Compressive Spectrum Sensing Augmented by Geo-location Database

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Abstract—In cognitive radio (CR), white space devices (WSDs) need to have the knowledge of spectrum occupancy in TV white space (TVWS) before dynamic access. There are two common schemes proposed to achieve this: 1) geo-location database and 2) spectrum sensing. In geo-location database, calculating digital terrestrial television (DTT) location probability and maximum permitted power in each channel in an efficient way becomes important as the database is supposed to give a quick response once a request comes. Spectrum sensing is a scheme which can provide a more reliable and real-time results for spectrum occupancy. However, the high sampling rate is a big challenge in spectrum sensing for power limited WSDs. In this paper, we proposed to combine the location probability based geo-location database with compressive sensing (CS) based spectrum sensing to achieve sub-Nyquist sampling rates for WSDs. The history data from geo-location database is utilized to support the signal recovery for the spectrum sensing. In addition, a new method to calculate DTT location probability efficiently is proposed. Theoretical analysis of the proposed algorithm are tested in TVWS and it shows that performance of the proposed algorithm outperforms the traditional algorithm.

I. INTRODUCTION

Cognitive Radio (CR) is being viewed as a new intelligent wireless communication technology to solve the inefficiency of the fixed spectrum assignment policy [1]. TV white space (TVWS) refers to TV channels that are not used by any licensed services at a certain location and a particular time due to the digital-switch-over. To successfully implement CR in TVWS, there are mainly two goals to be achieved: 1) to protect incumbent licensed users from harmful interference, 2) to utilize the available spectrum efficiently [2].

In order to avoid any harmful interference to primary services in TVWS, white space devices (WSDs) should have the knowledge of spectrum occupancy status. Two schemes have been proposed to make WSDs aware of the spectrum occupancy. One is geo-location database which is based on a central database to store the information such as Digital Video Broadcasting-Terrestrial (DVB-T) protected areas, specifications of DVB-T transmitters, advanced propagation models and protection rules that can be used to compute the maximum transmit power for each vacant transmission channel in a specic location and time [1], [2]. With a database, part of the complexity associated with sensing and maximum power computation is transferred to the core network, decreasing complexity and power demand of WSDs. However, a database approach can only protect registered systems. Public Making and Special Events (PMSE) devices operate mostly on an unlicensed basis, without any record, which may pose significant challenges to a geo-location database. Therefore, the way to protect unregistered PMSE applications, wireless microphone (WM) or any other use, is through sensing. It requires WSDs to have the capability to detect spectrum holes that are not occupied by primary users (PUs). This approach can provide the instant channel occupancy information but may cause interference to some reserved channels if they are determined as vacant and SU gets access to them. Therefore, it is more efficient and accurate if results from the the two schemes can be combined to obtain the channel occupancy status. So far, such a hybrid scheme has been implemented into an experimental platform that combines wireless microphone sensors with a web-based geo-location database access for PMSE to show its efficiency in [3].

For geo-location databases, power control model can be used to calculate the maximum permitted power for WSDs. It makes use of a two-ray path loss model to measure power attenuation. However, there are two main problems in this model. Firstly, the digital terrestrial television (DTT) receivers cannot be located precisely. All known information is the number of households located somewhere within a 100m × 100m pixel [4]. This location uncertainty means that the path loss cannot be calculated by the actual distance between WSDs and DTT receivers. Secondly, different environmental scenarios lead to different tolerance levels for DTT receivers. The power attenuation in urban areas is much higher than open areas. As a result, the maximum allowable equivalent isotropic radiated power (EIRP) of a specific WSD in open areas is higher than that in urban areas.

To solve these problems, the location probability model introduces the concept of location uncertainty, which classifies the location relationship between WSDs and DTT receivers into four different scenarios. In addition, transmission environment can be classified as open, suburban and urban areas. Power attenuation is measured by coupling gain, depending on both location relationship and transmission environment. Two prevailing algorithms for calculating DTT location probability are Monte Carlo simulation and Schwartz-Yeh’s method. However, these two algorithms are either unstable or too complex for the DTT location probability calculation. Therefore, we propose an efficient algorithm to perform the calculation of location probability by utilizing Wilkinson’s method, which is much more computationally efficient.

In spectrum sensing, sampling rates are difficult to achieve as many WSDs are power-limited with weak sensing capabilities. Compressive sensing (CS) was proposed to make it
where the received power of wanted DTT signal is labelled as \( P \). The average power on the edge area of a DTT base station, which receives the location database. It is assumed that a DTT reception is located A. Location probability model

B. System model of compressive spectrum sensing

In the CS based spectrum sensing mode, it is assumed that the bandwidth of the whole spectrum is \( B \) Hz. The received signal in a WSD can be expressed as:

\[
r(t) = h(t) \ast s(t) + n(t),
\]

at the point where DTT receiver fails. \( \Delta f = f_{WSD} - f_{DTT} \), where \( f_{WSD} \) is the frequency in which a WSD device operates and \( f_{DTT} \) is the DTT carrier frequency. The mean value of received power of wanted DTT signal \( P_s \) is labelled as \( m_S \).

The DTT receiver’s location probability in the absence of interference from systems other than DTT is labelled as \( q_1 \), \( q_2 \) is the DTT receiver’s location probability when considering the WSD interferer power, which causing a reduction in location probability \( \Delta q \). To identify the maximum allowable EIRP \( P_{IB} \), and \( \Delta q \) is set to be a maximum value \( \Delta q_{TT} \). There are two prevailing algorithms to calculate the DTT location probability described as follows, namely Monte Carlo simulation and Schwartz-Yeh’s method.

1) Monte Carlo simulation: Monte Carlo simulation makes use of statistical principles to generate a series of random points of \( P_s \) and \( V \) \( V = \sum_{k=1}^{K} r_{U,k} P_{U,k} \) respectively, which follows normal distribution. The number of points that satisfies \( P_s \geq V + P_{s,\min} \) is recorded. DTT location probability \( q_1 \) can be calculated by dividing the total number of available points. The method to achieve DTT location probability \( q_2 \) which includes WSD interference power follows the same logic.

2) Schwartz-Yeh’s method: Schwartz-Yeh’s method is an approximate algorithm to calculate the mean and standard deviation values of log-normal distribution variables \( \delta \). We can express (1) in decibel domain as follows \([9]\):

\[
q_1 = \Pr \left\{ P_s \geq P_{s,\min} + \sum_{k=1}^{K} r_{U,k} P_{U,k} \right\}
= \Pr \left\{ P_s \geq P_{s,\min} + V \right\}
= \Pr \left\{ 1 \geq \frac{P_{s,\min}}{P_s} + \frac{V}{P_s} \right\},
\]

\[
q_2 = \Pr \left\{ P_s \geq P_{s,\min} + \sum_{k=1}^{K} r_{U,k} P_{U,k} + r \left( \Delta f, m_S \right) P_{IB} \right\}
= \Pr \left\{ 1 \geq \frac{P_{s,\min}}{P_s} + \frac{V + r \left( \Delta f, m_S \right) P_{IB}}{P_s} \right\}
= \Pr \left\{ 1 \geq A + \frac{V + C}{P_s} \right\}
= \Pr \left\{ 1 \geq A + \frac{E}{P_s} \right\}
= \Pr \left\{ 1 \geq F \right\}
= \Pr \left\{ 0 \geq Y_{(dB)} \right\}
\]

where \( m_X \) and \( \sigma_X \) refer to the mean and standard deviation value of \( X \).

Similarly, \( q_2 \) can be expressed in decibel domain as follows \([9]\):

\[
q_1 = \Pr \left\{ P_s \geq P_{s,\min} + \sum_{k=1}^{K} r_{U,k} P_{U,k} \right\}
= \Pr \left\{ P_s \geq P_{s,\min} + V \right\}
= \Pr \left\{ 1 \geq A + B \right\}
= \Pr \left\{ 1 \geq X \right\}
= \Pr \left\{ 0 \geq X_{(dB)} \right\}
\]

where \( m_Y \) and \( \sigma_Y \) refer to the mean and standard deviation value of \( Y \).

B. System model of compressive spectrum sensing

In the CS based spectrum sensing mode, it is assumed that the bandwidth of the whole spectrum is \( B \) Hz. The received signal in a WSD can be expressed as:

\[
r(t) = h(t) \ast s(t) + n(t),
\]
where \( s(t) \in C_{N}^{N \times 1} \) is the time domain representation of the transmitted signal, and \( h(t) \) is the channel gain between transmitter and receiver, and \( n(t) \sim CN(0, \sigma^2 I_p) \) refers to additive white Gaussian noise (AWGN) in the transmission channels.

As a large percentage of the spectrum is underutilized, it exhibits the sparse property in frequency domain, which makes it possible to reduce sampling rates by implementing CS theory in WSDs. After CS theory is applied, a WSD would only collect compressed measurements by implementing a measurement matrix:

\[
x = \Phi (r(t) + n(t)) = \Phi F^{-1} (r_f + n_f) = \Theta r_f + \Theta n_f, \tag{6}
\]

where \( \Phi \in C_{P \times N}^P \) \((P < N)\) is a Gaussian distributed measurement matrix utilized to collect the compressed measurements \( x \in C_{P \times 1}^P \), and \( \Theta n_f = \Phi n(t) \sim CN(0, \sigma^2 I_p) \) where \( \sigma^2_n = \frac{\lambda}{N} \sigma^2 \). There are three conditions for \( \Theta \): (1) each column of \( \Theta \) is normalized, (2) each row of it has approximately equal norm, and (3) the rows of \( \Theta \) are orthogonal [10]. In addition, \( \Theta = \Phi F^{-1} \), where \( F^{-1} \) is inverse discrete Fourier transform (IDFT) matrix.

When CS theory is performed in a WSD, the sampling rates would be reduced. However, in order to make accurate decisions about spectrum occupancy, the original signals \( r_f \) should be reconstructed firstly. Signal recovery process can be formulated as a convex optimization problem and solved as [11]:

\[
\min \| \tilde{r}_f \|_1 \text{subject to } \| \Theta : \tilde{r}_f - x \|_2^2 \leq \sigma_n^2. \tag{7}
\]

When the reconstructed signal \( \tilde{r}_f \) is obtained by solving the above convex optimization problem, energy detection is performed to determine the spectrum occupancy. In the energy detection scheme, energy of the recovered signals is compared with a predefined threshold. If it is higher than threshold, the corresponding channel is determined as occupied by PUs, and WSDs are forbidden to access it. Otherwise, the corresponding channel is determined as vacant. As a result, WSDs can access it to transmit the unlicensed signals.

III. THE PROPOSED COMPRESSIVE SPECTRUM SENSING AUGMENTED BY GEO-LOCATION DATABASE

As aforementioned, in order to improve the CS based spectrum sensing results, we propose to utilize the history information in geo-location database as the prior information to construct the weights for signal recovery in the reweighted compressive spectrum sensing. In addition, the Wilkinson’s method is proposed to calculate the maximum permitted EIRP \( P_{1B} \) of each channel in TVWS efficiently for geo-location database. The whole procedure of the proposed algorithm can be illustrated in Fig. 2.

A. The proposed Wilkinson’s method based DTT location probability calculation

As aforementioned, it is noticed that both Monte Carlo and Schwartz-Yeh’s method have drawbacks for DTT location probability calculation. For Monte Carlo’s method, the brute-force simulation is not an ideal solution to achieve an exact result for DTT location probability calculation. For Schwartz-Yeh’s method, infinite loops are used in the process of calculating the mean and standard deviation values of log-normal distribution variables. As a result, the large computational complexity and low efficiency of the two methods are difficult to overcome in a low power WSD. In this paper, we propose to use Wilkinson’s method to calculate the mean and standard deviation values of log-normal distribution variables for DTT location probability calculation in a much simpler way. It can be described as follows [12].

Assuming \( M \) interference signals \( I_k \) \((k = 1, 2, \ldots, M)\) arrive at the receiver, it can be changed to decibel domain as \( X_k = 10 \log_{10} I_k \) and \( X = 10 \log_{10} \left( \sum_{k=1}^{K} 10^{X_k/10} \right) \). It is assumed that \( e^{Y_1} + e^{Y_2} + \cdots + e^{Y_M} = e^Z = 10^X \) and \( Z = \lambda X \), \( \lambda = \frac{1}{10} \ln 10 = 0.2302 \), the mean and standard deviation of parameter \( X \) could be calculated by bringing in two parameters \( \mu_1 \) and \( \mu_2 \).

\[
\mu_1 = \exp \left( m_Z + \frac{1}{2} \sigma_Z^2 \right) = \sum_{i=1}^{N} \exp \left( m_{Y_i} + \frac{1}{2} \sigma_{Y_i}^2 \right), \tag{8}
\]

\[
\mu_2 = \exp \left( 2 m_Z + 2 \sigma_Z^2 \right) = \sum_{i=1}^{N} \exp \left( m_{Y_i} + \frac{1}{2} \sigma_{Y_i}^2 \right) + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} \exp \left( m_{Y_i} + m_{Y_j} \right) \times \exp \left[ \frac{1}{2} \left( \sigma_{Y_i}^2 + \sigma_{Y_j}^2 + 2 r_{ij} \sigma_{Y_i} \sigma_{Y_j} \right) \right], \tag{9}
\]

\[
m_X = \left( \frac{1}{\lambda} \right) \left( 2 \ln \mu_1 - \frac{1}{2} \ln \mu_2 \right), \tag{10}
\]

\[
\sigma_X = \left( \frac{1}{\lambda} \right) \left( 2 \ln \mu_2 - \frac{1}{2} \ln \mu_1 \right). \tag{11}
\]

In (3), \( 0 \geq \frac{P_{\text{min}}}{P_s} + \frac{V}{P_r} = A + B \), and then \( 0 \geq 10 \log_{10} \left( 10^{A_{dBm}} + 10^{B_{dBm}} \right) \). Furthermore, it can be fitted into the precondition of Wilkinson’s method to get \( 10^{A_{dBm}} + 10^{B_{dBm}} = 10^X = e^{Y_1} + e^{Y_2} = e^Z \). Then the relevant correlation coefficient of \( A \) and \( B \) can be given as:

\[
r_{A,B} = \frac{\text{cov}(A_{dBm}, B_{dBm})}{\sqrt{\text{var}(A_{dBm}) \text{ var}(B_{dBm})}} = \frac{\sigma_S}{\sqrt{\sigma_S^2 + \sigma_Y^2}}. \tag{12}
\]
Once the DTT location probability in geo-location database is calculated efficiently. In addition, we propose to utilize the history information of allowed EIRP in each vacant channel from the geo-location database as prior information for the signal recovery process in a reweighted compressive spectrum sensing algorithm.

In fact, when the permitted EIRP is available, we propose to introduce a set of weight parameters \( W = \sum_{n=1}^{N} w_n \) to improve the signal recovery accuracy, where \( w_n = \frac{1}{\varepsilon + |\pi|} \). In addition, \( p_i = \frac{1}{T} \left( \sum_{t=0}^{T} p_i(t) \right) \) is the average power of the recorded permitted EIRP that can be used for WSDs in channel \( i \), and \( w_n \) is the weight for the bins locating in channel \( i \). \( \varepsilon \) is a positive constant used to make sure no error would happened in case of \( p_i = 0 \). Therefore, the reconstructed signal can be obtained by solving the following problem:

\[
\begin{align*}
\min \ |W \cdot \hat{r}_f|_1 \\
\text{subject to } \| \Theta \cdot \hat{r}_f - x \|_2^2 & \leq \sigma_n^2.
\end{align*}
\]

After the signal is recovered, the normal process for energy detection as aforementioned would be performed to get a more realizable decision about the spectrum occupancy.

IV. Numerical Results

A. Validation of the proposed Wilkinson’s method

Given the values \( m_V = -60 \text{dBm} \), \( \sigma_V = 5.0 \text{dB} \), the DTT location probability \( q_1 \) and the maximum allowable EIRP \( P_{1B} \) calculated by Monte Carlo method, Schwartz-Yeh’s method and Wilkinson’s method are shown in Fig. 4 and Fig. 5. From these two figures, we can see that both the \( q_1 \) and \( P_{1B} \) calculated by Wilkinson’s method are more closer to that of Monte Carlo simulation when compared with Schwartz-Yeh’s method.

Since Monte Carlo simulation is based on no assumption and approximation, the results could be precise as long as the number of trials is large enough. With 10,000 points, Monte Carlo simulation shows a relatively stable performance. Taken these values as a standard, the accuracy of the Schwartz-Yeh’s method and Wilkinson’s method could be measured by error rate \( \Delta X / X_{(Monte\ Carlo)} \). The error rates of \( q_1 \), \( q_2 \) and \( P_{1B} \) for the Schwartz-Yeh’s method and Wilkinson’s method are shown in Table I.

Table II shows the running time comparison for the three methods when calculating \( q_1 \), \( q_2 \) and \( P_{1B} \). It shows Wilkinson’s method can reduce the running time significantly. Therefore, the proposed method is efficient in the calculation of \( q_1 \), \( q_2 \) and \( P_{1B} \).
As shown in the above table, there are 11 available channels totally at SP515065. In location probability model, the transmission environment is classified into three situations: open, suburban and urban. Coupling gain in different situations is treated differently, leading to different interference toleration levels of DTT receivers. It is obvious that the power attenuation in open areas is much lower than suburban and urban areas. As a result, the actual maximum allowable EIRP in open areas is lower than the other two situations for a certain NGR geo-location. Take channel 51 as an example, the $P_{1B}$ is 0.0002 Watts in power control model, while the power could be utilized more effectively if the environment is classified into different scenarios in location probability model. It is 0.3981 Watts in open areas, 1.2589 Watts in suburban areas and 4.0000 Watts in urban areas.

C. Proposed compressive spectrum sensing algorithm

In the simulation part, the proposed compressive spectrum sensing augmented by geo-location database is tested in TVWS at NGR location SP515065 in Oxford. The $P_{1B}$ for each available channel shown in Table III is used as the prior information for the proposed algorithm. The transmission channel for signals is modeled as AWGN channel and $SNR$ is defined as the ratio of received signal power and noise power in each channel of TVWS.

The detection performance comparison of the proposed algorithm is shown in Fig. 6. Firstly, it is observed that the detection performance of spectrum sensing algorithm without CS implemented in a WSD is matched with the theoretical curves. In addition, the detection probability is also matched with theoretical values when 100% of samples are collected in a WSD for both the traditional compressive spectrum sensing algorithm and the proposed compressive spectrum sensing algorithm augmented by geo-location database. Furthermore, when the available number of samples is reduced to 5% in a WSD, we can see that the detection performance of the proposed hybrid scheme is higher than the traditional CS based spectrum sensing only scheme, which indicates that the proposed algorithm requires less number of samples to achieve the same detection performance as the CS based sensing only algorithm.

V. Conclusions

In this paper, we proposed a compressive spectrum sensing algorithm augmented by the location probability based geo-

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**Fig. 5:** Maximum radiated EIRP of white space device $P_{1B}$ with $m_{V} = -60$dBm, $\sigma_{V} = 5.0$dB.

**Fig. 6:** Probability of detection $P_{d}$ comparison under different SNR values.

<table>
<thead>
<tr>
<th>Available Channel</th>
<th>Open</th>
<th>Suburban</th>
<th>Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>28</td>
<td>0</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>29</td>
<td>0.0025</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>43</td>
<td>0</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>46</td>
<td>0</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>49</td>
<td>0.0013</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>51</td>
<td>0.3981</td>
<td>1.2589</td>
<td>4.0000</td>
</tr>
<tr>
<td>54</td>
<td>0.0013</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>58</td>
<td>0.0013</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
</tbody>
</table>

**TABLE III:** Comparison of actual maximum allowable EIRP $P_{1B}$ in Oxford.

<table>
<thead>
<tr>
<th>Power control model</th>
<th>Wilkinson’s method</th>
<th>Schwartz-Yeh’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{1}$</td>
<td>51.25%</td>
<td>4.76%</td>
</tr>
<tr>
<td>$q_{2}$</td>
<td>9.36%</td>
<td>1.25%</td>
</tr>
</tbody>
</table>

**TABLE I:** Error rate comparison.

<table>
<thead>
<tr>
<th>Power control model</th>
<th>Wilkinson’s method</th>
<th>Schwartz-Yeh’s method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{1}$</td>
<td>1 second</td>
<td>15 seconds</td>
</tr>
<tr>
<td>$q_{2}$</td>
<td>1 second</td>
<td>1 second</td>
</tr>
</tbody>
</table>

**TABLE II:** Running time comparison.
location database. For the geo-location database, a new semi-analytical solution was proposed by making use of Wilkinson’s method in order to calculate the location probability efficiently. In contrast with Schwartz-Yeh’s method, the computational complexity in DTT location probability calculation was largely reduced as well as maintaining the accuracy by Wilkinson’s method. Once the permitted EIRP of each vacant channel in TVWS is obtained, it is recorded and the average permitted EIRP of each channel would be calculated and utilized as the prior information for signal recovery in the proposed compressive spectrum sensing to improve the detection performance. The simulation results showed that the proposed algorithm had a better performance than the conventional algorithm.

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REFERENCES