A Product Mix Decision Model Based on Time-Driven Activity-Based Costing with Capacity Expansion

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Abstract

This study proposes a mixed integer programming (MIP) model based on the time-driven activity-based costing (TDABC) while encountering numerous resource limitations and allowing overtime-working and machine-leasing scenarios. The proposed model not only can improve the problems of the multi-stage solution process and the discontinuation of long-term profitable products used in the Theory of constraints (TOC) model, but also can avoid the subjective estimation used in the activity-based costing (ABC) model and highlight the difference between capacity supplied and capacity used. Finally, some illustrative examples are provided to show the usefulness of the proposed TDABC product mix model and to compare the difference among TDABC, ABC and TOC from the resource-used-based profit, the resource-supplied-based profit and the cash flow.

Keywords: Product mix, time-driven activity-based costing (TDABC), integer programming, theory of constraints (TOC), activity-based costing (ABC)
1. Introduction

Product mix is an important policy for a manufacturer facing massive market demand but shortage in production capacity. With the company’s limited capacity, it is to allocate the resources for producing different products that can maximize the expected profit, given the consideration of market demand.

The two usual approaches to determine a product mix under capacity limitations are Theory of Constraints (TOC) and Activity-based Costing (ABC). A number of studies have discussed or applied either the ABC or the TOC system. Luebbe and Finch (1992) compared TOC and LP in solving a product mix problem and concluded that LP is better than TOC. Spoede et al. (1994) showed that TOC is better than ABC in achieving profit goal. Malik and Sullivan (1995) developed a MIP model based on the information provided by ABC. Kee (1995) pointed out that TOC can possibly bring up sub-optimal decisions and claimed that ABC would be better. Campbell et al. (1997) suggested that both ABC and TOC can work under different circumstances and they are complementary. Kaplan and Cooper (1998) said that TOC and ABC are not conflicting approaches, as long as the long-term or short-term aim can be told. Shapiro (1999) modeled the problem in mathematical programming (MP) by resource-based view. Kee and Schmidt (2000) have thought about that the decision of whether to adopt TOC or ABC depends upon the discretionary power of managers on controlling the labor cost and the manufacturing overhead cost. Gupta (2001) proposed a model to take into account ABC and TOC at the same time, with a goal of maximum profit improvement. Perkins et al. (2002) used a model built by Excel, TOC and ABC to solve the problem with multiple resource constraints. Gupta et al. (2002) integrated ABC and TOC under limited resource allocations and established a model to prioritize the possible process flow improvements. Souren et al. (2005) pointed out that TOC might obtain sub-optimal solutions under a multi-constraint and integer-solution setting. Kirche et al. (2005) demonstrated that ABC can perform better than TOC when the direct production cost is relatively small portion of the total production cost. Kee (2008) considered the product mix problem with resource flexibility. Gong and Hu (2008) developed a flexible model to solve the problem with uncertain concerns, showing that the bottleneck factor decides the degree of flexibility.

As an improvement to ABC, the Time-driven ABC approach (TDABC), in which cost allocation is based on total activities time, has been proposed (Kaplan and Anderson 2004, 2007). TDABC adopts a time driver from resource to the cost objects. Without relying on any human judgment, TDABC is an objective approach in that it skips the first stage of ABC that are time-costly and in particular, the estimation of activities’ time proportions. As can be observed, with TDABC, the employees could therefore concentrate more on the production time and thus the company could gain sustainable competitive advantages.
Pernot et al. (2007) provided a supporting evidence for TDABC, pointing out that ABC has two drawbacks: building the ABC system is quite costly and periodically updating the system is also costly, although it is the widely used approach by most university libraries. Varila et al. (2007) mentioned that some complicated activities cannot be measured by one single activity driver and thus ABC is inadequate. Kaplan and Norton (2008) stated that TDABC can be used to forecast the required capacity according to the defined sales goal for the future. Everaert et al. (2008) found that ABC oversimplifies 64% of the activities and tends to improperly allocate the indirect costs. Kaplan and Anderson (2004, 2007) claimed that with ABC, the determination of the cost driver rate is subjective, thus it is unable to identify the unproductive flows so as to improve the idle capacity. This results in the over-estimations of both unit-cost and sale price that can induce possible market loss.

Also, Öker and Adigüzel (2010) used an illustrative example of a manufacturer to indicate that TDABC is more informative than TCAS in that it provides a closer scrutiny to the profitability of the products and the utilization rate of the capacity. Gervais et al. (2010) showed the long-term benefit of appropriating TDABC with an example of a shipping company. Stout and Propri (2011) proved that TDABC can offer more precise information than ABC, with the case of a company in the electronics industry. Ratnatunga et al. (2012) compared TDABC with ABC, but they did not examine whether the unused capacity can be used to perform decision analysis. Tanis and Özyapici (2012) stated that TDABC can provide more reliable information in detail for the decision-making process. Mortaji et al. (2013) proposed a fuzzy TDABC model to support a decision with uncertain parameter estimations.

This study aims to apply TDABC and to propose a model for suggesting a product mix that is most profitable to a company with limited production resources. The proposed model is established by mixed-integer programming (MIP) based on the TDABC system. It can, precisely, attribute only the used resources to the cost objects and display the unused resources as reference information for decision purpose. Also, with salient features of TDABC, the model avoids any subjective cost estimation of activities’ time proportions so that it obtains a better optimal solution that might achieve higher profit than ABC. Moreover, as is the advantage of using any programming model, using MIP as the modeling tool, the decision maker (DM) is able to easily extend the model by appending some additional constraints depending on the real operational concerns.

2. Assumptions and Notations

In this study, it is assumed that material supply is always sufficient. That is, only the other two usual factors, machine capacity and the number of workers, are taken into account as constraints. It is also assumed that workers’ overtime and machine leasing, rather than machine purchase, outsourcing or new employment, are the two main policies taken by a manufacturer for a flexible production when the capacity is insufficient.
Parameters:

- $C_i$: per-hour cost of workers in machinery (of the production department);
- $C_m$: per-hour cost of machine operations (of the production department);
- $C_o$: per-hour order handling cost of employees (of the order-handling department);
- $C_s$: per-hour shipping cost paid to workers (of the shipping department);
- $C_e$: per-hour engineering design cost paid to employees (of the engineering department);
- $oh$: time required to process the orders per batch;
- $h_i$: time required for each shipping;
- $hp$: time required to assembly/packaging each unit of product;
- $U_i$: capacity of the production department measured in workers’ hours;
- $U_m$: capacity of the production department measured in machine hours;
- $U_o$: capacity of the order-handling department on handling orders;
- $U_s$: capacity of the shipping department;
- $U_e$: capacity of the engineering/design department;
- $FC$: the marketing and administrative cost;
- $CO_{ol}$: per-hour cost for one overtime worker hour;
- $C_k$: per-machine cost for leasing one additional machine;
- $hk$: hours able to produce per leased machine.

Variables:

- $IP$: sale price per product $i$ unit
- $is$: product quantity per shipping
- $ID$: product demand
- $CI$: raw material cost
- $h^d_i$: direct working hours required to produce per product unit
- $h^m_i$: machine hours required to produce per product unit
- $h^b_i$: the setup time required by the machine to manufacture product $i$
- $h^c_i$: the per-batch time required for moving materials to the production department
- $h^e_i$: the time spent on designing product $i$

- $X_i$: quantity of the $i$-th type product to be manufactured;
- $B_i$: # of batches to produce the $i$-th type product;
- $S_i$: the number of shipments to ship the $i$-th product;
- $\omega$: a binary (0-1) variable indicating whether to carry out over-time policy or not;
- $\zeta$: a binary (0-1) variable indicating whether to carry out machine-leasing policy or not;
- $z_i$: a binary (0-1) variable indicating whether product $i$ is to be produced or not.
- $K$: number of additional machines to be leased.
3. The Proposed Model

Firstly, with the overtime setting, the cost function of workers for the production department can be formulated as:

$$c_w(X, B, \omega) = C_i \times \sum (h_i^k \times X_i + h_i^b \times B_i) + (C_{ol} - C_i) \omega \times [\sum (h_i^k \times X_i + h_i^b \times B_i) - U_j],$$

s.t. $M (\omega - 1) \leq \sum (h_i^k \times X_i + h_i^b \times B_i) - U_j \leq M \omega$.

where $M$ is a large number.

As can be seen, when $\omega = 0$, $\sum (h_i^k \times X_i + h_i^b \times B_i) \leq U_j$ implies that no overtime is required and the value of the cost function is $c_w(X, B, \omega) = C_i \times \sum (h_i^k \times X_i + h_i^b \times B_i) = A$. But when $\omega = 1$, $\sum (h_i^k \times X_i + h_i^b \times B_i) \geq U_j$, overtime is to be carried out and at the same time, $c_w(X, B, \omega) = A + (C_{ol} - C_i) \omega \times [\sum (h_i^k \times X_i + h_i^b \times B_i) - U_j]$.

Secondly, with the machine-leasing setting, the cost function of machines costs (i.e., the costs for machine operation and setup) can be formulated as:

$$c_m(X, B, \zeta) = C_m \times [(\sum h_i^m \times X_i + \sum h_i^b \times B_i) \times (1 - \zeta) + U_m \zeta] + C_k \times K \times \zeta$$

s.t. $U_m \zeta \leq \sum h_i^m \times X_i + \sum h_i^b \times B_i \leq U_m + K \times h_k \times \zeta$.

As can be seen, when $\zeta = 0$, $0 \leq \sum h_i^m \times X_i + \sum h_i^b \times B_i \leq U_m$ implies that no additional machine is required and the cost function value is $c_m(X, B, \omega) = C_m (\sum h_i^m \times X_i + \sum h_i^b \times B_i)$. But when $\zeta = 1$, $\sum h_i^m \times X_i + \sum h_i^b \times B_i \geq U_m$, additional machines must be leased and in the meanwhile, $c_m(X, B, \omega) = C_m \times U_m + C_k \times K$.

Then, the objective function of the MIP model that is to be maximized is defined as:

Profit = (Sales revenue) – (Material cost + Direct labor cost + Labor setup cost + Machine cost + Machines’ setup cost + Order handling cost + Shipping cost + Design/engineering cost + Marketing and administration cost).

$$Profit = \sum P_i \times X_i - \sum C_i^f \times X_i - C_i \times \sum (h_i^k \times X_i + h_i^b \times B_i)$$

$$- (C_{ol} - C_i) \omega \times [\sum (h_i^k \times X_i + h_i^b \times B_i) - U_j]$$

$$- C_m \times [(\sum h_i^m \times X_i + \sum h_i^b \times B_i) \times (1 - \zeta) + U_m \zeta] - C_k \times K \times \zeta$$

$$- \sum (h_p + h_j) \times C_o \times B_i - \sum (h_i \times S_i + h_p \times X_i) \times C_s - \sum h_i^e \times C_e \times z_i - FC$$

Therefore, the proposed model is:

(The MIP Model)

Max Profit

s.t. \(X_i = b_i \times B_i\), for all \(i\)

$$M (\omega - 1) \leq \sum (h_i^k \times X_i + h_i^b \times B_i) - U_j \leq M \omega, \text{ for all } i$$

$$U_m \zeta \leq \sum (h_i^m \times X_i + h_i^b \times B_i) \leq U_m + K \times h_k \times \zeta, \text{ for all } i$$
\( \sum (h_i + h_{ei}) \times B_i \leq U_0 \) \hspace{1cm} (7)
\( \hat{X}_i = s_i \times S_i, \text{ for all } i \) \hspace{1cm} (8)
\( \sum (h_i \times S_i + h_{ei} \times X_i) \leq U_e \) \hspace{1cm} (9)
\( \sum h_{ei} \times z_i \leq U_e \) \hspace{1cm} (10)
\( 0 \leq X_i \leq D_i \times z_i, \text{ for all } i \) \hspace{1cm} (11)

where \( X_i, B_i, S_i \) and \( K \) are the decision variables that are non-negative integers; \( z_i, \omega \) and \( \zeta \) are binary (0-1) variables; \( M \) is a large number.

In the above model, except that the Profit term to be maximized is as defined by Eq. (3), the equations from (4) to (11) are constraints. Eq. (4) is a hard constraint to ensure that the manufactured product quantity, \( X_i \), is a multiple of \# batches. Eq. (5) determines the value of \( \omega \), which represents a decision for whether to ask the workers for overtime working or not, according to Eq. (1). Eq. (6) determines the value of \( \zeta \), which represents a decision for whether to carry out machine-leasing or not, according to Eq. (2). Eq. (7) avoids the order handling activities (done by the order-handling department) from exceeding capacity limits; Eq. (8) is a hard constraint to ensure that \( X_i \) is a multiple of the number of shipments. Eq. (9) prevents the shipping activities (by the shipping department) from exceeding the capacity limit and Eq. (10) limits the design/engineering activities with the possible upper bound. Eq. (11) guarantees that the quantity to be produced for each product type does not exceed the demand.

The MIP model is constructed based on the TDABC system to solve for a product mix by mixed integer programming. For a linearization of the whole model, constraints that contain 0-1 binary variables can be further linearized.

**4. Numerical Example**

To demonstrate how the proposed MIP model works, an illustrative example is given. Suppose that a company manufactures and sells three kinds of products, namely, Types 1, 2 and 3, as shown in Table 1. Type 1 product is a normal consumer product with a mass amount of demand. It requires least material cost, working hours, setup time and design/engineering time. And the production department can produce most Type 1 products per batch but the per-unit machine hour is longest. In contrast, the market demands Type 3 product far lesser than Type 1 but it requires most working hours per unit and setup time. The material cost, the sale price and the required resources of Type 2 product are middle.

<table>
<thead>
<tr>
<th>Product Type</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Cost /Unit</td>
<td>$16</td>
<td>$20</td>
<td>$22</td>
</tr>
<tr>
<td>Direct Labor Hours /Unit</td>
<td>0.3</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>Machine Hours /Unit</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Setup Hours /Batch</td>
<td>5</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Produced #Products /Batch</td>
<td>360</td>
<td>125</td>
<td>50</td>
</tr>
<tr>
<td>Shipped #Products /Shipping</td>
<td>180</td>
<td>125</td>
<td>25</td>
</tr>
</tbody>
</table>

*Table 1. Basic data for the 3 types of product*
The allocation rates of the resources about this example case are as shown in Table 2.

Table 2. The allocation rates of the resources

<table>
<thead>
<tr>
<th>Resource</th>
<th>Capacity (Hours)</th>
<th>Cost ($)</th>
<th>Allocation Rate (/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Works</td>
<td>12,240</td>
<td>397,800</td>
<td>32.5</td>
</tr>
<tr>
<td>Machine</td>
<td>14,880</td>
<td>334,800</td>
<td>22.5</td>
</tr>
<tr>
<td>Order Handling</td>
<td>260</td>
<td>7,800</td>
<td>30</td>
</tr>
<tr>
<td>Shipping</td>
<td>4,550</td>
<td>136,500</td>
<td>30</td>
</tr>
<tr>
<td>Design/Engineering</td>
<td>960</td>
<td>78,000</td>
<td>81.25</td>
</tr>
</tbody>
</table>

The additional parameters are as follows: (I-1) The shipping department is in charge of packaging and shipping. The packaging time per product unit is 8 minutes for all types of products. The shipping time is 50 minutes per shipping process. (I-2) The marketing and administration cost of the company is $350,000. (I-3) Overtime wage is $48.75 per working hour. (I-4) Machine leasing fee is $8,400 per machine. Each leased machine offers 240 effective production hours.

By taking the proposed MIP model, the case problem can be formulated as follows:

(Model MIP-TDABC)

\[
\begin{align*}
\text{Max} \quad & \text{Profit} = 65X_1 + 80X_2 + 110X_3 - 16X_1 - 20X_2 - 22X_3 \\
& \quad - 3.25 \times (0.3X_1 + 0.5X_2 + 0.6X_3 + 5B_1 + 6B_2 + 12B_3) \\
& \quad - 16.25 \times \omega \times (0.3X_1 + 0.5X_2 + 0.6X_3 + 5B_1 + 6B_2 + 12B_3 - 12240) \\
& \quad - 22.5 \times [(0.5X_1 + 0.4X_2 + 0.3X_3 + 5B_1 + 6B_2 + 12B_3) \times (1 - \zeta) + 14880\zeta] \\
& \quad - 8400 \times K \times \zeta - 15 \times 30 \times (B_1 + B_2 + B_3)/60 - 30 \times (50S_1 + 55S_2 + 60B_3)/60 \\
& \quad - 30 \times (50S_1 + 8X_1)/60 - 30 \times (50S_2 + 8X_2)/60 - 30 \times (50S_3 + 8X_3)/60 \\
& \quad - 81.25 \times (60z_1 + 240z_2 + 600z_3) - 350000
\end{align*}
\]

s.t.

\[X_1 = 360B_1\] (13)
\[X_2 = 125B_2\] (14)
\[X_3 = 50B_3\] (15)
\[M(\omega - 1) \leq 0.3X_1 + 0.5X_2 + 0.6X_3 + 5B_1 + 6B_2 + 12B_3 - 12240 \leq M\omega\] (16)
\[14880\zeta \leq 0.5X_1 + 0.4X_2 + 0.3X_3 + 5B_1 + 6B_2 + 12B_3 \leq 14880 + 240 \times K\zeta\] (17)
\[15 \times (B_1 + B_2 + B_3) + 50B_1 + 55B_2 + 60B_3 \leq 260 \times 60\] (18)
\[X_1 = 180S_1\] (19)
\[X_2 = 125S_2\] (20)
\[ X_3 = 25S_3 \]  
\[ \left[ 50 \times (S_1 + S_2 + S_3) + 8 \times (X_1 + X_2 + X_3) \right] / 60 \leq 4550 \]  
\[ 60z_1 + 240z_2 + 600z_3 \leq 960 \]  
\[ X_1 \leq 18000z_1 \]  
\[ X_2 \leq 10000z_2 \]  
\[ X_3 \leq 2500z_3 \]

(21)  
(22)  
(23)  
(24)  
(25)  
(26)

where all symbols are as defined previously.

By solving the MIP-TDABC model by LINGO (Schrage 2002), an optimal solution connoting an optimal decision when the objective function achieves its maximal value (i.e., \( \text{Profit} = $354,313 \)) is obtained, which is:  
\[ (X_1, X_2, X_3)^* = (18000, 10000, 2500), \quad (B_1, B_2, B_3)^* = (50, 80, 50), \quad (S_1, S_2, S_3)^* = (100, 80, 100), \quad (z_1, z_2, z_3)^* = (1, 1, 1), \quad \omega, \zeta)^* = (1, 1) \]  
and \( K^* = 1 \). The solution indicates that all the three types of products should be manufactured with their proper quantities (i.e., \( X_1, X_2, X_3)^* = (18000, 10000, 2500) \) to be manufactured. Besides, overtime working is required and machine-leasing must be carried out, while one additional machine is to be leased (i.e., \( K^* = 1 \)).

To understand the expected profitability of each type of product and to show the utilization of the various resources, the budgeted income statement is shown in Table 3.

<table>
<thead>
<tr>
<th>Cost Item</th>
<th>Type 1 Product</th>
<th>Type 2 Product</th>
<th>Type 3 Product</th>
<th>Capacity Expansion</th>
<th>Resource Used</th>
<th>Unused Capacity</th>
<th>Resource Supplied</th>
<th>Utilization Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Revenue</td>
<td>$1,170,000</td>
<td>$800,000</td>
<td>$275,000</td>
<td>$2,245,000</td>
<td></td>
<td>$2,245,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct Material Cost</td>
<td>288,000</td>
<td>200,000</td>
<td>55,000</td>
<td>543,000</td>
<td></td>
<td></td>
<td>543,000</td>
<td></td>
</tr>
<tr>
<td>Direct Labor Cost</td>
<td>175,500</td>
<td>162,500</td>
<td>48,750</td>
<td>402,837</td>
<td>0</td>
<td>402,837</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>Machine Runtime Cost</td>
<td>202,500</td>
<td>90,000</td>
<td>16,875</td>
<td>3,900</td>
<td>313,275</td>
<td>313,275</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>Labs' Setup Cost</td>
<td>8,125</td>
<td>15,600</td>
<td>19,500</td>
<td>43,225</td>
<td></td>
<td>43,225</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine Setup Cost</td>
<td>5,625</td>
<td>10,800</td>
<td>13,500</td>
<td>29,925</td>
<td></td>
<td>29,925</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order Handling</td>
<td>1,625</td>
<td>2,800</td>
<td>1,875</td>
<td>6,300</td>
<td>1,500</td>
<td>7,800</td>
<td></td>
<td>81%</td>
</tr>
<tr>
<td>Design/Engineering</td>
<td>4,875</td>
<td>19,500</td>
<td>48,750</td>
<td>73,125</td>
<td>4,875</td>
<td>78,000</td>
<td></td>
<td>94%</td>
</tr>
<tr>
<td>Packaging/Shipping</td>
<td>74,500</td>
<td>42,000</td>
<td>12,500</td>
<td>129,000</td>
<td>7,500</td>
<td>136,500</td>
<td></td>
<td>95%</td>
</tr>
<tr>
<td>Gross Margin</td>
<td>$409,250</td>
<td>$256,800</td>
<td>$58,250</td>
<td>$704,313</td>
<td>$690,438</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Administration Cost</td>
<td></td>
<td>$350,000</td>
<td></td>
<td>$350,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating Income</td>
<td>$354,313</td>
<td>$340,438</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on the results, although there are still unused capacity, the suggested product mix decision does raise the rate of capacity utilization. While a total cost of $13,975 for those unused capacities is shown, the top management can watch this fact and see if such situation has occurred for a long time, and then decide whether, for example, to shift part of the employees from the order-handling department to the production department so that the worker’s overtime cost can be further reduced. Note that the operating incomes calculated based on resource-used and resource-supplied views are, also, shown different.
5. Comparison and Implications

The differences among these three systems can be salient from applying them individually to solve the same product mix decision problem and observing the results.

For this purpose, the problem case in Section 4 is extended with the following settings (i.e., (a) to (d)). It is extended for the reason that with ABC, the time proportions for how much the various department resources are put into each activity should be estimated before the second stage cost-allocation can begin. These works not only demonstrates the additional and burdensome jobs when taking ABC, but also form a base to further comparisons in that all drivers adopted by every approach are duration-typed.

(a) The order-handling department spends 25% of the time on order-handling activities and 75% on moving the required materials from the warehouse to the production site.
(b) In the production department, in average, the workers spend 85% of their time on manufacturing the products while the rest 15% is on setting up the machines.
(c) For any machine, in average, 90% of the total operation time is for production while the setup of it takes 10%.
(d) The shipping department spends 90% of its time in packaging and 10% in delivering.

The product mix problem is solved with the ABC system at first. Before the second stage of ABC can begin, part of the constraint equations, Eqs. (16) – (18) and Eq. (22) in MIP-TDABC, must be reformulated according to those ‘time proportions’, as follows:

\[
M (\omega - 1) \leq 0.3X_1 + 0.5X_2 + 0.6X_3 - 12240 \times 0.85 \leq M \omega
\]  
\[
5B_1 + 6B_2 + 12B_3 \leq 12,240 \times 0.15
\]  
\[
14880 \times 0.9 \times \zeta \leq 0.5X_1 + 0.4X_2 + 0.3X_3 \leq 14880 \times 0.9 + 240 \times K \zeta
\]  
\[
5B_1 + 6B_2 + 12B_3 \leq 14,880 \times 0.1
\]  
\[
15 \times (B_1 + B_2 + B_3) \leq 260 \times 60 \times 0.25
\]  
\[
50B_1 + 55B_2 + 60B_3 \leq 260 \times 60 \times 0.75
\]  
\[
50 \times (S_1 + S_2 + S_3)/60 \leq 4,550 \times 0.1
\]  
\[
8 \times (X_1 + X_2 + X_3)/60 \leq 4,550 \times 0.9
\]

And by doing so, a new model, which is named MIP-ABC, is established. Note that the objective function of MIP-ABC is the same as that of MIP-TDABC. Next, the same problem is modeled and solved with the TOC system. Due to the fact that TOC takes a resource-supplied view, which is different from the resource-used view taken by TDABC, the only difference between TOC and TDABC is in their objective function.

By solving the abovementioned models, MIP-TOC and MIP-ABC, results in similar forms as those solved by MIP-TDABC in Section 4, are obtained. In this subsection, only the meaningful parts of these results are summarized in Table 4 for comparison.
Table 4 Solutions and expected profits by the models

<table>
<thead>
<tr>
<th>Product Mix Decision</th>
<th>MIP-TOC</th>
<th>MIP-ABC</th>
<th>MIP-TDABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>18,000</td>
<td>18,000</td>
<td>18,000</td>
</tr>
<tr>
<td>$X_2$</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>$X_3$</td>
<td>2,500</td>
<td>2,500</td>
<td>2,500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Product-Mix Profits</th>
<th>MIP-TOC</th>
<th>MIP-ABC</th>
<th>MIP-TDABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource-used-based profit</td>
<td>340,438</td>
<td>341,245</td>
<td>$354,313</td>
</tr>
<tr>
<td>Cost of unused resource</td>
<td>0</td>
<td>33,875</td>
<td>13,875</td>
</tr>
<tr>
<td>Resource-supplied-based profit</td>
<td>340,438</td>
<td>307,370</td>
<td>340,438</td>
</tr>
<tr>
<td>Adjust the non-cash item</td>
<td>334,800</td>
<td>334,800</td>
<td>334,800</td>
</tr>
<tr>
<td>Cash flow</td>
<td>675,238</td>
<td>642,170</td>
<td>675,238</td>
</tr>
</tbody>
</table>

As can be observed, the decided product mixes are just the same, although MIP-TOC takes a resource-supplied view but either MIP-ABC or MIP-TDABC takes the resource-used view. Note that the obtained machine runtime cost and the direct labor cost are not shown and they are actually the same for each column because the solved mixes are identical.

However, the expected profit is varying among the three approaches and there are two main and interesting reasons for such a fact.

Firstly, the degree to which the unused resources are controlled results in the difference of the expected profits assessed by MIP-TOC and MIP-TDABC. TDABC assumes that all the resources are controllable and the MIP-TDABC model always tries to depress the final level of unused capacity by shifting the existing unused capacities to other activities, to make the profit number greater. In contrast, TOC assumes that the resources are uncontrollable and all the unused resources are viewed as period cost. These results in the different expected profit by MIP-TOC and MIP-TDABC.

Secondly, the difference in the expected profits assessed by MIP-ABC and MIP-TDABC is simply because that in fact, MIP-ABC and MIP-TDABC have two different sets of constraints. MIP-ABC has more constraints than MIP-TDABC, which have required a time-costly investigation and lead to more ‘limitations’ during model solving. This is true from the comparison in that the expected profit by MIP-ABC, $341,245, is lesser than $354,313, which is assessed by MIP-TDABC. This implies that TDABC, compared with ABC, can be more efficient in calculation and more efficacious in determining a proper product mix to pursue a better profit.

6. Conclusion

This study proposes a model that takes the programming approach based on the TDABC system to decide a suitable product mix. The proposed model, compared with TOC, allows the scenarios of over-time working and machine leasing, obtains a long-term decision instead of a sub-optimal solution which is to discontinue producing a product that is long-term profitable. It also avoids solving the problem with multiple stages until all constraints are met, which is required by TOC while numerous resource limitations are present. In addition, taking the model avoids obtaining different product-mix decisions subject to human
judgments, as is the possible case of ABC, which also tends to neglect the potential unused capacity. The proposed, TDABC-based MIP model is not only able to attribute the unused resources to the relevant cost objects, but also to provide significant information about the idle capacities to the DMs. Moreover, except for suggesting an appropriate product mix decision, the MIP model automatically manifests a decision pertaining to overtime working and machine leasing, helping DMs in making a proper production plan when the existing capacities are insufficient to meet the orders.

References


