A Novel Price-Based Power Control Algorithm in Cognitive Radio Networks
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Abstract—This letter investigates price-based power control problem in the spectrum sharing cognitive radio networks (CRNs). The base station (BS) of primary users (PUs) can admit secondary users (SUs) to access if their interference power is under the interference power constraint (IPC). In order to access the spectrum, the SUs need to pay for their interference power. The BS first decides the price for each SU to maximize its revenue. Then, each SU controls its transmit power to maximize its revenue based on non-cooperative game. We model the interaction between the BS and the SUs as a Stackelberg game. Using backward induction, the revenue function of the BS is expressed as a function of the transmit power of the SUs. After we depict the property of optimal transmit power of the SUs, we propose a novel price-based power control algorithm to maximize the revenue of the BS. Simulation results show the proposed algorithm improves the revenue of both the BS and SUs compared with the proportionate pricing algorithm.

Index Terms—Power control, cognitive radio, price, Stackelberg Game.

I. INTRODUCTION

WITH the increasing demand for wireless services, spectrum becomes scarce and increasingly crowded, and it needs to be used efficiently. The Federal Communications Commission (FCC) found the utilization of the spectrum is low most of the time [1]. Thus, the technology of cognitive radio networks (CRNs) [2] is proposed to solve the problem of spectrum scarcity and improve spectrum efficiency.

The issue about pricing in CRNs was investigated in [3]–[6]. In [3], the authors proposed a joint spectrum bidding and service pricing model for IEEE 802.22-based cognitive wireless networks. In [4], the authors investigated the pricing issue for the power control problem in code division multiple access (CDMA) based CRNs. In [5], the optimal investment and pricing decisions in CRNs under spectrum supply uncertainty were addressed. In [6], the authors proposed a joint pricing and power allocation scheme for CRNs. Another important issue of power control is to reduce power consumption to extend terminal’s life-time [7], [8], we will consider to model the energy efficient CRN by Stackelberg game in our future works.

In this letter, we will focus our study on the price-based power control in CDMA based CRNs as [4]. Since the utility of the base station (BS) is non-convex function, it is difficult to find the optimal pricing scheme, and the authors proposed a sub-optimal proportionate pricing algorithm to maximize the revenue of the BS. By characterizing the property of the transmit power of SUs under the optimal pricing scheme, we propose a novel price-based power control algorithm to find the optimal price for each SU. Simulation results show that the proposed pricing scheme can improve the utilities of both the BS and SUs compared with the pricing scheme proposed in [4].

II. SYSTEM MODEL

We consider the uplink transmission for the CDMA based CRNs. As shown in Fig. 1, the primary users (PUs) are licensed to transmit to the BS, and the n SUs need to pay the BS for their uplink transmissions. Link gain between SU and the BS is denoted by \( h_i(i = 1, \ldots, n) \). \( L \) is the spreading gain. The interference power constraint (IPC) [9] of SUs to the BS is \( T \). The BS will charge the \( i \)-th SU \( \lambda_i \) per unit interference power.

We model the strategy between the BS and SUs as a Stackelberg game. The BS is the leader in this game. It chooses a price for each SU to maximize its own revenue under IPC. The SUs are the followers of the game. After the BS chooses the price for each SU, the SU will decide the transmit power to maximize its utility based on non-cooperative power control game. The problem of the BS is as follows:

\[
\begin{align*}
\text{maximize } & \quad \upsilon_p(\lambda_1, \ldots, \lambda_n) = \sum_{i=1}^{n} \lambda_i h_i p_i \\
\text{subject to } & \quad \sum_{j=1}^{n} h_j p_j \leq T, \quad p_j \geq 0, \quad j = 1, \ldots, n, \\
& \quad h_j p_j \leq P_{\text{max}}, \quad j = 1, \ldots, n,
\end{align*}
\]

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where $T$ is the IPC at the BS, $\lambda_i$ is the price charged for the $i$-th SU per unit interference power and $P_{\text{max}}$ is the maximal allowed receive power for each SU at the BS. Constraint (2) means that the total interference power made by SUs should be below a given threshold $T$ to ensure the SUs’ transmission would not cause unendurable interference to the PUs. Constraint (3) means that the interference power of each SU to the BS should be less than $P_{\text{max}}$ to guarantee the fairness among SUs. The utility of the $i$-th SU has two parts: one is the income from the transmit rate achieved at the BS when it transmits at a given power $p_i(i = 1, \ldots, n)$, the other is the payment to the BS. The SINR of the $i$-th SU at the BS is as follows:

$$
\gamma_i(p) = \frac{L_i p_i}{\sum_{j \neq i} h_j p_j + \sigma^2}
$$

where $p_i$ is the transmit power of the $i$-th SU, $p = (p_1, \ldots, p_n)$ is the transmit power of all SUs and $\sigma^2$ is the interference caused by the PUs and the ambient noise at the BS. Thus, the utility of SU $i$ is given by

$$
u_i(p, \lambda_i) = w_i \log(1 + \gamma_i(p)) - \lambda_i p_i,
$$

where $w_i$ is the preference factor of the $i$-th SU for the unit rate. The optimization problem for the $i$-th SUs is as follows:

$$
\begin{align*}
\text{maximize} & \quad \nu_i(p_i, p_{-i}, \lambda_i) \\
\text{subject to} & \quad p_i \geq 0,
\end{align*}
$$

where $p_{-i}$ denotes the transmit power of the SUs except the $i$-th SU.

III. Novel Price-Based Power Control Algorithm

In this section, we give a novel pricing algorithm for the BS to maximize its revenue according to the property of the transmit power of SUs under the optimal price. The relationship among the transmit power of SUs for the given price $\lambda_i$ ($i = 1, \ldots, n$) is as follows:

Lemma 1: Let $(p_1, \ldots, p_n)$ be the transmit power of the SUs when the BS charges the $i$-th SU for a given price $\lambda_i$ such that $\lambda_i$ is less than or equal to $w_iL/\sigma^2(i = 1, \ldots, n)$, then the receive power $h_ip_i(i = 1, \ldots, n)$ of the SUs at the BS satisfies the following equations:

$$
\begin{bmatrix}
1 & \cdots & 1/L \\
\vdots & \ddots & \vdots \\
1/L & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
h_1 p_1 \\
\vdots \\
h_n p_n
\end{bmatrix}
= \begin{bmatrix}
w_1 / \lambda_1 - \sigma^2 / L \\
\vdots \\
w_n / \lambda_n - \sigma^2 / L
\end{bmatrix}.
$$

Proof: Use the optimal condition for the $i$-th SU in (6), we have

$$
\frac{\partial \nu_i(p_i, p_{-i})}{\partial p_i} = \frac{w_i L h_i}{\sum_{j \neq i} h_j p_j + \sigma^2 + L h_i p_i} - \lambda_i = 0,
$$

then we can get the following equations:

$$
\sum_{j \neq i} h_j p_j / L + \sigma^2 / L + h_i p_i = w_i / \lambda_i
$$

for SU $i (i = 1, \ldots, n)$. (7) is derived by rewriting (9) in the matrix form.

From (8), we know that the transmit power of SU $i$ is zero if the price for the $i$-th SU is larger than $w_iL/\sigma^2$. This means that the SU $i$ will not buy the interference power when the price is larger than $w_iL/\sigma^2$. Therefore, we only need to find the optimal price for the BS in the price region such that $\lambda_i$ is less than or equal to $w_iL/\sigma^2(i = 1, \ldots, n)$. By lemma 1, we have the following important identities:

$$
\frac{L w_i h_i p_i}{\sum_{j \neq i} h_j p_j + \sigma^2 + L h_i p_i} = \lambda_i h_i p_i (i = 1, \ldots, n).
$$

(10)

It means that the revenue of the BS gets from the $i$-th SU can be expressed as:

$$
\sum_{j \neq i} h_j p_j + L h_i p_i + \sigma^2
$$

subject to $h_j p_j \leq T, p_j \geq 0, j = 1, \ldots, n$.

(12)

The transmit power of each SU under the optimal price of the BS is the solution to (11). The constraint (12) and (13) are coupled, and constraint (13) will not be active constraints when $P_{\text{max}}$ is large enough. Therefore, we first consider the solution to (11) without (13):

$$
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{n} \frac{L w_i h_i p_i}{\sum_{k \neq i} h_k p_k + L h_i p_i + \sigma^2} \\
\text{subject to} & \quad h_j p_j \leq T, j = 1, \ldots, n.
\end{align*}
$$

(14)

First, we give the optimal solution to (14), then we characterize the property of the optimal solution to (11) with (13), and finally we propose a novel pricing algorithm to maximize the revenue of the BS. The optimal solution to (14) satisfies the equality constraint by the following lemma.

Lemma 2: Let $(p_1^*, \ldots, p_n^*)$ be the optimal solution to (14) for a given $T$, the following condition is satisfied:

$$
\begin{align*}
\sum_{j=1}^{n} h_j p_j^* = T.
\end{align*}
$$

(15)

Proof: We prove it by contradiction. Let $P = (p_1, \ldots, p_n)$ be the optimal solution to (14) such that $\sum_{j=1}^{n} h_j p_j$ is less than $T$. Let $a = \frac{T}{\sum_{j=1}^{n} h_j p_j}$, $p_i^* = a p_i (i = 1, \ldots, n)$, we can verify that the following conditions are satisfied:

$$
\begin{align*}
\sum_{j=1}^{n} h_j p_j^* = \frac{T}{\sum_{j=1}^{n} h_j p_j} = T, \\
\frac{L w_i h_i p_i^*}{\sum_{k \neq i} h_k p_k + L h_i p_i + \sigma^2} = \frac{L w_i h_i p_i}{\sum_{k \neq i} h_k p_k + L h_i p_i + \sigma^2}.
\end{align*}
$$

Because $a$ is greater than one, we can get:

$$
\begin{align*}
\frac{L w_i h_i p_i^*}{\sum_{k \neq i} h_k p_k + L h_i p_i + \sigma^2} > \frac{L w_i h_i p_i}{\sum_{k \neq i} h_k p_k + L h_i p_i + \sigma^2}.
\end{align*}
$$

(17)

Therefore, we have

$$
\begin{align*}
\sum_{j=1}^{n} \frac{L w_i h_i p_i^*}{\sum_{k \neq i} h_k p_k + L h_i p_i + \sigma^2} > \sum_{i=1}^{n} \frac{L w_i h_i p_i}{\sum_{k \neq i} h_k p_k + L h_i p_i + \sigma^2}.
\end{align*}
$$

(18)
This contradicts the assumption that \((p_1, \ldots, p_n)\) is the optimal solution to (11). Thus, lemma 2 is proved.

Using lemma 2, we substitute \(\sum_{j=1}^{n} h_j p_j = T\) into (14). Therefore, the optimal solution to (14) is equivalent to the following problem:

\[
\begin{align*}
\max & \sum_{i=1}^{n} \frac{L w_i h_i p_i}{T + (L - 1) h_i p_i + \sigma^2} \\
\text{subject to} & \sum_{j=1}^{n} h_j p_j = T, p_j \geq 0, \quad j = 1, \ldots, n.
\end{align*}
\] (19)

It can be verified that the object function of (19) is a concave function. So (19) is a concave maximization problem. Using Karush-Kuhn- Tucker (KKT) conditions [10], we can derive the solution to (19) as follows:

**Theorem 1:** Let \(p_1, \ldots, p_n\) be the optimal solution to (19), the transmit power of the \(i\)-th SU is given by

\[
p_i = \frac{\lambda}{h_i} \frac{\sqrt{L w_i (T + \sigma^2)}}{(T + \sigma^2)}, 0 \leq p_i \leq P_{\max},
\] (20)

where \(\lambda\) is the solution to the following equation:

\[
\sum_{i=1}^{n} \frac{L w_i}{\sqrt{L w_i (T + \sigma^2)}} - \frac{L - 1}{T + \sigma^2} = T.
\] (21)

Theorem 1 gives the optimal power for each SU without considering the constraint (13). The optimal solution to (11)-(13) is given by theorem 1 when it satisfies constraint (13). However, the constraint (13) may become an active constraint if \(P_{\max}\) is much smaller than \(T\). Next, we characterize the property of the optimal solution to (11)-(13) before we give an algorithm to find the solution to it.

**Lemma 3:** Let \(p_1, \ldots, p_n\) be the optimal solution to (11)-(13). If the preference factor of any two SUs \(i\) and \(j\) satisfies \(w_i > w_j\), the receive power of the \(i\)-th SU and \(j\)-th SU will satisfy:

\[h_i p_i \leq h_j p_j\] (22)

**Proof:** We prove it by contradiction. Let \(P = (p_1, \ldots, p_n)\) be the optimal solution to (11)-(13), and two SUs \(i\) and \(j\) satisfy \(w_i > w_j\) such that the receive power satisfies \(h_i p_i < h_j p_j\). Let \(p_k^* = p_k\) for \(k \neq i, j\), \(p_i^* = h_i p_i / h_i\), and \(p_j^* = h_j p_j / h_j\). We can verify that \(P^* = (p_1^*, \ldots, p_n^*)\) satisfies the constraints (12) and (13), and the following inequality satisfies:

\[\sum_{i=1}^{n} \frac{L w_i h_i p_i}{\sum_{k \neq i} h_k p_k + \sum_{i=1}^{n} h_i p_i^* + \sum_{i=1}^{n} h_i p_i^* + \sigma^2} \leq \sum_{i=1}^{n} \frac{L w_i h_i p_i}{\sum_{k \neq i} h_k p_k + \sum_{i=1}^{n} h_i p_i^* + \sum_{i=1}^{n} h_i p_i^* + \sigma^2}.
\] (23)

This contradicts the hypothesis that \(P = (p_1, \ldots, p_n)\) is the optimal solution to (11)-(13). Therefore, we complete the proof.

From lemma 3, we know that if the constraint (13) for SU \(i\) is an active constraint, then the constraint (13) for the users whose preference factor is larger than the user \(i\) will also be an active constraint. Without loss of generality, we assume \(w_1 \geq \cdots \geq w_n\) and so is the optimal solution to (11). If the solution from theorem 1 does not satisfy the constraint (13), there must exist \(k \in [1, \min(n, [T/P_{\max}])]\) such that \(h_i p_i^\ast = P_{\max}, \cdots, h_k p_k^\ast = P_{\max}\) from lemma 3. Therefore, there exists \(T_k\) such that the optimal transmit power of SUs \(k\) to \(n\) is the solution to the following problem:

\[
\begin{align*}
\max & \sum_{i=1}^{n} \frac{w_i h_i p_i}{L w_i h_i p_i + \sigma^2}, 0 \leq p_i \leq P_{\max} \\
\text{s.t.} & \sum_{j=1}^{n} h_j p_j = T_k, p_j \geq 0, \quad j = 1, \ldots, n.
\end{align*}
\] (24)

where \(T_k\) is determined such that the optimal solution \((p_1^*, \ldots, p_n^*)\) to (24) satisfies:

\[
\begin{align*}
\left\{ \begin{array}{ll}
\max_{i=1}^{n} \frac{\sqrt{w_i}}{T} - a_k &= 0, \\
\max_{i=1}^{n} \frac{\sqrt{w_i}}{T} - a_k &= P_{\max}.
\end{array} \right.
\] (25)

where \(a_k = \frac{w_k}{T} + (k - 1) \cdot P_{\max}\) and \(b = \frac{w_k}{T} + (k - 1) \cdot P_{\max}\) for a given \(k\), and \(k\) can be determined by search in \([1, \min(n, [T/P_{\max}])]\) to find the best one that maximize the revenue of the BS by (11).

From theorem 1 and (25), we give a novel price-based power control algorithm for the BS and the corresponding power allocation for each SU by Algorithm 1. From (20), the optimal power for some SUs with the lower preference factor may be zero. Therefore, the BS might not admit some SUs to maximize its own revenue.

**IV. Simulation Results**

In this section, we evaluate the performance of the proposed pricing algorithm by comparing it with the scheme proposed in [4]. The scheme proposed in [4] is a proportionate pricing (PP) algorithm which confines the prices for SUs to be linearly proportional form, and search for the sub-optimal price in the confined price region. We compare the system performance obtained by the proposed algorithm with the PP algorithm versus IPC. The simulation parameters are as follows: \(P_{\max} = 1, \quad L = 32, \sigma^2 = 10\), the number of SUs is 10, \(w_i\) is the uniform distribution in \([0, 300]\), \(h\) is uniform distribution in \([0, 1]\), \(T\)
Algorithm 1: Novel Price-based Power Control Algorithm (NPPC)

Initialization: set $p_i = \max(\frac{\lambda(1+c^2) - T}{\lambda(1+c^2) - (T+c^2)} - 0)/h_i$

\[ i = 1, \ldots, n \]

where $\lambda$ is the solution to

\[ \sum_{i=1}^{n} \max(\frac{\lambda(1+c^2) - T}{\lambda(1+c^2) - (T+c^2)} - 0) = T, \text{ and } k = 1 \]

if $h_i p_i \leq P_{\text{max}}$ for $i = 1, \ldots, n$ then

\[ p^*_i = p_i, \]

\[ \lambda^*_i = \frac{L_{wi}}{\sum_{j} h_j p^*_j + \sigma^2 + L_h p_i} \text{ for } i = 1, \ldots, n \]

else

\[ \text{while } k \leq \min(n, \lceil T/P_{\text{max}} \rceil) \text{ do} \]

\[ i = k - 1, p^*_j = P_{\text{max}}/h_i \text{ for } j = 1, \ldots, i - 1 \]

\[ p^*_j = \max(\frac{\lambda(1+c^2) - T}{\lambda(1+c^2) - (T+c^2)} - 0)/h_j \text{ for } j \geq i, \text{ where } T_k \]

\[ \text{is the solution to (25)} \]

\[ k = k + 1 \]

end while

\[ \text{for all } j \in (1, \ldots, \min(n, \lceil T/P_{\text{max}} \rceil)) \text{ do} \]

\[ j = \arg \max_{k \in (1, \ldots, \min(n, T/P_{\text{max}}))} \sum_{i} \frac{w_{ki} p^*_i}{\sum_{j} h_j p^*_j + \sigma^2 + L_h p_i} \]

\[ p^*_i = p^*_j, \]

\[ \lambda^*_i = \sum_{j} h_j p^*_j + \sigma^2 + L_h p_i \text{ for } i = 1, \ldots, n \]

end for

end if

Output: For $i = 1, \ldots, n$, the optimal price for SU $i$ is given by $\lambda^*_i$, and the power transmit by SU $i$ is given by $p^*_i$.

Changes from 0 to 20. All simulations are averaged by $10^4$ realizations.

Fig. 2 shows the revenue of the BS versus the IPC. Our algorithm has the same performance with PP algorithm when $T$ is less than 5 and superior to it when $T$ is greater than 5. This is because when $T$ is greater than 5, the constraint of $P_{\text{max}}$ will be an active constraint for some SUs, and the problem for the BS will become a non-convex problem. The PP algorithm can only find a sub-optimal pricing scheme. Our algorithm will gain more than 7% profit comparing with the PP algorithm.

The revenue of two algorithms will remain unchanged when $T$ is greater than 10. The reason is that $T$ becomes an inactive constraint when it is large enough.

Fig. 3 shows the sum utility of the SUs versus IPC. The sum utility of both algorithms increases as IPC increases. Our algorithm is better than the PP algorithm when IPC is greater than 5. This is because the proposed algorithm improves the rate of each SU more rapidly than the payment to the BS.

The sum rate of SUs is plotted in Fig. 4. Our algorithm is getting better and better than the PP algorithm as IPC increases from 5 to 10. Comparing with the PP algorithm, the sum rate of SUs has improved by our algorithm as much as 26% when $T$ is 10. The sum rate of both algorithms will be saturated when IPC is more than 10. This is because that $T$ becomes an inactive constraint when it is more than 10.

V. CONCLUSION

In this letter, a novel price-based power control algorithm is proposed in CDMA-based CRNs. We characterize the revenue of the BS as a function of the transmit power of the SUs. The optimal condition of maximizing the revenue of the BS is described by considering different active power constraint for SUs. Simulation results show that the proposed pricing algorithm improves the revenue of both the BS and SUs compared with the proportionate pricing algorithm.

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