On Some Spectrum Estimation Methods for Analysis of Non-stationary Signals in Power Systems
Part II: Numerical Applications

A. Bracale, G. Carpinelli, Member, IEEE, D. Lauria, Z. Leonowicz, T. Lobos J. Rezmer

Abstract: This is a companion paper to a paper of the same title, Part I, in which the theoretical aspects of some spectrum estimation methods for analysis of non-stationary voltages and currents in power systems with nonlinear loads are analysed. In particular, the Prony model and rootMusic method are taken into account. In this paper, numerical applications of the methods presented in Part I are discussed, in order to illustrate their implementation problems and practical applications. To investigate the methods, several experiments were performed using non-stationary voltage and current waveforms in a supply system of a dc arc furnace. For comparison, similar experiments were repeated using the Short-Time Fourier Transform. The comparison proved the effectiveness of the new methods; however, their computational efforts can be much more complex. Conclusions and recommendations are made concerning the framework of each method.

Keywords: Dc Arc Furnaces Measurements, Waveform Distortion Analysis, Subspace Methods, Prony Method.

I. INTRODUCTION

A companion paper [1] has presented the theoretical aspects for the spectrum estimation of non-stationary voltage and current waveforms in electrical power systems with nonlinear loads. In particular, it has provided for the spectrum estimation by Prony method, where the sampled data are represented as a linear combination of exponentials, and by rootMusic method, where the waveforms are modelled as a sum of sinusoids in the background of noise of known covariance function.

In this paper, several numerical applications of the methods presented in the companion paper [1] are effectuated, in order to illustrate implementation problems and practical applications of each method. The illustrative examples refer to non-stationary voltage and current waveforms in a supply system of a dc arc furnace.

The results obtained with Prony and rootMusic methods are compared with the ones obtained with the Short-Time Fourier Transform [2].

The comparison is effected on the basis of the values of the IEC harmonic and interharmonic subgroups introduced by the Standards drafts [3-4], of the Total Harmonic Distortion, of the Load Nonlinearity Indicator and, finally, of spectral component frequency time variation.

The paper is organized such that the characteristics of the supply system of the actual dc arc furnace to which the numerical applications refer are briefly presented at first. This is followed by some considerations about the assumptions adopted in order to apply the considered spectrum estimation methods to the recorded current and voltage waveforms. Then, the numerical experiments are effectuated. Finally, considerations are given about the result accuracy and the computational efforts each method suffers.

II. DC ARC FURNACE

A typical dc arc furnace plant is shown in Fig. 1. It consists of a dc arc connected to a medium voltage ac busbar with two parallel thyristor rectifiers that are fed by transformer secondary winding with Δ and Y connection, respectively.

The medium voltage busbar is connected to the high voltage busbar with a HV/MV transformer whose windings are Y-Δ connected. The power of the furnace is 80 MW. The other parameters are: Transformer $T_1$ - 80 MVA, 220kV/21kV; Transformer $T_2$ - 87 MVA, 21kV/0.638kV/0.638kV. Some filters are provided in the plant.

Fig. 1 - Dc arc furnace plant
In the dc arc furnaces the presence of the ac/dc static converter and the random motion of the electric arc, whose non linear and time-varying nature is known\(^1\), are responsible for dangerous perturbations, in particular waveform distortions and voltage fluctuations. These perturbations are time-varying.

The voltage and current waveforms recorded at the MV busbar of Fig. 1 have been sampled with the frequency of 5000 Hz. As an example, Fig. 2 shows a MV current waveform and its spectrum. The amplitudes of the frequency components are relative to the maximum value in the spectrum.

![Current and Spectrum](image)

Fig. 2 - Current waveform at MV busbar of the dc arc furnace in Fig. 1 (a) and its spectrum (b)

The analysis of Fig. 2 clearly shows the presence of the ac/dc converter characteristic harmonics \((h = 12p±1, p = 1, 2,\ldots)\). There are several interharmonics around the characteristic harmonics and around the fundamental component due to the arc fluctuations. Non characteristic harmonics appear too.

### III. GENERAL ASSUMPTIONS

The Prony and rootMusic methods are compared with Short-Time Fourier Transform on the basis of the values of the IEC harmonic and interharmonic subgroups, of the Total Harmonic Distortion, of the Load Nonlinearity Indicator and, finally, of the spectral component frequency time variation. The theoretical background on these quantities are recalled in the companion paper [1].

Some assumptions have been effected in order to apply all the methods at best. In the following, these assumptions are evidenced.

#### a) IEC harmonic and interharmonic subgroups

The evaluation of harmonic and interharmonic subgroups has been effected on the basis of the following assumptions:
- the window length is 200 ms and the successive windows till 3 s have not overlap;
- for each window, the \(n\)th harmonic subgroup includes all the spectral components inside the frequency interval \([nf_1-7.5, nf_1+7.5]\);
- for each window, the \((n+0.5)\)th interharmonic subgroup includes all the spectral components inside the frequency interval \([nf_1+7.5, (n+1)f_1-7.5]\).

In applying Prony and rootMusic methods some filters have been applied to pre-processing data. In particular:
- a bandstop Butterworth IIR filter cuts out the main (50Hz) component;
- a lowpass (40 Hz) Butterworth IIR filter has been applied for analyzing interharmonics grouping \(n = 0.5\);
- bandpass Butterworth IIR filters have been applied for the other subgroups.

Moreover, in order to compare the different processing techniques, a reference technique is adopted. We assumed as reference the technique proposed in [6], named as “Ideal IEC”, based on the extension of IEC grouping to high resolution spectral analysis performed on \(T_W = 3\) s, that is the whole interval of very short time measurements.

#### b) Total Harmonic Distortion

The evaluation of Total Harmonic Distortion has been effected on the basis of the following assumptions:
- the window length is assumed to be 200 ms and the successive windows until 3 s have not overlap;
- for each window, the THD includes all harmonic and interharmonic components until 1000 Hz.

In applying Prony and rootMusic methods the same assumptions as for the grouping calculations have been effected.

#### c) Load Nonlinearity Indicator

The evaluation of Load Nonlinearity Indicator has been effected assuming a window length for the calculation of the fundamental current and voltage components of 20 ms. A 1 s signal length has been considered to handle an assigned condition.

#### d) Spectral Component Frequency Time Variation

For parameter estimation of harmonics and interharmonic frequencies some filters have been applied. In particular:
- a bandstop Butterworth IIR filter cuts out the main (50Hz) component;

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\(^1\) The phenomena correlated to the arc behavior are very complex. In [5] it has been shown that the DC arc voltage waveform has the aperiodic and irregular behavior that characterizes every chaotic phenomenon. This has been demonstrated with the analysis of proper delay plots and the evaluation of the largest Lyapunov exponent.
• a lowpass (40 Hz) Butterworth IIR filter has been applied for analyzing subharmonics (frequencies under 50Hz);
• a bandpass Butterworth IIR filter has been applied for each assigned range of frequencies.

IV. INVESTIGATIONS

Several numerical experiments have been effected on the current and voltage waveforms recorded at MV busbar of the dc arc furnace shown in Sect. II. Some results are provided in the next.

a) IEC harmonic and interharmonic subgroups

The comparison of the harmonic and interharmonic subgroups values over 3 s interval with the “Ideal IEC” has shown that:
• both Prony and rootMusic methods generally give an acceptable estimation of harmonic groupings and, in some cases, furnish results more accurate than the ones obtained with STFT;
• the STFT generally gives a better estimation in case of interharmonic groupings, while Prony and rootMusic methods in some cases furnish results quite different from the ones of STFT and in some cases similar results;
• Prony and rootMusic method results are influenced by the number of components and of samples taken into account.

As an example of the obtained results, Figs 3 and 4 report the amplitudes of some harmonic and interharmonic subgroups obtained with the different techniques, for both current (Fig. 3) and voltage (Fig. 4) waveforms. The results in Figs. 3 and 4 have been obtained assuming for both Prony and rootMusic methods 3 (7) components for the harmonic (interharmonic) subgroups.

Moreover, as a further example, Figs. 5 a) and b) compare the Ideal IEC values with the progressive average of the amplitudes of the interharmonic group \( n = 1.5 \) obtained with the different techniques, for both current (Fig. 5 a) and voltage (Fig. 5 b). The progressive average has been calculated properly averaging values obtained with successive 200ms-groups with the following expression:

\[
C_{1.5\text{-mean}}(k) = \sqrt{\frac{1}{k} \sum_{m=1}^{k} C_{1.5\text{-}(200ms)}^2}, \quad k = 1,...,15. \quad (1)
\]

As obvious, similar relations can be obtained for all the harmonic and interharmonic subgroups.

The analysis of Figs. 5 shows as some techniques approach better the Ideal IEC values when the averaging approaches the 3 s interval (\( k = 15 \)).
b) **Total Harmonic Distortion**

The comparison of the Total Harmonic Distortion values has shown that:
- STFT, Prony and rootMusic methods generally give similar estimation of THD values;
- once again Prony and rootMusic method results are influenced by the number of components and of samples taken into account.

As an example of the obtained results, Figs 6 report the THD values obtained with the different techniques, for both current (Fig. 6 a) and voltage (Fig. 6 b) waveforms. The results reported have been obtained using the spectral components calculated during the grouping analysis of Section a).

As clearly shown by Fig. 7, as foreseeable, the values obtained with the two different methods are characterized by a constant difference, linked to the different estimation of the current and voltage fundamental components in the first cycle.

c) **Load Nonlinearity Indicator**

Fig. 7 shows the values of LNI calculated with FFT and Prony method (see rel. 31 of the companion paper [1]), for 50 successive cycles.

The values of the Load Nonlinearity Indicator averaged on all the 50 cycles are 0.46 and 0.43, respectively.

As clearly shown by Fig. 7, as foreseeable, the values obtained with the two different methods are characterized by a constant difference, linked to the different estimation of the current and voltage fundamental components in the first cycle.

![Fig. 6: THD of the current (a) and voltage (b)](image)

![Fig. 7: Load Nonlinearity Indicator](image)
**d) Spectral Component Frequency Time Variation**

With reference to the current waveforms, the components in the range 300-500 Hz has been analysed, since this range seems to be interesting (Fig.2). Figs. 8 shows the time-frequency characteristics calculated using the Prony and the rootMusic methods. The number of 3 signal components and a sliding sampling window have been assumed. In the case of the Prony method, the sampling window with 22 samples and a median filter with 50 samples at the output have been applied. The sliding step was 10 samples. In the case of the rootMusic methods the sampling window was 500 samples and the sliding step 20 samples.

Figs. 9 show the voltage waveform and its spectrum. In this case the frequency range 100-300 Hz seemed to be more interesting. Figs. 10 shows the time-frequency characteristics calculated using the Prony and the rootMusic methods. The same number of components have been considered.

Finally, the subharmonics (frequencies under 50Hz) have been analysed. The investigations show fluctuations of frequencies. For the Prony method the fluctuations are larger because of a smaller sampling window. Because of the properties of the investigations it was necessary to apply different sampling windows. Each method delivers a certain middle value over a sampling window. The non stationarity of the subharmonic components is especially visible on the Figs. 11 an 12. The fundamental component (50) Hz has a constant frequency and the frequency of the subharmonic fluctuates, because of the non stationarity of the arc. Nevertheless, the results obtained with the Prony and the rootMusic methods show a certain similarity.
It has been shown that some high-resolution spectrum estimation methods (Prony and rootMusic) could be effectively used for parameter estimation of distorted non-stationary signals. This conclusion is based on the results of several applications of these techniques referred to the Power Quality disturbance characterization.

The proposed methods were investigated under different conditions and found to be valuable and efficient tools for detection of all higher harmonics existing in a signal. They also make it possible the estimation of interharmonics and the calculation of the time-frequency characteristics of signal components.

In particular, the application of the proposed advanced methods makes possible to estimate the changing in time the signal component parameters, even in situations when the frequencies of the components differ insignificantly. In the time-frequency characteristics of signal components fluctuations of frequency characteristics is visible, mainly when comparing the frequency of the main component and the subharmonic. The time-frequency characteristics obtained by both the proposed methods (Prony, rootMusic) show a certain similarity.

However, the accuracy of the estimation depends on the signal distortion characteristics, the sampling window, the number of samples and components taken into the estimation process. In particular, the numerical experiments have clearly shown a strong sensitivity on the number of components and of samples, so that optimal numbers can be forecasted for the different power quality aspects to be taken into account. Moreover, the computation of the methods can be more complex than STFT.

Works in progress on these techniques are concentrated on the evaluation of the aforementioned optimal numbers, with particular reference to the Total Harmonic Distortion index due to its importance in characterizing nonsinusoidal waveforms. Moreover, their accuracy will be tested on other actual current and voltage waveforms to better validate their accuracy.

**VI. REFERENCES**


**VII. BIOGRAPHIES**

See Companion paper [1]