ is a common decision variable. All decision variables are all column vectors. For notational convenience, let $(x, y, z)$ denote $(x^T, y^T, z^T)^T$.

The bilevel programming is neither continuous anywhere nor convex even if the objective functions of the upper level and lower level and the constraints are all linear. Bard, Ben-Ayed and Blair proved bilevel linear programming (BLLP) is a NP-Hard problem [2,4]. Besides, Hansen, Jaumard and Savard proved BLPP is a strongly NP-Hard problem [7]. Later, Vicente, Savard and Judice declared it is a NP-Hard problem to find the local optimal solution of BLPP [21]. Furthermore, the upper and lower level's objective functions are generally conflict each other and they have no communications, which is not always reasonable in practice because the decision makers will behave cooperatively and have some communications, even though the information between them is incomplete and vague.

So we develop an algorithm for obtaining a satisfactory solution to the decentralized bilevel programming problem with a common decision variable based on the idea that the leader chooses a solution satisfying the minimal satisfactory level specified by the leader with consideration to the overall satisfactory balance between the both levels, and then the followers maximize their degrees of satisfactory on the condition that the leader obtains its minimal satisfactory level.

Compared with problem (BLP), we use the following formulation to solve the decentralized bilevel programming problem with cooperative DMs:

$$(BP) \quad \min F_0(x, y, z)$$
$$\min F_1(x, y, z)$$
$$\vdots$$
$$\min F_M(x, y, z)$$
$$s.t. \quad g(x, z) \leq 0$$
$$h(x, y, z) \leq 0.$$ 

Let $S = \{(x, y, z) : g(x, z) \leq 0, h(x, y, z) \leq 0\}$ denote the constraint region of the (BP) problem.

**Definition 1** $(x^*, y^*, z^*) \in S$ is called to be a complete optimal solution, if and only if $F_0(x^*, y^*, z^*) \leq F_0(x, y, z)$ and $F_j(x^*, y^*, z^*) \leq F_j(x, y, z)(j = 1, \cdots, M)$ for all $(x, y, z) \in S$.

However, in general, there does not always exist such a complete optimal solution which simultaneously minimizes both the leader’s and followers’ objective functions. Thus, the Pareto optimality [14] is introduced in the (BP) problem as follows:
Definition 2 \((x^*, y^*, z^*) \in S\) is called to be a Pareto optimal solution, if and only if there does not exist \((x, y, z) \in S\) such that \(F_0(x, y, z) \leq F_0(x^*, y^*, z^*), F_j(x, y, z) \leq F_j(x^*, y^*, z^*) (j = 1, \cdots, M)\) and at least one inequality holds strictly.

Supposing that \(S\) is nonempty and compact, then there exists a Pareto optimal solution for the (BP) problem [15].

It is natural that the leader and followers have fuzzy goals for their objective functions when they take fuzziness of human judgments into consideration [8,10,13]. Thus, we assume that the leader and followers have fuzzy goals using fuzzy theory [9,25] for each of the objective functions of the (BP) problem. Then, through the interaction between decision makers, their fuzzy goals can be quantified by eliciting the corresponding membership functions, which are denoted by \(\mu(F_0(x, y, z)))\) for the leader and \(\mu(F_j(x, y, z)) (j = 1, 2, \cdots, M)\) for the followers, respectively.

To facilitate computation for obtaining solutions, we use the following linear function as membership function

\[
\mu(F_j) = \begin{cases} 
0; & \text{if } F_j > F_j^U \\
\frac{F_j^U - F_j^L}{F_j^U - F_j^L}; & \text{if } F_j^L < F_j \leq F_j^U \\
1; & \text{if } F_j \leq F_j^L 
\end{cases} \tag{6}
\]

where \(F_j^U\) and \(F_j^L\) denote the upper bound and lower bound of the objective function \(F_j(x, y, z)\), such that the degrees of membership function are 0 and 1, respectively. Without loss of generality, we can assume that \(F_j^U\) and \(F_j^L\) \((j = 0, 1, \cdots, M)\) are optimal values of the following programming problem, respectively:

\[
\begin{align*}
F_j^U &= \max_{x,y,z} F_j(x, y, z) \\
\text{s.t.} & \quad g(x, y) \leq 0 \quad (7) \\
& \quad h(x, y, z) \leq 0 \\
F_j^L &= \min_{x,y,z} F_j(x, y, z) \\
\text{s.t.} & \quad g(x, y) \leq 0 \\
& \quad h(x, y, z) \leq 0 \quad (8)
\end{align*}
\]

### 3 Fuzzy Interactive Decision Making Approach

Here, the M-Pareto optimal solution, instead of the Pareto optimal solution, is introduced, where M refers to membership.

Definition 3 \((x^*, y^*, z^*) \in f(\alpha_0) \overset{df}{=} \{(x, y, z) \in S : \mu(F_0(x, y, z)) \geq \alpha_0\}\) is said to be an M-Pareto optimal solution to the (BP) problem, if and only if there does not exist \((x, y, z) \in f(\alpha_0)\), and \((x, y, z) \neq (x^*, y^*, z^*)\), such that \(\mu(F_j(x, y, z)) \geq \mu(F_j(x^*, y^*, z^*)) (j = 1, 2, \cdots, M)\) and at least one inequality holds strictly.
After the leader specifies the minimal acceptable level $\alpha_0$ for his/her membership function, the satisfying solution of each follower can be generated by solving the following programming:

$$(P) \quad \min t$$

s.t. 

$$\alpha_0 \leq \mu(F_0(x, y, z))$$

$$\alpha_j - \mu(F_j(x, y, z)) \leq t$$

$$(x, y, z) \in S, \ |t| \leq 1, \ j = 1, \ldots, M.$$ 

where $\alpha_j$ is the minimal acceptable reference level specified by the $j^{th}$ follower for his/her membership function.

The problem $(P)$ can be written as an equivalent programming problem by the definition of the membership function (6):

$$\min t$$

s.t. 

$$F_0(x, y, z) \leq F_0^U - \alpha_0(F_0^U - F_0^L)$$

$$F_j(x, y, z) \leq F_j^U + t(F_j^U - F_j^L) - \alpha_j(F_j^U - F_j^L)$$

$$(x, y, z) \in S, \ |t| \leq 1, \ j = 1, \ldots, M.$$ 

We have the following theorem, which is not difficult to prove:

**Theorem 1** Suppose that $(x^*, y^*, z^*, t^*)$ is an optimal solution of the problem $(P)$ and $t^* = 0$, then $(x^*, y^*, z^*)$ is an M-Pareto optimal solution to the problem $(BP)$.

**Proof:** We use the eduction to absurdity. Supposing that $(x^*, y^*, z^*)$ is not an M-Pareto optimal solution to the (BP) problem even though $(x^*, y^*, z^*, t^*)$ is an optimal solution of the problem (P) and $t^* = 0$. According to the definition of the M-Pareto optimal solution, there exists $(\bar{x}, \bar{y}, \bar{z}) \neq (x^*, y^*, z^*)$, such that $\mu(F_j(\bar{x}, \bar{y}, \bar{z})) \geq \mu(F_j(x^*, y^*, z^*))(j = 1, 2, \ldots, M)$ and at least one inequality holds strictly. Because $(x^*, y^*, z^*, t^*)$ is an optimal solution of the problem (P), then $(x^*, y^*, z^*, t^*)$ satisfy $\alpha_j - \mu(F_j(x^*, y^*, z^*)) \leq t^*(j = 1, 2, \ldots, M)$, then $\alpha_j - \mu(F_j(\bar{x}, \bar{y}, \bar{z})) \leq \alpha_j - \mu(F_j(x^*, y^*, z^*)) \leq t^* = 0(j = 1, 2, \ldots, M)$ with the above conclusion $\mu(F_j(\bar{x}, \bar{y}, \bar{z})) \geq \mu(F_j(x^*, y^*, z^*))$ $(j = 1, 2, \ldots, M)$. With the condition that $(\bar{x}, \bar{y}, \bar{z}) \in f(\alpha_0), (x, y, z, \bar{t})$ is the feasible solution of the problem (P) by choosing $\bar{t} = \max\{\alpha_j - \mu(F_j(\bar{x}, \bar{y}, \bar{z})), j = 1, 2, \ldots, M\}$. From the above, it can be seen that $\bar{t} \leq t^*$, thus $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ is the optimal solution of the problem (P), which is contradiction with that $(x^*, y^*, z^*, t^*)$ is an optimal solution of the problem (P). Therefore, the suppose is not correct, that is, $(x^*, y^*, z^*)$ is an M-Pareto optimal solution to the (BP) problem. 

Since the decision makers subjectively specify membership functions and the minimal acceptable levels, the balance among the both degree of satisfaction between the leader and the followers should be considered. Therefore we must consider the overall satisfactory balance between both levels.
To take account of the overall satisfactory balance between the leader and the followers, the leader needs to compromise with each follower on his/her own minimal satisfactory level. So the following ratio of satisfactory degree of the leader and each of the followers is used:

$$\delta_j = \frac{\mu(F_j(x, y, z))}{\mu(F_0(x, y, z))}, \quad j = 1, 2, \ldots, M$$  \hspace{1cm} (11)

For convenience, we introduce the following definition of autonomy level [8]:

$$\delta_L = \min_{j=1, \ldots, M} \frac{\mu(F_j(x, y, z))}{\mu(F_0(x, y, z))}.$$  \hspace{1cm} (12)

and the maximal ratio of satisfactory degree between the leader and each follower:

$$\delta_U = \max_{j=1, \ldots, M} \frac{\mu(F_j(x, y, z))}{\mu(F_0(x, y, z))}.$$  \hspace{1cm} (13)

Let $\Delta_L$ and $\Delta_U$, specified by the leader, denote the lower bound and the upper bound of $\delta_j (j = 1, 2, \ldots, M)$. Unless all $\delta_j (j = 1, 2, \ldots, M)$ are in the interval $[\Delta_L, \Delta_U]$, the leader or the followers need to update his/her or their minimal satisfactory level to reinforce their decision making power in accordance with the following procedure:

**Procedure for updating the minimal satisfactory level**

1. If $\delta_L > \Delta_U$, the leader needs to increase his/her satisfactory level $\alpha_0$ to reinforce his/her decision making power.

2. If $\delta_U < \Delta_L$, the leader needs to reduce his/her satisfactory level $\alpha_0$ to reinforce the followers’ decision making power.

3. Otherwise, the $i^{th}$ follower increases his/her minimal satisfactory level, while the $j^{th}$ follower decrease his/her minimal satisfactory level, where $i \in W = \{i|\delta_i < \Delta_L\}, \ j \in V = \{j|\delta_j > \Delta_U\}$.

We are now ready to construct a fuzzy interactive decision making approach for deriving the satisfactory solution for the leader and the followers from the problem (BP), which is summarized as follows:

**Interactive Fuzzy Decision Making Approach**

**Step 1.** The leader elicits the membership function $\mu(F_0(x, y, z))$ of the fuzzy goal of the leader and specifies $\Delta_L$ and $\Delta_U$, then the followers elicit the membership functions $\mu(F_j(x, y, z))$ of the followers, $j = 1, \ldots, M$. And set $\epsilon$ specified to measure whether the followers satisfy the solution to the problem (P).

**Step 2.** After the leader sets the initial minimal acceptable level $\alpha_0$ for his/her membership function $\mu(F_0(x, y, z))$, the followers set the initial reference membership value $\alpha_j$, $j = 1, \ldots, M$. 

Step 3. Solve the problem (P). If there does not exist a solution to the problem (P), the leader adjusts the minimal acceptable level $\alpha_0$ by reducing $\alpha_0$, until a solution $(\hat{x}, \hat{y}, \hat{z}, \hat{t})$ is obtained for the problem (P).

Step 4. If the followers satisfy the solution $(\hat{x}, \hat{y}, \hat{z})$, then goto Step 6. Otherwise, goto the next step.

Step 5. If $t < 0$, increase $\alpha_j$, $j = 1, \cdots, M$. If $t > 0$, then ask the leader to update the current minimal acceptable level $\alpha_0$. Goto Step 3.

Step 6. Calculate the individual ratio of satisfactory degree $\delta_j$, $j = 1, 2, \cdots, M$, and $\delta_L$ and $\delta_U$ are computed. If $\delta_L, \delta_U \in [\Delta_L, \Delta_u]$, namely $\delta_j \in [\Delta_L, \Delta_u], j = 1, 2, \cdots, M$, then stop. We already obtain an M-Pareto optimal solution $(x^*, y^*, z^*) = (\hat{x}, \hat{y}, \hat{z})$, which is the satisfying solution for the leader and the followers. Otherwise, update the minimal acceptable level $\alpha_j (j = 1, 2, \cdots, M)$ according to the procedure of updating minimal satisfactory level and then goto Step 3.

4 Numerical example

To demonstrate the feasibility and efficiency of the proposed fuzzy interactive decision making algorithm, we consider the following linear bilevel program:

\[ \min_{x, y, z} \begin{aligned} f_0(x, y, z) &= 18x_1 - 10x_2 - 18y_{11} + 11y_{12} - 8y_{21} - 23y_{22} + 32z_1 - 40z_2 \\ f_1(x, y, z) &= 17x_1 - 40x_2 + 45y_{11} - 31y_{12} + 31y_{21} - 49y_{22} + 4z_1 + 15z_2 \\ f_2(x, y, z) &= 19x_1 - 8x_2 + 30y_{11} - 3y_{12} - 40y_{21} + 40y_{22} + z_1 - 36z_2 \end{aligned} \]

s.t.

\[ \begin{align*} -9x_1 - 18x_2 + 12y_{11} + 13y_{12} - 13y_{21} + 37y_{22} + 19z_1 - 11z_2 &\leq 18 \\ 47x_1 - 14x_2 - y_{11} + 4y_{12} + 21y_{21} + y_{22} + 16z_1 - 49z_2 &\leq 12 \\ -23x_1 + 2x_2 + 45y_{11} - 35y_{12} - 17y_{21} + 12y_{22} + 13z_1 + 41z_2 &\leq 22 \\ 6x_1 - 19x_2 - y_{11} - 2y_{12} + 3y_{21} - 49y_{22} + 31z_1 - 11z_2 &\leq -23 \\ -31x_1 - 8x_2 + 2y_{11} + 17y_{12} + 5y_{21} + 47y_{22} - 2z_1 - 25z_2 &\leq 5 \\ 46x_1 + 3x_2 - 28y_{11} + 17y_{12} + 41y_{21} - 36y_{22} + 2z_1 - 3z_2 &\leq 24 \\ -45x_1 + 34x_2 - 44y_{11} + 44y_{12} + 42y_{21} + 16y_{22} + 39z_1 - 2z_2 &\leq 50 \\ 29x_1 - 13x_2 + 38y_{11} + 19y_{12} - 10y_{21} - 2y_{22} - 29z_1 + 7z_2 &\leq 20 \\ 13x_1 + 10x_2 + 27y_{11} - 29y_{12} - 12y_{21} - 49y_{22} + 23z_1 - 38z_2 &\leq -33 \end{align*} \]

\[ x = (x_1, x_2)^T \geq 0, \quad y = (y_{11}, y_{12}, y_{21}, y_{22})^T \geq 0, \quad z = (z_1, z_2)^T \geq 0. \]

Iteration 1.

Step 1. The membership functions (6) of the fuzzy goal are assessed by using (7) and (8). The optimal values and the corresponding optimal
solutions are shown in table 1. And then specify $\Delta_L = 0.75$ and $\Delta_U = 0.85$ and set $\epsilon = 0.05$.

Table 1. The initial computation results

<table>
<thead>
<tr>
<th>$(x, y, z)$</th>
<th>$F_0^*$</th>
<th>$F_1^*$</th>
<th>$F_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(x, y, z)$</td>
<td>1.995713</td>
<td>2.971589</td>
<td>0.000000</td>
</tr>
<tr>
<td>$(x, y, z)$</td>
<td>0.5164339</td>
<td>0.592149</td>
<td>0.000000</td>
</tr>
<tr>
<td>$(x, y, z)$</td>
<td>2.060751</td>
<td>3.17676</td>
<td>0.000000</td>
</tr>
<tr>
<td>$(x, y, z)$</td>
<td>41.50743</td>
<td>0.322018</td>
<td>2.279073</td>
</tr>
<tr>
<td>$(x, y, z)$</td>
<td>76.68814</td>
<td>0.0529739</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Step 2. The leader sets the initial minimal acceptable level $\alpha_0 = 1.0$, while the followers set the initial reference membership value $\alpha_1 = 0.4$ and $\alpha_2 = 0.4$, respectively.

Step 3. The problem (P) for the numerical example can be formulated as

$$\begin{align*}
(P) \quad & \min t \\
\text{s.t.} \quad & \frac{2.377733 - F_0}{2.377733 - 82.25461} \geq \alpha_0 \\
& \frac{41.50743 - F_1}{41.50743 + 190.6531} \leq t \\
& \frac{76.68814 - F_2}{76.68814 + 61.39379} \leq t \\
& (x, y, z) \in S, \ |t| \leq 1.
\end{align*}$$

where $S$ denotes the constraint region of the problem. There does not exist a feasible solution for the problem (14) with $\alpha_0 = 1.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.4$. So the leader adjusts the minimal acceptable level $\alpha_1$ by reducing $\alpha_0 = 1.0$ to $\alpha_0 = 0.9$. The problem (14) is solved with $\alpha_0 = 0.9$, $\alpha_1 = 0.4$, $\alpha_2 = 0.4$ and the optimal solution is $(\hat{x}, \hat{y}, \hat{z}) = (1.168495, 1.394113, 0.0000, 0.2515213, 0.5112497, 1.552246, 0.0000, 1.096456)$. The results are listed in table 2.

Table 2. The first iteration

<table>
<thead>
<tr>
<th>$t$</th>
<th>$-0.5513538E - 01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$</td>
<td>73.7914</td>
</tr>
<tr>
<td>$F_1$</td>
<td>87.4617</td>
</tr>
<tr>
<td>$F_2$</td>
<td>12.4614</td>
</tr>
</tbody>
</table>

Table 3. The second iteration

<table>
<thead>
<tr>
<th>$t$</th>
<th>$-0.1107760E - 01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$</td>
<td>73.7914</td>
</tr>
<tr>
<td>$F_1$</td>
<td>88.7526</td>
</tr>
<tr>
<td>$F_2$</td>
<td>13.0216</td>
</tr>
</tbody>
</table>

Step 4. $|t| = 0.5513538E - 01 > 0.05 = \epsilon$, the followers do not satisfy the solution, then goto step 5.

Step 5. $t = -0.6513538E - 01 < 0$, then increase $\alpha_1 = 0.4$ to $\alpha_1 = 0.55$ and $\alpha_2 = 0.4$ to $\alpha_2 = 0.45$. Goto step 3.

Iteration 2
Step 3. The problem (14) is solved with $\alpha_0 = 0.9, \alpha_1 = 0.55, \alpha_2 = 0.45$ and the optimal solution is $(\hat{x}, \hat{y}, \hat{z}) = (1.178220, 1.419919, 0.0000, 0.2500594, 0.5012161, 1.554870, 0.0000, 1.094477)$. The results are listed in table 3.

Step 4. $|\hat{t}| = 0.1107760E - 01 < 0.05$, then goto step 6.

Step 6. Calculate $\delta_j(j = 1, 2), \delta_1 = 0.6234, \delta_2 = 0.5123$, so $\delta_U = \max\{\delta_1, \delta_2\} = 0.6234 < 0.75 = \Delta_L$. Therefore, decrease $\alpha_0 = 0.9$ to $\alpha_0 = 0.8$. Goto step 3.

**Iteration 3**

step 3. The problem (14) is solved with $\alpha_0 = 0.8, \alpha_1 = 0.55, \alpha_2 = 0.45$ and the optimal solution is $(\hat{x}, \hat{y}, \hat{z}) = (1.108843, 1.853427, 0.0000, 0.3876602, 0.1019916, 1.110834, 0.0000, 1.116305)$. The results are listed in table 4.

<table>
<thead>
<tr>
<th>Table 4. The third iteration</th>
<th>Table 5. The fourth iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$-0.6740129E - 01$</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$-65.3281$</td>
</tr>
<tr>
<td>$F_1$</td>
<td>$-101.8288$</td>
</tr>
<tr>
<td>$F_2$</td>
<td>$5.2443$</td>
</tr>
</tbody>
</table>

Step 4. $|\hat{t}| = 0.6740129E - 01 > 0.05$, the followers do not satisfy the solution, then goto step 5.

Step 5. $\hat{t} = -0.6740129E - 01 < 0.05$, so increase $\alpha_1 = 0.55$ to $\alpha_1 = 0.6$ and $\alpha_2 = 0.45$ to $\alpha_2 = 0.5$. Goto step 3.

**Iteration 4**

Step 3. The problem (14) is solved with $\alpha_0 = 0.8, \alpha_1 = 0.6, \alpha_2 = 0.5$ and the optimal solution is $(\hat{x}, \hat{y}, \hat{z}) = (1.100382, 1.874932, 0.0000, 0.1848270, 0.2518467, 1.188325, 0.0000, 0.9768137)$. The results are listed in table 5.

Step 4. $|\hat{t}| = 0$, then goto step 6.

Step 6. Calculate $\delta_j(j = 1, 2), \delta_1 = 0.75, \delta_2 = 0.6250$, so $\delta_2 < \delta_L = 0.75$, therefore, increase $\alpha_2 = 0.5$ to $\alpha_2 = 0.55$. Goto step 3.

**Iteration 5**

Step 3. The problem (14) is solved with $\alpha_0 = 0.8, \alpha_1 = 0.6, \alpha_2 = 0.55$ and the optimal solution is $(\hat{x}, \hat{y}, \hat{z}) = (1.058288, 1.719276, 0.0000, 0.3952597, 0.1541502, 1.097191, 0.0000, 1.126596)$. The results are listed in table 6.

<table>
<thead>
<tr>
<th>Table 6. The fifth iteration</th>
<th>Table 7. The sixth iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$0.1150473E - 01$</td>
</tr>
<tr>
<td>$F_0$</td>
<td>$-65.3282$</td>
</tr>
<tr>
<td>$F_1$</td>
<td>$-95.1180$</td>
</tr>
<tr>
<td>$F_2$</td>
<td>$2.3317$</td>
</tr>
</tbody>
</table>

Step 4. $t = 0.1150473E - 01 < 0.05$, then goto step 6.

Step 6. Calculate $\delta_j(j = 1, 2), \delta_1 = 0.7356, \delta_2 = 0.6731$, so $\delta_U = \max\{\delta_1, \delta_2\} = 0.7356 < \Delta_L = 0.75$. Therefore, decrease $\alpha_0 = 0.8$ to $\alpha_0 = 0.7$. Then goto step 3.

**Iteration 6**

Step 3. The problem (14) is solved with $\alpha_0 = 0.7, \alpha_1 = 0.6, \alpha_2 = 0.55$ and the optimal solution is $(\hat{x}, \hat{y}, \hat{z}) = (0.9750742, 2.187953, 0.0000, 0.2011452)$. The results are listed in table 7.
0.0000 0.7798844 0.0000 0.9202992). The results are listed in table 7.

Step 4. $|t| = 0.1636073E - 01 < 0.05$, then goto step 6.

Step 6. Calculate $\delta_j(j = 1, 2)$, $\delta_1 = 0.8806, \delta_2 = 0.8091$, so $\delta_1 > \delta_U = 0.85$, therefore, decrease $\alpha_1 = 0.6$ to $\alpha_1 = 0.55$. Then goto step 3.

Iteration 7.

Step 3. The problem (14) is solved with $\alpha_0 = 0.7, \alpha_1 = 0.55, \alpha_2 = 0.55$ and the optimal solution is $(\hat{x}, \hat{y}, \hat{z}) = (0.9274641, 2.046317, 0.0000, 0.2793429, 0.0000, 0.7392696, 0.0000, 0.9791416)$. The results are listed in table 8.

Table 8. The seventh iteration

<table>
<thead>
<tr>
<th>$t$</th>
<th>$-0.4351141E - 01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$</td>
<td>$-56.8649$</td>
</tr>
<tr>
<td>$F_1$</td>
<td>$-96.2825$</td>
</tr>
<tr>
<td>$F_2$</td>
<td>$-5.2651$</td>
</tr>
</tbody>
</table>

Step 4. $|t| = 0.4351141E - 01 < 0.05$, then goto step 6.

Step 6. Calculate $\delta_j(j = 1, 2)$, $\delta_1 = 0.8479 \in [\Delta_L, \Delta_U]$, then stop. Thus, the M-Pareto optimal solution is $(x^*, y^*, z^*) = (0.9274641, 2.046317, 0.0000, 0.2793429, 0.0000, 0.7392696, 0.0000, 0.9791416)$. And all satisfied degree are $\mu(F_0) = 0.7$, $\mu(F_1) = \mu(F_2) = 0.5935$.

From these results above, we can see that the leader’s membership function value is 0.7, the followers’ membership function values are all 0.5935 and the satisfactory degree between the leader and each follower is 0.8479, so our algorithm considers not only the leader’s domination but also the satisfactory degree between the leader and the followers, thus the proposed algorithm is feasible and efficient.

5 Conclusions

In this paper, we have constructed a fuzzy interactive approach for solving the decentralized bilevel programming problem with a common decision variable in the upper level and lower level problem. In our interactive approach, by updating the minimal acceptable level of the leader and the followers, an overall satisfactory solution has been efficiently derived with considerations to overall satisfactory balance between the two levels. And the illustrative numerical example has been provided to demonstrate the feasibility and efficiency of the proposed algorithm.

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7 References


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