Understanding herding based on a co-evolutionary model for strategy and game structure

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Abstract
So far, there has been no conclusion on the mechanism for herding, which is often discussed in the academia. Assuming escaping behavior of individuals in emergency is rational rather than out of panic according to recent findings in social psychology, we investigate the behavioral evolution of large crowds from the perspective of evolutionary game theory. Specifically, evolution of the whole population divided into two subpopulations, namely the co-evolution of strategy and game structure, is numerically simulated based on the game theoretical models built and the evolutionary rule designed, and a series of phenomena including extinction of one subpopulation and herding effect are predicted in the proposed framework. Furthermore, if the rewarding for rational agents becomes significantly larger than that for emotional ones, herding effect will disappear. It is exciting that some phase transition points with interesting properties for the system can be found. In addition, our model framework is able to explain the fact that it is difficult for mavericks to prevail in society. The current results of this work will be helpful in understanding and restraining herding effect in real life.

1. Introduction
Herding effect (i.e., herding behavior), considered as a common phenomenon in various fields such as emergency evacuation of large crowds, has caught much interest of scholars [1–5]. For large population to escape from danger in a closed building with two symmetrically located exits or paths, herding effect means that the great majority of people adopt the same one in escaping, leaving the other one vacant. It is worth noting that herding effect usually means inefficient utilization of resources, thus often leading to inferior outcomes in real life. For example, the asymmetric utilization of escaping exits in emergency due to herding effect will decrease evacuation efficiency and bring disastrous consequences. Therefore it is worthwhile to get a deep and exhaustive understanding of the mechanism for herding effect in real scenarios, the formation of which is definitely very complicated.

There have been a plenty of studies on herding effect proposed by researchers from different disciplines [6–9]. Cognitive psychologists hold the opinions that herding effect emerges because of local interactions between individuals instead of central coordination [6]. Socialists think herding effect is caused by social networks [10–12], while some economists believe that it is due to the fact that people are incapable of dealing with new information effectively [13,14]. Recent researches in social psychology about herding effect in emergency [15–17] indicate that, escaping behaviors among individuals are rational actions instead of crowd panic and a series of phenomena including herding effect are the result of rational choices in behaviors for escaping agents. However, most microscopic simulation models [5,18–20] in the field of emergency evacuation up to now are generally based on the assumption that panic instead of rational actions induces herding...
effect. Moreover, modeling methods on herding effect in emergency are mainly twofold [1,21]: one regards the crowd as a whole [22,23] while the other is agent-based [2,18,24,25]. Among the agent-based approaches, methods based on game theory have been extensively focused recently.

According to recent researches in psychology stated above, the viewpoint that behaviors in emergency are rational and not out of panic is adopted and a new co-evolutionary model for both strategy and game structure [12] on a square lattice as underlying topology is established. Specifically, crowds in emergency are divided into two subpopulations represented by different game models and the mechanism for herding effect is provided through agent-based method from the viewpoint of evolutionary game theory [26–32]. As both the behavior and subpopulation identity evolve with time, our proposed model is indeed a co-evolutionary model for strategy and game structure. By numerically simulating behavioral evolution of individuals in emergency based on game theoretical models built and the evolutionary rule designed, proportions of the two subpopulations and fractions of the two behaviors after evolution varying with the only game parameter are obtained. Notably, our model predicts herding effect and asymmetric utilization of escaping exits in some cases and there exist phase transition points with interesting properties for the system of population.

2. Model

The following situation is discussed in the work: high density of crowds in a closed building in emergency (such as fire) have to escape quickly, and coincidently there are two symmetrically located and identical exits in the building. According to the difference in the way of thinking when choosing the two exits, the large-scale crowds in emergency evacuation are divided into subpopulations 1 and 2, namely emotional subpopulation and rational subpopulation. While individuals in subpopulation 1 emphasize emotional needs and tend to follow the crowd, subpopulation 2 is made up of people who often analyze cases rationally and avoid selecting the escaping exit chosen by most individuals to get out of danger fast. In the rest of this work, choosing escaping exit 1 is denoted by behavior 1, while behavior 2 stands for adopting exit 2. As stated above, agents in subpopulation 1 tend to select the same behavior with others nearby, however, individuals in subpopulation 2 are apt to adopting the behavior different from the choice of most people.

According to the concept of utility in game theory, the heterogeneity between individuals can be represented by payoff matrices. Suppose that the payoff or utility of a player belonging to subpopulation $u$ with behavior $i$ when meeting an agent from subpopulation $v$ with behavior $j$ is denoted as $p_{ij}^{uv}$. Here, $u, v \in \{1, 2\}$ and $i, j \in \{1, 2\}$. After the representation of the payoffs, the results of games within and between subpopulations can be obtained. For example, the payoff values for two agents coming from subpopulations 1 and 2 respectively after interacting with each other are shown in Table 1.

In order to simplify the analysis, assume that $p_{ij}^{11} = p_{ij}^{22}$, $p_{ij}^{12} = p_{ij}^{21}$, $\forall i, j \in \{1, 2\}$. The above assumption implies that, for each player the utility obtained after interacting with another agent relies on only the subpopulation identity of the focal player and the behaviors of both, which is independent of the subpopulation identity of the game partner.

As mentioned above, subpopulation 1 is the group of individuals who are apt to imitating the behaviors of others. In emergency evacuation, people in subpopulation 1 attach great importance to emotional needs, and they would like to choose the same escaping exit with the majority nearby in order to reinforce psychological sense of security and comfort. A general cooperative game [33] is adopted to represent the interactions for agents in subpopulation 1, and the payoffs for individuals in subpopulation 1 are expressed as in Table 2, where $a > 0$. According to the payoff matrix in Table 2, it can be concluded that agents in subpopulation 1 prefer neither of the two behaviors and they will get a positive benefit $a$ if choosing the same behavior with the game partner. In this study, game parameter $a$ is called the rewarding of following the crowd for emotional agents. In the game represented in Table 2, R-reciprocity [34] is expected instead of ST-reciprocity [35], since choosing the same behavior as the opponent brings a higher utility.

On the contrary, subpopulation 2 is composed of rational agents who prefer to behave differently. Therefore, the interactions for subpopulation 2 are modeled as the so-called minority game [33], which is expressed in Table 3 with parameter $b > 0$. From the payoff matrix in Table 3, it can be found that agents in subpopulation 2 interacting with many people tend to choose the escaping exit different from the choice of the majority in order to escape quickly, which is just the reason why they are considered rational. Similar to subpopulation 1, game parameter $b$ in Table 3 is named the rewarding of behaving differently for rational agents. Unlike the game type expressed in Table 2, the expected outcome for the game represented in Table 3 is ST-reciprocity [35], that is, players in subpopulation 2 tend to adopt different behaviors in order to obtain a higher payoff.

Based on the assumption mentioned above, the payoff matrix for two players from subpopulations 1 and 2 respectively can be derived straightforwardly according to Tables 2 and 3. It is worth to note, in this paper the

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<th>Table 1</th>
<th>Payoffs for two players in subpopulations 1 and 2 respectively (general form).</th>
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<tr>
<td>Behavior 1</td>
<td>(p_{ij}^{11}, p_{ij}^{12})</td>
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<tr>
<td>Behavior 2</td>
<td>(p_{ij}^{21}, p_{ij}^{22})</td>
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<th>Table 2</th>
<th>Payoffs for two players in subpopulation 1.</th>
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<tr>
<td>Behavior 1</td>
<td>(a, a)</td>
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<td>Behavior 2</td>
<td>(0, 0)</td>
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<th>Table 3</th>
<th>Payoffs for two players in subpopulation 2.</th>
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<tr>
<td>Behavior 1</td>
<td>(0, 0)</td>
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<tr>
<td>Behavior 2</td>
<td>(a, a)</td>
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escaping behavior and way of thinking (namely subpopulation identity) evolve dynamically for the whole population in emergency evacuation. Without loss of generality, the normalization treatment is made, namely $a = 1$. Therefore there is only one game parameter $b$ and in this case the parameter is called the relative rewarding for rational agents.

3. Results

In this section, agent-based method is adopted to numerically simulate the behavioral evolution of the whole population in emergency evacuation, which is composed of subpopulations 1 and 2. Above all, it is assumed that the number of individuals in the whole population is $N$, and they are distributed on a $L \times L$ square lattice, which is one of the simplest network structure in evolutionary game theory [36–38]. Regarding the interaction topology of the game, a series of complex networks [39–42] such as scale-free networks [43,44], interdependent networks [45,46] have been proposed to illustrate the evolution of cooperative behavior. Moreover, periodic boundary conditions are applied in the simulations so as to weaken the influence of lattice size. At the initial instant, individuals from subpopulations 1 and 2 are randomly distributed on the square lattice, with subpopulation 1 occupying the proportion $f$ in the whole population, and within each subpopulation the number of agents with behavior 1 is equal to that of entities selecting behavior 2.

In the evolution process, the state to be updated in each step includes two parts: behavior and subpopulation identity. In other words, the model proposed in this work is a co-evolution model for strategy and game structure [12,38]. For the individuals having to escape in emergency evacuation, evolution of the two parts should not be separated. Then, we will design a simple updating rule for the evolution of them. In the simulations, Moore style interaction neighborhood is chosen and in each round each player obtains its total payoff as the sum of 9 payoffs after playing games with 8 neighbors and itself in accordance with the game models built. On one hand, the famous Fermi updating rule [47,48] is adopted in the evolution of behavior. Specifically, each individual randomly chooses one of its neighbors and mimics the behavior of the chosen agent in the next round with probability $1/(1 + \exp(-\Delta \pi/K))$, which is determined by the payoff difference $\Delta \pi$. Here, the parameter $K$ stands for degree of irrationality or noise intensity in the decision process and values for the parameters are set as $K = 0.2$, $N = 10000$ and $L = 100$ in the numerical simulations. On the other hand, considering that agents are very likely to imitate the subpopulation identity of one neighbor with a higher total payoff and that it is more difficult to change subpopulation identity than behavior because the former corresponds to the way of thinking, we set a probability 0.1 for the agents to update its subpopulation identity. Namely, each agent imitates the subpopulation identity of one neighbor in the next step only with a small probability 0.1. In addition, the synchronous updating is used and the simulation result is obtained after the behavioral evolution is stabilized.

In the following, the symbols $y$, $u$ and $v$ are utilized to denote proportion of subpopulation 1 in steady state, fraction of behavior 1 in subpopulation 1 in steady state and fraction of behavior 1 in subpopulation 2 in steady state, respectively.

Fig. 1 exhibits how the proportion of subpopulation 1 in steady state varies with parameter $b$, in which the five curves in different colors correspond to distinct initial proportions of subpopulation 1 respectively, namely $f = 0.1, 0.3, 0.5, 0.7, 0.9$.

It can be concluded from Fig. 1 that, ratio of subpopulation 1 after behavioral evolution decreases monotonically with the increment of $b$, and the relationship curve between them is similar to the step function. To be concise, the result of each simulation run is that subpopulation 1 defeats subpopulation 2 when $b \leq 1.32$, while if $b \geq 1.42$ it turns out that only subpopulation 2 survives ultimately. Therefore, in conclusion the value of $b$ determines the evolution result of the two subpopulations, and there exist a narrow range of values for $b$ which can be regarded as phase transition points or critical points.

It is very surprising that subpopulation 1 will win in the competition with subpopulation 2 when $b = 1$, no matter what proportion this subpopulation occupies initially. In other words, although the two subpopulations seem to possess the same competitiveness, in fact the individuals

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<td>Payoffs for two players in subpopulation 2.</td>
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<tr>
<td>Behavior 1</td>
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<td>Behavior 1</td>
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Fig. 1. Proportion of subpopulation 1 in steady state varying with parameter $b$. In the figure, each data point is averaged over 10 independent numerical simulations, and the result of each simulation run is the average of the last 100 MC steps in the whole evolution including 20,000 MC steps. The five colors in yellow, red, green, blue and black correspond to five initial cases of $f = 0.1, 0.3, 0.5, 0.7, 0.9$ respectively. In the figure, each curve includes 22 data points, whose horizontal coordinates are 0.9, 1.1, 1.2, 1.3, 1.31:0.01:1.45, 1.5, 1.6. The sub-figure in the down left corner is the enlargement of the middle part of the figure. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
who prefer choosing the behavior different from that of the majority cannot survive ultimately even if they account for 90% of the whole population at the initial instant. According to the results shown in Fig. 1, agents in subpopulation 2 are able to survive only if the value of $b$ is significantly larger than 1. Hence, the simulation results can explain the reality that it is normally difficult for mavericks to prevail in society. As for the emergency evacuation of large crowds, it implies that if the relative rewarding for rational agents is not sufficiently high, agents who initially analyze the conditions rationally to avoid being blocked in the crowded exit will eventually follow the crowd.

Moreover, it can be seen from Fig. 1 that the stable proportion of subpopulation 1 will increase with the increment of $f$. However, compared to $b$, the influence of $f$ on the evolution result is weak due to the fact that $f$ plays a role only when $b$ lies near the critical points. Therefore, our numerical simulations indicate that initial fractions of the two subpopulations have little effects on the evolution result of large-scale population in emergency evacuation.

Notably, the evolution result becomes unstable and sensitive to randomness when parameter $b$ lies near critical points. In these cases, sometimes subpopulation 1 wins, sometimes subpopulation 2 survives, and sometimes the two groups coexist. For example, the results corresponding to $f = 0.9$ are depicted in Fig. 2. It can be obtained from Fig. 2 that, as a critical point $b = 1.4$ will lead to various probable proportions of subpopulation 1 in steady state, therefore it is difficult for us to judge the evolution result.

In addition to proportion of subpopulation 1, ratios of behavior 1 in subpopulations 1 and 2 respectively are also recorded. Figs. 3 and 4 exhibit the fraction of behavior 1 in subpopulation 1 after behavioral evolution becomes stable which varies with parameter $b$. In particular, in Fig. 3 each data point is averaged over 10 independent simulation runs, while in Fig. 4 the steady state fraction of behavior 1 in subpopulation 1 for each simulation in case of $f = 0.1$ is depicted.

It can be concluded from Figs. 3 and 4 that fraction of behavior 1 in subpopulation 1 after evolution when the value of $b$ is smaller than the critical point often deviates severely from 0.5. For the emergency evacuation of...
individuals crowded in a closed building with two symmetrically located doors, the results for behavioral evolution indicate that the prevailing of subpopulation 1 often leads to symmetry breaking in the use of two escaping exits and even herding effect, which will definitely decrease the evacuation efficiency.

In order to describe the evolution process in detail when \( b \) is relatively small, Fig. 5 depicts the time course of one numerical simulation in which the parameters are set as \( b = 1.2 \) and \( f = 0.5 \). According to the three curves in Fig. 5, it can be obtained that in this simulation subpopulation 2 becomes extinct since about the 300th MC step, and that all the individuals in subpopulation 1 adopt behavior 2 ever since about the 2000th MC step. In conclusion, the result of this computation simulation indicates all the entities ultimately choose escaping exit 2, which means herding effect arises in emergency evacuation.

Furthermore, Fig. 6 depicts fraction of behavior 1 in subpopulation 2 in steady state varying with game parameter \( b \), which is obtained by averaging over 10 independent simulations. According to Fig. 6 and the data for each simulation run, it can be established that ratio of behavior 1 in subpopulation 2 after evolution when subpopulation 2 wins ultimately is very close to 0.5 for each simulation, with the deviation less than 0.005. Therefore, the simulation results imply that if the mavericks who are rational in exit choice prevail at last, the two escaping exits will be adopted equally and hence the evacuation efficiency can be enhanced.

Interestingly, the result of behavioral evolution for the population becomes vulnerable and sensitive to randomness when the value of \( b \) approaches critical points, as mentioned above. To provide a vivid description of the property of critical points (phase transition points), Figs. 7–9 present a few snapshots of two evolution processes respectively, all of the three corresponding to the same parameter setting of \( b = 1.39 \) and \( f = 0.5 \). At the initial step in both simulations, the four types of agents, differing in which subpopulation they belong to and which behavior they choose, are placed on the square lattice randomly and equally. In the simulation depicted in Fig. 7, subpopulation 1 laughs last, with all the individuals adopting behavior 1 ultimately. Different from Fig. 7, the evolution process described in Fig. 8 ends with the coexisting of the two behaviors in subpopulation 1. In addition, form Fig. 8 it can be observed that agents adopting behavior 1 or behavior 2 aggregate together and form a big cluster, and the two clusters always compete with each other in the end. According to Fig. 8 and the results depicted in Fig. 4, it can be found that the behavioral co-existing in subpopulation 1, as a supplement of bi-stable tendency, is likely to happen, the reason for which may lie in the fact that the two behaviors form large clusters respectively and are divided into two sides by a boundary when subpopulation 2 disappears. However, according to the behavioral evolution process described in Fig. 9, it turns out that agents in subpopulation 1 disappear at last and when the evolution comes to an end the two behaviors are always alternating in subpopulation 2, with each one accounting for a half constantly. Notably, in the evolution process the two behaviors in subpopulation 1 normally gather in clusters respectively, as the clusters in blue and red in Figs. 7–9 indicate, while behaviors 1 and 2 in subpopulation 2 do not aggregate in clusters.

According to the evolution results shown in Figs. 7 and 8, it can be derived that \( b = 1.39 \) is a critical point of the system in which subpopulation 1 accounts for a half initially, and that the result of the behavioral evolution for the whole population will be difficult to decide when the relative rewarding for rational agents approaches the narrow range of critical points.

Concerning the game type represented in Table 2, by mean field analysis we can obtain that the result of behavioral evolution in subpopulation 1 should exhibit a bi-stable tendency. Yet, the co-existing of the two
behaviors in subpopulation 1 takes place sometimes, just as Figs. 4 and 8 depict. In order to avoid the co-existing phase, we have introduced a noise $\sigma$ in payoffs for agents in subpopulation 1. In detail, in the game model we presume \((1 + \sigma, 1 + \sigma)\) for (behavior 1, behavior 1) and \((1 - \sigma, 1 - \sigma)\) for (behavior 2, behavior 2), where $\sigma$ is a random variable obeying uniform distribution with mean 0. For the new model considering noise with suitable magnitude in payoffs, after numerical simulations we find that the bi-stable tendency in subpopulation 1 does emerge, namely, herding effect that all individuals choose behavior 1 or 2 happens in each simulation when subpopulation 1 wins. However, it turns out that introducing this noise does not affect the proportion of subpopulation 1 in steady state and the positions of the phase transition points, therefore the primary conclusions in the aforementioned text about which subpopulation wins and whether or not herding effect emerges are not altered.

### 4. Conclusions

In the work large crowds in emergency to escape from a closed building that has two symmetrically located exits are divided into two subpopulations, namely emotional group imitating the behavior of others and rational group choosing the escaping exit different from the choice of most people. According to the game models built and the simple evolution rule designed in this paper, the behavioral evolution of the whole population in emergency is simulated numerically. Several interesting results can be obtained through the computation simulations. First, if the relative rewarding for rational agents is not significantly larger than 1, all individuals will become emotional and the two escaping exits will be adopted asymmetrically ultimately, and even herding effect that all people use the same exit can take place. Second, the system exhibits a phase transition from only emotional agents left to only...
rational agents left who choose the exits evenly when the relative rewarding for rational agents increases monotonically and crosses the range of critical points. Interestingly, the result of behavioral evolution is unstable and sensitive to randomness near critical points. Third, for the evolutionary result of the population in emergency, the influence of initial proportion of one certain subpopulation is weak compared to that of the relative rewarding for rational agents. In addition, our model framework is able to explain the fact that it is difficult for mavericks to prevail in society. To sum up, from the perspective of evolutionary game theory, the results provided in this study shed new light on how to understand and repress herding effect in emergency, which may seriously decrease evacuation efficiency and cause great casualties.

Acknowledgments

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References

[34] Wakiyama M, Tanimoto J. Reciprocity phase in various 2 × 2 games by agents equipped with two-memory length strategy encouraged by grouping for interaction and adaptation. Biosystems 2011;103:93–104.