Fuzzy JPDAF Approach for Vehicle Tracking in Road Situation

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Abstract - Data association was an important content in Multi-target tracking. Typical algorithms to deal with such problems are the joint probabilities data association filter (JPDAF) proposed by Bar-Shalom and his team. The basis of JPDAF is the calculus of the joint probabilities between the measurements and the tracks. The algorithm assigns weights for reasonable measurements and uses a weighted centroid of those measurements to update the track. In this paper, a new weight assignment method based on fuzzy c-means methodology was proposed, and the general methodology of JPDAF remains unchanged. This leads to a fruitful combination between fuzzy and probability approaches. It is proved that the method is simple and fast by simulation, and suits for automotive radar multi-target tracking.

Keywords: Multi-target tracking, data association, fuzzy clustering

1 Introduction

A significant problem in multiple target tracking is the measurement-to-track data association. The problem was derived from Sittler’s work [1], and there have many different algorithms. Typical algorithms to deal with such problems are the probabilities data association filter (PDAF) in case of single target tracking and the joint probabilities data association filter (JPDAF) in case of multi-target tracking proposed by Bar-Shalom and his team [2][3]. The basis of JPDAF is the calculus of the joint probabilities between the tracks and the measurements. The algorithm assigns weights for reasonable measurements and uses a weighted centroid of those measurements to update the track. However, the calculation of joint probabilities in JPDAF seems complicated even if the used formulas are well established in probability theory, this motivates some authors to investigate simplifications of these formulations. This includes, for instance, Fitzgerald’s ad hoc proposal [4], which seems to work well in some cases of crossing targets, and the computation cost is smaller than standard JPDAF method. Another weak of JPDAF is that the JPDAF method tends to result in track convergence for closely spaced targets [5], and in road situation, the targets are close to each other.

In this paper, a new weight assignment method based on fuzzy c-means methodology was proposed, the joint probability in JPDAF method was substituted by the fuzzy membership, but the general methodology of JPDAF remains unchanged. This leads to a fruitful combination between fuzzy and probability approaches. To overcome the weak of JPDAF method, decision logic was introduced. It is proved that the method is simple and fast by simulation, and suits for automotive radar multi-target tracking.

This paper is organized as follows. Section 2. introduces problem description and JPDAF method. Section 3. provides the basics of fuzzy clustering means algorithm, the possibility using FCM for data association, and the realization of the proposed method. Section 4 the simulation results of four targets tracking in clutter are given. Section 5 presents the conclusions.

2 Problem description and JPDAF methodology

Suppose there are multiple targets in front of our vehicle, the discrete state equation of each target model is described by:

\[ x'(k+1) = A'(k)x'(k) + B'(k)w'(k) \quad (t = 1, 2, \ldots, T) \]  (1)

Measuring the position and state of targets by using the laser radar or millimeter wave radar in vehicle, the measurement equation is:

\[ z(k) = H'(k)x'(k) + v'(k) \]  (2)

Where \( x'(k+1) \) is the n-dimensional state vector at time \( k \), represents the relative position, relative velocity and relative acceleration between the targets and the vehicle. \( A'(k) \) is the transition matrix of target \( k \); \( z(k) \) is the measurement vector of target \( k \) at time \( k \), \( H'(k) \) is measurement gain matrix of target \( k \), \( w'(k) \) and \( v'(k) \) are process noise and measurement noise of target \( k \), which are independent zero mean and known variance.

\[ E[w'(k)(w'(j))^\top] = Q'(k)\delta_{kj} \]  (3)

\[ E[v'(k)(v'(j))^\top] = R'(k)\delta_{kj} \]  (4)

Where \( \delta_{kj} \) is the Kronecker delta function.

The set of validated measurement (target \( t \)) received at time \( k \) is defined as: \( Z'(k) = \{z'_t(k)\}_{t=1}^{N_k} \), \( m_k \) is the
numbers of validated measurement, that is there are \( m_k \)
measurement inside the tracking gate. If we use kalman filter to estimate the state of targets, then the update equation is:
\[
\hat{x}'(k|k) = \hat{x}'(k|k-1) + W'(k)v'(k) \quad t=1,2,...,T
\]
Where \( \hat{x}'(k|k) \) is the state estimation of target \( t \) at time \( k \), and \( \hat{x}'(k|k-1) \) is the state prediction of target \( t \), \( v'(k) \) is known as the combined innovation, which is given as:
\[
v'(k) = \sum_{j=1}^{m} \beta_j(k)v_j'(k), \quad t=1,2,...,T
\]
Where \( \beta_j(k) \) is the joint probability between the measurement \( j \) and the target \( t \), \( v_j'(k) \) is the innovation between the measurement \( j \) and the target \( t \).
\[
v_j'(k) = z_j'(k) - \hat{z}'(k|k-1)
\]
\[j=1,2,...,m \quad t=1,2,...,T\]
\[z_j'(k) \text{ represents the } k \text{th measurement of target } t \text{ at time } k, \hat{z}'(k|k-1) \text{ is the prediction measurement of target } t. \]
\[W'(k) = \text{ the gain matrix of kalman filte :}
\]
\[W'(k) = P'(k|k-1)H^T [H'P'(k|k-1)H'^T + R'(k)]^{-1}
\]
The error covariance associated with the estimation Eq.5 is:
\[P'(k|k) = \beta_j(k)\mu_j(k)P_j(k|k-1) + (1-\beta_j(k))\mu_j(k) + P_0(k)
\]
\[P_j(k|k) = [1-W'(k)H(k)]\mu_j(k)P_j(k|k-1)
\]
\[P_0(k|k) = W'(k)[\sum_{j=1}^{m} \beta_j(k)v_j'(k)\mu_j(k) - v_j'(k)v_j'(k)]W'(k)
\]
The predictions of error covariance, state and measurement are:
\[P'(k|k-1) = A'(k-1)P'(k|k-1)A'(k-1)^T + Q
\]
\[\hat{x}'(k|k-1) = A'(k-1)\hat{x}'(k-1|k-1)
\]
\[\hat{z}'(k|k-1) = A'(k)\hat{x}'(k|k-1)
\]
The constrain of joint probability for target \( t \) is:
\[\sum_{j=0}^{m} \beta_j(k) = 1
\]
Where \( \beta_j(k) \) represent the probability of no measurement associated with the target \( t \).

The general approach to calculate the joint probability in JPDAF methodology is: firstly determining the validated matrix according to gate techniques, then calculating the conditional probability of all possible feasible events, and finally the posterior probability \( \beta_j(k) \) is gotten[3]. When the number of targets is big, the compute cost in making the feasible events is too big for some applications, so some sub-optimize JPDAF method were introduced, among them, Fitzgerald[4] gave an cheap JPDAF method, and the formula of joint probability is:
\[
\beta_j(k) = \frac{G_j}{\sum_{j=1}^{m} G_j - G_j + B}
\]
\[G_j = N[v_j(k)] = \frac{1}{|S_j'(k)|^{1/2}} \exp\left\{ -\frac{1}{2} v_j(k)^T S_j'(k)^{-1} v_j(k) \right\}
\]
Where \( B \) is a constant.
However, the computation cost of the cheap JPDAF method is also too big for some applications such as target tracking in road situation, in order to handle the multiple target matching problem in automotive radar tracking, we tried to use fuzzy c-means methodology in data association by using the fuzzy membership to get the weight for state estimation, and compared with the cheap JPDAF method, the result is satisfied.

3 Realization of data association based on fuzzy c-means methodology

3.1 Basics of fuzzy c-mans methodology

The most widely used clustering algorithm is the fuzzy clustering means algorithm (FCM) developed by Bezdek [6]. The clustering algorithms produce a degree (grade) of membership for each data in each cluster. Unlike conventional clustering, which involves a partitioning of objects into disjoint clusters, fuzzy clustering allows a data point \( x \) to have a partial degree of membership in more than one set. This may represent the reality distribution of data.

The fuzzy c-means functional given by Bezdek is:
\[
J_n(U, V) = \sum_{i=1}^{n} \sum_{j=1}^{c} u_{ij}^m d_{ij}^2
\]
subject to:
\[
\left\{ \begin{array}{l}
\sum_{j=1}^{c} u_{ij} = 1 \quad \forall i \quad (1 \leq i \leq n, 1 \leq j \leq c) \\
u_{ij} \in [0,1] \quad \forall i, j
\end{array} \right.
\]
Where \( X = \{x_1, x_2, ..., x_n\} \subset \mathbb{R}^o \) is the set of data, \( n \) is the number of data, \( U = \{u_{ij}\} \subset M \) stands for the fuzzy c-means partition of \( X \), \( u_{ij} \) represents the degree of
membership of data point \( i \) in fuzzy cluster center \( j \),
\[ V = \{v_1, v_2, ..., v_n\} \subset \mathbb{R}^p \] is cluster center vector, \( c \)
stands for the number of clusters. (\( 2 \leq c \leq n \)),
\[ d_{ij} = \|x_i - v_j\| \] stands for the distance from the
datum \( x_i \) to the cluster center \( v_j \), \( m \in [1, \infty) \) refers to a
parameter that models the degree of fuzziness, called the
fuzzification constant (or weight exponent).

In Eq.18 the function \( J_m(U, V) \) represents the sum
of the squared errors weighted by the \( m \)th power of the
corresponding degree of membership, the goal of the
fuzzy clustering algorithm is to determine the optimum
degrees of membership \( u_{ij} \) and the optimum fuzzy cluster
centers \( v_j \), such that the sum of the square
errors \( J_m(U, V) \) is minimum. The results are given as
below:

\[
u_{ij} = \frac{1}{\sum_{k=1}^{n} (d_{ij} / d_{ik})^{2/(m-1)}} \quad \forall i, j \quad (20)
\]

\[
v_j = \frac{\sum_{i=1}^{n} (u_{ij})^m x_i}{\sum_{i=1}^{n} (u_{ij})^m} \quad \forall j \quad (21)
\]

In multi-target multi-sensor tracking system, \( c \) is the
number of targets, \( n \) is the total number of received
measurements, \( x_i \) is the \( s \)-dimensional measurement
vector \( (i = 1, 2, ..., n) \) and \( v_j \) is the \( s \)-dimensional
predicted vector for target \( j \) \( (j = 1, 2, ..., c) \). The fuzzy c-
means clustering algorithm or the Picard algorithm is
guaranteed to converge to a local minimum.

### 3.2 Possibility of substitution

Clearly, the meaning of the grade \( u_{ij} \) as a degree of
agreement between the datum \( i \) and the cluster \( j \), as well
as the fulfillment of a constraint as Eq.19, makes it a
potential candidate and an alternative to \( \beta^t_j \) in JPDAF.

But there are some different between Eq.15 and Eq.19.
In Eq.19, the sum of the memberships between
measurement \( i \) and all the clusters is equal to 1, and in
Eq.15, the sum of joint probabilities between
target \( j \) and the valuated measurements is equal to 1, so it
must be taken a transformation before substitution.

Strictly speaking, the fact of substituting joint
probabilities by other values even outside the framework
of probability in the same process is not quite strange as it
may sound at first glance. Indeed, the justification of such
substitution can be carried out from different viewpoints:
(i) the existence of different proposals like Fitzgerald’s ad
hoc formulations for the joint probabilities proves the
non-uniqueness of the solution. (ii) The interpretation of
the joint probability quantities as degrees of agreement
between measurements and targets means that both joint
probabilities and membership grades have the same
interpretative setting. (iii) The general constraints
governing the construction of the joint probabilities are
kept preserved in the fuzzy setting. So, we can conclude
that it is possible to substitute the probability by
membership.

### 3.3 Generation of weight matrix

The data association problem between the measurement
and the tracks is an assignment problem in reality,
assigning the measurements to the tracks at time \( k \). If the
predictions of the targets are looked as the cluster center
vector, the measurements are looked as the data to be
partitioned, then the association problem may transform to
the cluster partition problem. It is different from the
conventional cluster problem in that the cluster center and
membership are calculated by prediction technique (such
as kalman filter), not by recursion technique. In order to
satisfy the constrains of Equ.15, the membership matrix
\( U = \{u_{ij}\} \) is reassigned according to the follow logic,
and only assigning two measurements to one target for
reducing the computation complexity.

The memberships in membership matrix \( U = \{u_{ij}\} \)
are re-expressed according to the following decision logic.
From the memberships at each target, two maximum
values are selected, one corresponds the dominant
measurement, the other the minor measurement. The
values of the membership of the target \( t \) being associated
with each measurements can be rescaled by a factor \( \alpha \)
\( (\alpha > 1) \), which is used to avoid the track coalescence
of neighbored tracks. If a measurement is used as the
dominant measurement of any track, the weight of the
measurement for other tracks is reduced by \( \alpha \). The
weight matrix \( \hat{U} = \{\hat{u}_{ij}\} \) is reconstructed by using the
normalization constant. And if there no measurement in
the gate of target \( t \), then \( \hat{u}_{0t} = 1 \), otherwise \( \hat{u}_{0t} = 0 \).
The constraints will become:

\[
\sum_{j=0}^{m_t} \hat{u}_{jt} = 1 \quad (t = 1, 2, ..., T) \quad (22)
\]

It is same with Eq.15 in form, and then it is possible
to substitute \( \beta^t_j \) by \( \hat{u}_{jt} \).

### 3.4 Application FCM for target tracking

Now we are able to address the whole algorithm mixing
JPDAF scheme and the new elaborated modified FCM
algorithm called fuzzy JPDAF algorithm.

Step 1. Starting from previous estimation
\( X(k-1|k-1) \) and \( P(k-1|k-1) \) for \( t = 1 \) to \( T \),
determine the prediction \( X'(k|k-1) \) and
\( P'(k|k-1) \) using Eq.12 and Eq.13.
Step 2. Calculating partition matrix $U$ using fuzzy c-means, which represents the membership between the prediction vector $V = \{v_1, v_2, ..., v_c\}$ and the measurement vector $X = \{v_1, v_2, ..., v_n\}$.

Step 3. Determining maximum matrix from $U_m$, finally the weight matrix $\hat{U}$ is gotten by making a transformation and normalization.

Step 4. Updating the state and covariance by using Equ.5 to Equ.11.

Step 5. Repeat Steps 1–4 for different time increments.

4 Simulation results

The aim in this section is to test the performance of the proposed method, and compared with the cheap JPDAF method. The measurement data of the range and the velocity were generated by adding random noise and clutter to the true track measurements. The model trajectories of the four tracks[7] are shown in Fig.1 along with the noisy range and clutter measurements (with the line represents the actual track, and "*" represents measurements, "." represents clutter). The four targets have the same state transition and measurement transition matrices, the actual trajectories have the constant velocities of -4.2m/s, -5m/s, 5m/s, and 4m/s, and the initial ranges are 95m, 92m, 70m, and 65m. Using millimeter wave radar detects the position of targets, the deviation of the measurement noise has been selected as $\sigma^2 = 0.5m^2$, and the sample interval is $T = 0.1s$.

Where $x(k), \hat{x}^j(k)$ stand for the true state of target and its estimation in the running $j$th at time $k$. $M$ is the times of Monte Carlo running.

The definition of TM error is:

$$T = \frac{1}{N} \sum_{k=1}^{K} e(k)$$

(24)

Where $N$ represents the sample number.

![Fig2. Position error of cheap JPDAF method](image)

![Fig3. Position error of proposed method](image)

![Fig4. Velocity error of cheap JPDAF method](image)
The monte carlo simulation has been tried for M=50. Fig.2 and Fig.3 depict the RMS position error, and Fig.4 and Fig.5 depict the RMS velocity error. $T_j$ represents the TM error of target $j$. From the above results, we can conclude that the RMS error of the proposed method almost equal the error of the cheap JPDAF method, but the computational complexity has decreased, and it needn’t know the prior knowledge such as detection probability and clutter density etc.

5 Conclusion

The accurate tracking of multiple vehicles is a very important consideration in the design of active safety system. In this paper, an improved JPDAF method was introduced; the proposed approach is based on the fuzzy clustering means algorithm, substituting the joint probabilities by memberships, while maintaining the advantage of the JPDAF method in the presence of heavy clutter. Performance evaluation has been done and compared to the cheap JPDAF method by simulation. The new approach requires fewer computations than the cheap JPDAF method.

6 References


