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A Fast Linear Programming Algorithm for Blind Equalization

Zhi Ding and Zhi-Quan Luo

Abstract—A fast implementation of a special non-MSE cost function for blind equalization is presented here. This baud-rate equalization algorithm is based on a convex cost function coupled with a simple linear constraint on the equalizer parameters. For a generic class of channels with persistently exciting quadrature amplitude modulation input signals, this new algorithm allows the convergence of equalizer parameters to a unique global minimum achieving intersymbol interference suppression and carrier phase recovery.

Index Terms—Blind deconvolution, convexity, equalization, global convergence, intersymbol interference, linear programming.

1. INTRODUCTION

BLIND equalization of an unknown, nonminimum phase channel is an important problem in data communication systems. A blind equalizer achieves parameter adaptation based on the observed channel output and a priori knowledge of some distributional or statistical properties of the input sequence [1], [2], [4]. By eliminating the need for training signals, a blind receiver can begin its self-adaptation without disrupting the normal flow of data transmission. It can also recover from system failures resulted from disrupted connections that may have caused equalizers to lose its desired parameter settings.

Although fractionally spaced blind equalization algorithms have received much attention recently, baud-rate equalizers remain useful and practical for communication channels that either lack excess bandwidth or exhibit common zeros. In this paper, we focus on the problem of blind linear baud-rate equalization.

Although baud-rate equalizers such as the constant modulus algorithm (CMA) [2], [5] and the Shalvi-Weinstein algorithm [3] are not globally convergent for arbitrary initialization [9], [10], we have presented in [15] a linearly constrained blind equalizer that is globally convergent with a convex cost. This formulation does not attempt to directly identify the exact gain of the zero-forcing equalizer and focuses instead on generating an equalizer output that is a scaled version of the channel input. This approach of intersymbol interference (ISI) elimination without gain identification makes it straightforward to estimate independently the unknown scalar gain by, for example, matching the power of the channel input and equalizer output. Any additional constant complex phase ambiguity can also be readily resolved with differential encoding.

Unfortunately, the minimization of the $l_{\infty}$ cost in [13] and [15] was realized either via $l_{2}$ approximation or via iterative block data estimation. The resulting algorithms can be slow and tedious, making them undesirable for practical implementation. In this paper, we present a computationally efficient linear programming formulation to minimize the convex $l_{\infty}$ cost function. The new approach demonstrates a vast improvement of convergence speed, as evidenced by computer simulation examples. In addition, the linear programming approach requires substantially fewer data samples for training and much shorter equalizer length.

II. BASICS OF QUADRATURE AMPLITUDE MODULATION (QAM) BLIND EQUALIZATION

In a baseband representation of the QAM channel equalization system, the original symbol sequence $\{a_k\}$ takes values from a complex signal set $A$ (known as the QAM constellation) in a manner such that all possible finite symbol subsequences occur with nonzero probability. This persistent excitation assumption on the input is weaker than the typical and somewhat idealistic $i.i.d.$ assumption common in other works. The complex channel input $\{a_k\}$ is transmitted through a nonideal channel, assumed to be linear, causal, and (bounded-input bounded-output) stable. The channel transfer function can be written as

$$H(z^{-1}) = \sum_{i=0}^{\infty} h_i z^{-i}, \quad h_i \in \mathbb{C}$$

where $\mathbb{C}$ signifies the set of complex numbers and $\{h_i\}$ represents the channel impulse response sequence. When the channel in (1) is such that there is more than one nonzero element in $\{h_i\}$, the channel output

$$x_k = H(z^{-1}) a_k = \sum_{i=0}^{\infty} h_i a_{k-i}$$

suffers from ISI and the removal of this linear distortion is the task of channel equalizers.

A linear channel equalizer is a linear filter $\Theta(z^{-1})$ that is applied to the channel output $x_k$ in order to eliminate the ISI. Initially, we may take this filter as being stable and potentially noncausal (doubly infinite), and of the form

$$\Theta(z^{-1}) = \sum_{i=\infty}^{i=0} \theta_i(k) z^{-i}, \quad \theta_i \in \mathbb{C}$$

so as to deal with nonminimum phase channels. The time dependence of the equalizer parameters signifies that they are subject
to adaptation via an algorithm to be specified. By denoting the adaptive equalizer parameter vector as
\[ \theta(k) = [\cdots \theta_{-2}(k) \, \theta_{-1}(k) \, \theta_0(k) \, \theta_1(k) \, \theta_2(k) \, \cdots]^T \] (3)
and the equalizer input signal regressor as
\[ X_k = [\cdots x_{k+2} \, x_{k+1} \, x_k \, x_{k-1} \, x_{k-2} \, \cdots]^T \] (4)
the equalizer output can then be written as an inner product
\[ z_k = \sum_{i=-\infty}^{\infty} \theta_i(k) x_{k-i} = X'_k \theta(k). \] (5)
The ideal objective of any channel equalizer \( \Theta(z^{-1}) \) is to achieve distortionless reception
\[ z_k = \Theta(z^{-1}) x_k = a_{k-\nu}, \quad \nu \in \mathbb{Z}_+, \forall k \]
or simply
\[ H(z^{-1}) \Theta(z^{-1}) = z^{-\nu} \] (6)
where we can tolerate a fixed integer delay \( \nu \). (The strict form of this objective (6) will be modified subsequently.) Such an objective translates into the identification of channel inverse by the equalizer, i.e.,
\[ \Theta(z^{-1}) = z^{-\nu} h(z^{-1}) \]
where \( h(z^{-1}) \triangleq H^{-1}(z^{-1}) \) is understood to be the stable, (potentially) doubly infinite (noncausal) inverse of the channel \( H(z^{-1}) \). Note that the existence of \( h(z^{-1}) \) is only possible for channels without zeros on the unit circle. The goal of equalization could be achieved by adapting the equalizer parameters to minimize the mean square error (MSE) between the appropriately delayed channel input \( a_{k-\nu} \) and the equalizer output, when the original channel input signal is available to the receiver as a training sequence.

In blind equalization, the original sequence is unknown to the receiver except for its probabilistic or statistical properties over the known alphabet \( A \). Usually this signal constellation \( A \subset \mathbb{C} \) has symmetrical properties such that
\[ \exp(jm\pi/2)A = A, \quad m = 0, 1, 2, 3 \]
i.e., the constellation is 90° rotation invariant. If the statistics of the input data reflects this same symmetry (which is typical) over \( A \), then any \( m\pi/2 \) phase rotation does not cause any statistical changes in the channel output. Thus, the data recovered from blind equalization will exhibit an intrinsic phase ambiguity of \( m\pi/2 \) and the best possible equalization result would be
\[ H(z^{-1}) \Theta(z^{-1}) = e^{m\pi/2} z^{-\nu}, \quad m = 0, 1, 2, 3 \] (7)
which relaxes the objective (6). The remaining phase ambiguity can be resolved through differential encoding of the input data. A further relaxation of the ideal objective (6) is necessary since in practical applications only equalizers with a finite number of adjustable parameters can be implemented rather than those of (2). Typically, these finite parameters are arranged in the form of a causal transversal filter
\[ \Theta(z^{-1}) = z^{-N} \sum_{i=-N}^{N} \theta_i(k) z^{-i}. \] (8)
Since a finite impulse response equalizer can only approximate the desired impulse response (7), a quantizer should be used to recover the original channel input from the equalizer output \( z_k \). This is often regarded as the practical objective of blind equalization.

In designing our new equalizer based on linear programming, we do not attempt to identify the exact gain of channel inverse and focus instead on the elimination of ISI which is the primary objective of channel equalization [12]. Consequently, it is not essential to recover the exact (complex) gain of the channel inverse because once the ISI is removed such that the equalizer output is
\[ z_k = c a_{k-\nu} = |c|e^{j\phi_c} a_{k-\nu}, \quad c \neq 0 \]
then it is straightforward to estimate the unknown gain \( |c| \) by comparing the power of \( z_k \) with that of the \( a_k \). The constant phase ambiguity \( \phi_c \) can also be readily resolved by the utilization of differential encoding, especially if it is a multiple of \( \pi/2 \).

III. EQUALIZATION BASED ON LINEAR PROGRAMMING

We assume the QAM constellation is such that the real and imaginary components are independent and identical (e.g., the square type) with
\[ M \triangleq \max |\text{Re}\{a_k\}| = \max |\text{Im}\{a_k\}| \]
or can be transformed into such by a rotation. We can denote the total system (channel and equalizer) as
\[ T(z^{-1}) = H(z^{-1}) \Theta(z^{-1}) = \sum_{i=-\infty}^{\infty} t_i z^{-i} \]
where \( \{t_i\} \) is the impulse response of the combined system given by
\[ t_i = h_i(0) = \sum_{k=0}^{\infty} h_k \theta_{i-k} \]
with equalizer output \( z_k = \sum_{i=-\infty}^{\infty} t_i a_{k-i} \). The equalizer is taken to be doubly infinite and noncausal to permit complete removal of ISI. This assumption needs relaxation (as shown earlier) so that the equalizer remains in the truly implementable finite dimensional parameter space.

The cost function for minimization is chosen as
\[ J(0) \triangleq \max |\text{Re}\{z_k\}| + \max |\text{Im}\{z_k\}| \]
\[ = \max_{\{a_k\}} \left| \sum_{i} \text{Re}\{t_i\} \text{Re}\{a_{k-i}\} - \text{Im}\{t_i\} \text{Im}\{a_{k-i}\} \right| + \max_{\{a_k\}} \left| \sum_{i} \text{Re}\{t_i\} \text{Im}\{a_{k-i}\} + \text{Im}\{t_i\} \text{Re}\{a_{k-i}\} \right| \]
\[ = 2M \sum_{i=-\infty}^{\infty} \left| \text{Re}\{t_i\} + |\text{Im}\{t_i\}| \right|. \] (9)
The convexity of $J(\theta)$ with respect to $\theta$ can be seen as follows: both $\text{Re}\{t_i\}$ and $\text{Im}\{t_i\}$ are linear in 0; therefore $|\text{Re}\{t_i\}|$ and $|\text{Im}\{t_i\}|$ are convex functions of $\theta$. Since $J(\theta)$ is the sum of these convex functions, it is also convex. Without any constraint on $\theta$, the convex cost function achieves a trivial global minimum at $\theta = 0$, with equalizer output $z_k = 0$.

In order to make the cost function $J(\theta)$ useful, a linear constraint

$$\text{Re}\{\theta_0\} + \text{Im}\{\theta_0\} = 1 \quad (10)$$

was proposed in [15] on the equalizer parameter vector to avoid the all-zero solution without compromising the ability of the equalizer to remove ISI and carrier phase error. Due to the linearity of the constraint, the convexity of the cost function is maintained and global convergence is therefore assured.

We have shown [15] that, under this parameterization, simultaneous removal of $|S1|$ and carrier phase error globally minimizes the cost function. The main results are as follows.

**Theorem 3.1:** For a linear channel with a stable (noncausal) inverse impulse response sequence $\{h_k\}$ satisfying

$$h_i \cdot h_i = \delta(i)$$

doubly infinite noncausal equalizer with parameter setting

$$\theta_i = \frac{h_{i+m} e^{jk\pi/2}}{|\text{Re}\{h_m\}| + |\text{Im}\{h_m\}|} \quad (11)$$

minimizes the blind convex cost

$$\max_{\{a_k\}} |\text{Re}\{z_k\}| + \max_{\{a_k\}} |\text{Im}\{z_k\}|$$

where

$$m = \max_q \max_{\{a_k\}} \left\{|\text{Re}\{h_q\}| + |\text{Im}\{h_q\}|\right\}$$

and where $k \in \{1, 2, 3, 4\}$ is chosen such that

$$\text{Re}\{\theta_0\} + \text{Im}\{\theta_0\} = 1 \quad (12)$$

**Remarks and Comments:** Before proceeding onto implementation issues related to developing an algorithm based on the above convex function, we first examine the theoretical limitations and other considerations of the above scheme.

1) The linear constraint can be changed to almost any equation involving the real and the imaginary parts of $\delta$ [16].

2) Convexity is an essential property in the above formulation because it guarantees that when we use causal finite-dimensional practical equalizers, the global convergence tendencies of the algorithm, or unimodality, are preserved. This behavior is a consequence of convexity since truncation (setting many taps zero) is a form of linear constraint which does not destroy convexity. Convexity also ensures that one can approximate arbitrarily closely the performance of the ideal nonimplementable doubly infinite noncausal equalizer parameter setting.

3) The minimum (11) not only achieves zero $|S1|$ but also results in near-ideal phase recovery, i.e., except for an ambiguity of $\pi/2$ in the phase, the output of the equalizer is equal to the input since the channel/equalizer convolution is given by

$$h_i \otimes \theta_i(m, \phi^*) = \delta(i + m) \frac{e^{jk\pi/2}}{|\text{Re}\{h_m\}| + |\text{Im}\{h_m\}|}$$

Hence, our formulation achieves simultaneous channel equalization and carrier phase recovery.

**Linear Programming Formulations:** Suppose we restrict the total number of taps in the equalizer to be $2N + 1$. Then, the dimension of $\theta = (\theta_{-N}, \theta_{-N+1}, \ldots, \theta_0, \ldots, \theta_N)$ is $2N + 1$. Given the block of equalizer input data $\{X_k\}_{K=-\infty}^{K} \ll 0$, then the output of the equalizer is $z_k = X_k(\theta)$ where $X_k(\theta)$ is the signal regressor (4). Based on Theorem 3.1, the equalizer $\theta$ can be found by

$$\begin{align*}
\text{minimize} & \quad \max_{-K \leq k \leq K} |\text{Re}\{z_k\}| + \max_{-K \leq k \leq K} |\text{Im}\{z_k\}| \\
\text{subject to} & \quad \text{Re}\{\theta_0\} + \text{Im}\{\theta_0\} = 1.
\end{align*} \quad (13)$$

Note that

$$\text{Re}\{z_k\} = \text{Re}\{X_k(\theta)\} = \sum_{i=-N}^{N} \text{Re}\{x_{k-i}\} \text{Re}\{\theta_i\} - \text{Im}\{x_{k-i}\} \text{Im}\{\theta_i\}$$

and

$$\text{Im}\{z_k\} = \text{Re}\{X_k(\theta)\} = \sum_{i=-N}^{N} \text{Re}\{x_{k-i}\} \text{Im}\{\theta_i\} + \text{Im}\{x_{k-i}\} \text{Re}\{\theta_i\}.$$ 

Introducing two auxiliary variables $\tau_1$ and $\tau_2,$ we can reformulate the above minimization problem as the following linear program:

$$\begin{align*}
\text{minimize} & \quad \tau_1 + \tau_2 \\
\text{subject to} & \quad \text{Re}\{\theta_0\} + \text{Im}\{\theta_0\} = 1, \\
& \quad -\tau_1 \leq \sum_{i=-N}^{N} \text{Re}\{x_{k-i}\} \text{Re}\{\theta_i\} - \text{Im}\{x_{k-i}\} \text{Im}\{\theta_i\} \leq \tau_1, \\
& \quad -\tau_2 \leq \sum_{i=-N}^{N} \text{Re}\{x_{k-i}\} \text{Im}\{\theta_i\} + \text{Im}\{x_{k-i}\} \text{Re}\{\theta_i\} \leq \tau_2, \\
& \quad k = 0, \pm 1, \pm 2, \ldots, \pm K.
\end{align*} \quad (14)$$

The equivalence of (14) to (13) follows from the fact that, when at optimality, the variables $\tau_1$ and $\tau_2$ must be equal to $\max_{-K \leq k \leq K} |\text{Re}\{z_k\}|$ and $\max_{-K \leq k \leq K} |\text{Im}\{z_k\}|$, respectively.

The above linear program (14) has $4N + 4$ variables and $8K + 5$ constraints. It can be solved efficiently using the recently developed interior point methods. The number of iterations required to solve the linear program is $O(\sqrt{N}L)$, where $L$ is the number of bits required to encode the input data $\{X_k\}_{K=-\infty}^{K}$ in binary. Each iteration requires $O(K^{2.5})$ arithmetic operations.

An alternative, yet equally effective, formulation of the equalization problem is the following:

$$\begin{align*}
\text{minimize} & \quad \max_{-K \leq k \leq K} |\text{Re}\{z_k\}| \\
\text{subject to} & \quad \text{Re}\{\theta_0\} + \text{Im}\{\theta_0\} = 1.
\end{align*} \quad (15)$$
This is based on the fact that Theorem 3.1 remains true if we minimize the convex cost
\[
\max_{\{a_k\}} |\text{Re}(z_k)|
\]
instead of the convex cost
\[
\max_{\{a_k\}} |\text{Re}(z_k)| + \max_{\{a_k\}} |\text{Im}(z_k)|.
\]

The equivalent linear programming formulation of (15) is the following:

\[
\begin{align*}
\text{minimize} \quad & \tau_1 \\
\text{subject to} \quad & \text{Re} \{x_0\} + \text{Im} \{x_0\} = 1 \\
& -\tau_1 \leq \sum_{i=-N}^{N} \text{Re} \{x_{k-i}\} \text{Re} \{\theta_i\} - \text{Im} \{x_{k-i}\} \text{Im} \{\theta_i\} \leq \tau_1, \\
& k = 0, \pm 1, \pm 2, \ldots, \pm K.
\end{align*}
\]

Notice that this linear program has only \(4K + 3\) constraints and \(4N + 3\) variables. Thus, it is simpler than (14) and generally leads to faster convergence time for the equalizer. An entirely similar formulation is to minimize the imaginary part of the combined channel output only, leading to the following linear program:

\[
\begin{align*}
\text{minimize} \quad & \tau_2 \\
\text{subject to} \quad & \text{Re} \{x_0\} + \text{Im} \{x_0\} = 1 \\
& -\tau_2 \leq \sum_{i=-N}^{N} \text{Re} \{x_{k-i}\} \text{Im} \{\theta_i\} + \text{Im} \{x_{k-i}\} \text{Re} \{\theta_i\} \leq \tau_2, \\
& k = 0, \pm 1, \pm 2, \ldots, \pm K.
\end{align*}
\]

IV. IMPLEMENTATION AND SIMULATION

We demonstrate the effectiveness of our linear programming equalization algorithms through two simulation examples. In all of our examples, the linear programs are solved using an interior point optimization code SeDuMi [173 developed using Matlab. In our simulations, the severity of ISI (eye-closeness) can be described by the scatter-plots of equalizer output signal samples. The combined channel and equalizer is an open eye system if the signal constellation can be identified from the scatter plot.

In the first example, we tested the following all-pass channel with a phase error of \(\pi/4\):

\[
H(z) = e^{j\pi/4} \frac{0.7 - z^{-1}}{1 - 0.7z^{-1}}.
\]

The input sequence is an i.i.d. 16-QAM signal. This channel setting was used in [15] where an equalizer of length 21 and a total of 1000 data samples are needed for the so-called block algorithm based on 1, smoothing. In contrast, the linear programming channel equalization algorithm requires only 600 data samples and an equalizer of length 17. As seen from Fig. 1(a), the channel output before equalization is completely scrambled due to ISI. The real and imaginary parts of the channel impulse response are depicted in Fig. 1(b) and (c).

Fig. 2(a) shows the channel output after equalization, while Fig. 2(b) and (c) shows the real and imaginary parts of the combined impulse response of the channel and the equalizer. As can be seen, the channel has been well equalized since the eye diagram is completely open and the combined impulse response is very close to the ideal delta function. The phase error has also been corrected but, as Fig. 2(c) indicates, a \(-\pi/2\) phase shift remains. The improvement on the convergence speed is dramatic: the linear programming approach (SeDuMi) requires only about 20 iterations, while the gradient descent algorithm or the block algorithm of [15] (both based on \(l_p\) approximation) require from 200 up to 10 000 iterations.

We now test a complex channel whose real part and imaginary part of its impulse response are shown in the example of [4] and also in [15]. Suppose the input sequence is an i.i.d. QPSK signal. The channel output and impulse responses are given in
The linear programming equalization algorithm takes about 30 iterations to converge, as opposed to 300 iterations reported in [15]. The equalized channel output and the combined impulse response of the channel and the equalizer are given in Fig. 4. Clearly, the eye diagram is open and the combined impulse response is nearly ideal.

References