A New WSN Deployment Algorithm for Water Pollution Monitoring in Amazon Rainforest Rivers

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Abstract—In this paper, we study the wireless sensor network deployment for water pollution monitoring in the Amazon rainforest rivers. Our objective consists in minimising the number of deployed geographical field installations along the river, while ensuring the detection of the substance spilled in the given river regardless of the position of its source. A geographical field installation is formed by a set of barrier coverage underwater sensors which detect the pollutant if its molarity in the water is greater than a predefined threshold. Indeed, the substance molarity is inversely proportional to the moving distance. To generate the best topology, we propose a novel geographic Installation Field Deployment Algorithm based on the Backtracking heuristic named BT-FIDA. Since the river has a several forks, in order to reduce the number of installation fields, BT-FIDA minimises the rate of at least 2-covered river segments. The simulation results obtained show that our proposal minimises the number of field installations (i.e., deployment cost) while minimising the rate of areas which are miss-covered and over-covered.

Keywords: River monitoring, coverage, under-water WSN, optimisation, backtracking heuristic.

I. INTRODUCTION

Over 97% of all the water on earth is salty and most of the remaining 3% is frozen in the polar ice-caps [1]. The fresh water contained in rivers, lakes and underground represent less than 1%. Fresh water which is paramount necessity for human life is a precious resource and the increasing pollution of our rivers is a cause for alarm. Chemical waste products from industrial processes are willingly and/or accidentally discharged into rivers, which is catastrophic for the biodiversity of fauna and flora. We notice that in most of the industrial pollution river disasters, the spilled substances include cyanide, zinc, lead, copper, cadmium, mercury, etc. These substances may seep into the water in such high concentrations so that fish and other animals are immediately killed. Sometimes the pollutants impact food chains and accumulate until they reach toxic levels, eventually killing birds, fish and mammals.

In this paper, we tackle the underwater wireless sensor network deployment problem for water monitoring in rivers. In fact, this work is undertaken within the FP7 European project “GOLDFISH” [2], which is focused on the monitoring of water quality in the Amazon rainforest rivers. The objective is to reduce the deployment cost by minimising the number of geographical Field Installations (FI) along the river, while guaranteeing the detection of a pollutant chemical substance whatever the geographic area in which it is spilled (i.e., the pollution source). It is worth pointing out that each geographical FI ensures a barrier coverage of a river and any moving pollutant substance crossing a barrier is detected if its molarity is greater than a predefined threshold. Indeed, based on the propagation of a pollutant within water [3], the substance concentration in water is inversely proportional to its moving distance due to the river current. Based on the latter water propagation model, two succeeding FIs cannot be geographically separated by more than a predetermined distance calculated with respect to the river’s proprieties and the sensitivity of underwater sensors.

In order to generate the best FIs topology, we propose a novel heuristic named Deployment Algorithm based on Backtracking heuristic called BT-FIDA. Our proposal adopts a “divide and conquer” strategy and operates as follows. First, the river is modelised as a directed graph with many sources (located in mountains) and one sink (located in the sea or ocean). Then, BT-FIDA generates the prominent candidate positions which could host FIs by applying three basic heuristics: i) Max-FIDA, ii) Min-FIDA and iii) Avg-FIDA. Afterwards, BT-FIDA runs a backtracking algorithm to select the best candidate positions to deploy the FIs. Note that the above three basic heuristics (i.e., Max-FIDA, Min-FIDA and Avg-FIDA) make use of a recursive cut of the directed river graph to generate the candidate positions. Based on extensive simulations with a real river in the Amazon rainforest, BT-FIDA obtains a good level of performance in terms of i) number of FIs, ii) the rate of miss-covered segments and iii) the rate of K-covered segments with $K \geq 2$.

The rest of this paper is organised as follows. The next section will describe the architecture of the water river monitoring system. In Section III, we will formulate the water monitoring optimisation problem. Then, we will describe our field installation deployment algorithm (BT-FIDA) and the simulation results obtained, respectively in Section IV and Section V. In Section VI, we will summarise the main research papers related to the issue handled in this paper. Finally, Section VII will conclude the paper.

II. ARCHITECTURE OF THE WATER RIVER MONITORING SYSTEM

The water river monitoring system is composed of a set of FIs deployed along the river and a set of gateways. In fact,
each $FI$ is associated with a single gateway deployed on a bank of the river. Hereafter, we will describe the components of i) the Field Installation and ii) the Gateway.

- **The Field Installation**: It is formed by a set of clusters ensuring barrier coverage of a river. Indeed, we assume that any chemical substance crossing the $FI$ is detected. In fact, the micro-deployment of clusters within the $FI$ is out of the scope of this paper and can be achieved based on the related research work [4]. Each cluster is composed of a set of static underwater sensors and one surface water antenna. Indeed, the set of clusters within each $FI$ builds a static ad hoc network, which sends the monitored events to the gateway. Note that a reactive or proactive routing protocol can be installed within a cluster network.

- **The Gateway**: is a centralised node that collects all the detected events with an $FI$. Moreover, it sends the detected alarms to the control room via a cellular and/or satellite communication.

Fig. 1 summarises the global architecture of the water river monitoring system and the components of an $FI$.

### III. FORMULATION OF $FI$s DEPLOYMENT PROBLEM

In this section, we will formulate the geographical field installation deployment problem along the river. To do so, first we propose to model the river as a directed graph denoted by $\mathcal{G} = \left( V(\mathcal{G}), E(\mathcal{G}) \right)$, where $V(\mathcal{G})$ and $E(\mathcal{G})$ are, respectively, the sets of i) river’s forks and sources and ii) river’s segments connecting the sources and forks. It is worth pointing out that the direction of each segment $e \in E(\mathcal{G})$ is the same as the river’s current. In this work, we assume that each fork $w \in V(\mathcal{G})$ contains multiple incoming edges, denoted by $\{e_w\}$. Besides, we assume that it has only one outgoing segment denoted by $e_w^+$ and its next-hop node is denoted by $w^+$. In other words, many streams merge to form one larger canal. On the other hand, we assume that the river carries the water in only one sea or ocean area. Hence, the graph $\mathcal{G}$ has many sources (i.e., nodes without incoming edges) and only one sink (i.e., nodes without outgoing edges). Each segment $e \in E(\mathcal{G})$ is characterised by its distance and the average velocity of the water, respectively denoted by $D(e)$ and $\bar{V}(e)$.

The molarity level of a pollutant can be predicted as a function of time and space making it possible to model the behaviour of the solutes over the river course. As proposed in [5], we model the concentration (i.e., molarity) time and space function of a pollutant chemical substance in the river water as:

$$C(x, t) = \frac{M}{2 \cdot A \cdot \sqrt{\pi} \cdot D_L \cdot t} \cdot \exp\left[ \frac{-(x - U \cdot t)^2}{4 \cdot D_L \cdot t} \right]$$  \hspace{1cm} (III.1)

where i) $x$ is the distance in the downstream direction expressed in meters ($m$), ii) $t$ is the duration between the spill and the monitoring at point $x$ expressed in seconds ($s$), iii) $M$ is the mass of the contaminant expressed in kilograms, iv) $A$ is the cross-sectional area of the river expressed in $m^2$, v) $D_L$ and $U$ are, respectively, longitudinal dispersion and advective velocity expressed in $m^2/s$ and $m/s$.

We notice that the concentration decreases as the moving distance and the duration from the spill of the chemical substance increases. Hence, the risk consists in no detecting the pollutant since the underwater sensors can send the alarm if and only if the substance’s molarity is greater than a predefined threshold. Note that the latter reflects the hardware sensitivity of the underwater sensors. In this sense, we can define a full-river-coverage notion as the ability of the deployed $FI$s to detect in the river a substance spilled anywhere.

Our objective is thus to minimise the cost of deployment, while guaranteeing the full-river-coverage of the monitoring system. The above objective (i.e., cost) can be evaluated by the number of $FI$s denoted by $N_{FI}$. To minimise the cost, it is equivalent to maximise the distance between successive

![Fig. 1. Architecture of water river monitoring system](image)
deployment problem, as follows:

\[
\text{maximise} \left[ \min_{\mathcal{F}_i \in \mathcal{F}_I \setminus \{\mathcal{F}_j\}} \{d(\mathcal{F}_i, \mathcal{F}_j)\} \right], \forall \mathcal{F}_i \in \mathcal{F}_I
\]  

\text{(III.2)}

where \( \mathcal{F}_I \) is the set of deployed field installations and \( d(\mathcal{F}_i, \mathcal{F}_j) \) is the euclidean distance separating \( \mathcal{F}_i \) and \( \mathcal{F}_j \).

On the other hand, we must guarantee the full-river-coverage. To do so, the maximum distance between two successive \( \mathcal{F}_I \)s cannot exceed a predefined distance \( D_{th} \) which depends on the sensitivity of the hardware underwater sensor. Formally,

\[
\min_{\mathcal{F}_i \in \mathcal{F}_I \setminus \{\mathcal{F}_i\}} \{d(\mathcal{F}_i, \mathcal{F}_j)\} \leq D_{th}, \forall \mathcal{F}_i \in \mathcal{F}_I
\]  

\text{(III.3)}

We will now outline our geographic river field installation deployment problem, as follows:

\[
\text{maximise} \left[ \min_{\mathcal{F}_i \in \mathcal{F}_I \setminus \{\mathcal{F}_j\}} \{d(\mathcal{F}_i, \mathcal{F}_j)\} \right]
\]

\text{subject to:}

\[
\forall \mathcal{F}_i \in \mathcal{F}_I \text{ } \min_{\mathcal{F}_i \in \mathcal{F}_I \setminus \{\mathcal{F}_j\}} \{d(\mathcal{F}_i, \mathcal{F}_j)\} \leq D_{th}
\]

Our problem is a non-linear multi-objective combinatorial optimisation problem, which is NP-hard. In the next section, we will propose a novel geographic Installation Field Deployment Algorithm based on the Backtracking heuristic called BT-FIDA.

IV. PROPOSAL: FIELD INSTALLATION DEPLOYMENT ALGORITHM BASED ON BACKTRACKING HEURISTIC

The \( \mathcal{F}_I \) deployment problem is highly complex and the generation of the optimal solution within a polynomial time for a large scale river deployment is not possible. Thus, we propose here a novel algorithm named Backtracking based Installation Field Deployment Algorithm, BT-FIDA. We notice that our problem is a combinatorial optimisation problem. Thus, the size of the solution space is finite but is exponential. Our proposal BT-FIDA operates as follows. First, in order to minimise the number of combinations, hence the time complexity, BT-FIDA generates the prominent candidate positions (i.e., \( \mathcal{P}_S \)) to deploy \( \mathcal{F}_I \). To achieve this, BT-FIDA runs three basic heuristics denoted by i) Max-FIDA, ii) Min-FIDA and iii) Avg-FIDA. Then, BT-FIDA takes into account the union of the above positions and runs a beam search backtracking algorithm [6] [7] by building a partial solution tree and selects the best one. The best branches are explored by going down and going back up in the partial tree solution until BT-FIDA converges to the best leaf (i.e., the global solution). It is worth noting that the optimality is impacted by the width of the partial tree solution and the cost function \( \phi \) evaluating the solution in terms of i) number of \( \mathcal{F}_I \)s (i.e., \( N_{\mathcal{F}_I} \)), ii) rate of miss-covered (i.e., \( N_{\mathcal{M}_{\mathcal{M}_{\mathcal{M}_{\mathcal{C}}}}} \)) and ii) rate of K-covered (i.e., \( \mathcal{O}_{\mathcal{C}} \)) segments in the river with

Algorithm 1: BT-FIDA

1. \( \mathcal{P}_S_{\text{Min}} \leftarrow \text{Min-FIDA}(\mathcal{G}, D_{th}) \)
2. \( \mathcal{P}_S_{\text{Max}} \leftarrow \text{Max-FIDA}(\mathcal{G}, D_{th}) \)
3. \( \mathcal{P}_S_{\text{Avg}} \leftarrow \text{Avg-FIDA}(\mathcal{G}, D_{th}) \)
4. \( \mathcal{P}_S \leftarrow \mathcal{P}_S_{\text{Min}} \cup \mathcal{P}_S_{\text{Max}} \cup \mathcal{P}_S_{\text{Avg}} \)
5. FI-backtracking (\( \mathcal{G}, \mathcal{P}_S, \phi \) )

\( K \geq 2 \). The pseudo-algorithm of our proposal is summarised in Algorithm 1.

In the following, we will explain in depth i) Min-FIDA, ii) Max-FIDA, iii) Avg-FIDA and iv) FI-backtracking heuristics.

A. Min-FIDA heuristic

The main idea behind the Min-FIDA heuristic consists in deploying \( \mathcal{F}_I \)s recursively and by associating a level for each node \( w \in V(\mathcal{G}) \). In this aim, we note all the source nodes (i.e., without incoming edges) by \( V_0 = \{w_0\} \) and their level is initialised to 0. Next, the level of each fork node \( w \in V(\mathcal{G}) \) is denoted by \( L_w \) and is equal to:

\[
L_w = \max_{v \in W}(L_v) + 1, \quad \text{IV.4}
\]

where \( W \) is the set of predecessor nodes of \( w \) in directed graph \( \mathcal{G} \). We note the fork nodes with level 1 as \( V_1 = \{w_1\} \). Min-FIDA operates as follows. First, for each source node \( w_0 \in V_0 \), Min-FIDA deploys an \( \mathcal{F}_I \) every \( D_{th} \) along its corresponding outgoing edge \( e_{w_0}^+ \). The process will be deceased just after overtaken its successor fork node \( w_1 \). Note that \( e_{w_0}^+ \) is the link connecting \( w_0 \) to its successor node \( w_1 = w_1^+ \). The set of \( \mathcal{F}_I \)s deployed after the fork node \( w_1 \) is denoted by \( S_{w_1} \). The deployed \( \mathcal{F}_I \)s are added in the set \( \mathcal{P}_S_{\text{min}} \) but without including \( S_{w_1} \). Once all the source nodes have been explored, Min-FIDA deals with the fork nodes \( V_{w_1} \). In fact, two cases can arise for each fork node \( w_1 \in V_{w_1} \).

1) The maximum distance between the \( w_1^+ \) and \( \mathcal{F}_I_k \in S_{w_1} \) is located within \( e_{w_1}^+ \) (i.e., outgoing edge). We denote the furthest field installation in \( S_{w_1} \) from \( w_1^+ \) by \( \mathcal{F}_I_k \). Formally, \( \mathcal{F}_I_k \) satisfies the following constraint:

\[
d(w_1, \mathcal{F}_I_k) = \max_{\mathcal{F}_I_k \in S_{w_1}} \{d(w_1, \mathcal{F}_I_k)\}, \quad \text{IV.5}
\]

where \( d(w_1, \mathcal{F}_I_k) \) is the euclidean distance between the fork node \( w_1 \) and \( \mathcal{F}_I_k \). In this case, Min-FIDA deploys a field installation at \( \mathcal{F}_I_k \). Formally, \( \mathcal{P}_S_{\text{min}} \leftarrow \mathcal{P}_S_{\text{min}} \cup \{\mathcal{F}_I_k\} \text{ IV.6} \)

Then, Min-FIDA shrinks \( e_{w_1}^- \) (i.e., outgoing link) by moving the fork node \( w_1^+ \) to \( \mathcal{F}_I_k \). Moreover, all the edges \( \{e_{w_1}^-\} \) (i.e., incoming links) and their corresponding sources are removed from the directed graph \( \mathcal{G} \).
It is straightforward to see that \( F I_n \) is the nearest field installation to fork node \( w^f_i \). Moreover, we notice that with this strategy, we will certainly obtain over-covered (i.e., no hole of detection) river segments since we select the nearest field installation from the fork nodes.

### 3. V. Performance Evaluation

In this section, we will evaluate the performance of our proposal BT-FIDA. To do so, first we will detail the simulation environment. Then, we will recall the performance metrics. Afterward, we will briefly describe the baseline \( F I \) Deployment Algorithm, denoted by Base-FIDA, used for comparison. Finally, we will illustrate the main results obtained and highlight the benefit of our proposal BT-FIDA.

#### A. Simulation Environment

As mentioned in Section I, our proposal is validated within a real Amazon Rainforest river. The map of the river is illustrated in Fig. 3. We formulate the above river as a directed graph with parameters illustrated in Table I.
### Parameters of the Direct River Graph \( \vec{G} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>V(\vec{G})</td>
</tr>
<tr>
<td>(</td>
<td>E(\vec{G})</td>
</tr>
<tr>
<td>( \sum_{e \in E(\vec{G})} (D(e)) )</td>
<td>323.838 ( \text{Km} )</td>
</tr>
<tr>
<td>( \text{Avg}(D(e)) )</td>
<td>2.7 ( \text{Km} )</td>
</tr>
</tbody>
</table>

**Fig. 3. Amazon rainforest river map**

Based on the inputs of our partner CapSenze company [8] in FP7 GOLDFISH project according to their tests and hardware sensors, the maximum distance \( D_{th} \), in which a substance can move within the river, while remaining detectable, is set to 6 \( \text{Km} \). In fact, beyond \( D_{th} \), the pollution will be not detected. We set the parameters of the cost function \( \alpha \) and \( \beta \) to 2.

#### B. Performance Metrics

We recall hereafter, the main performance metrics to evaluate our deployment strategies of \( \mathcal{F} \)Is within a river.

- \( N_{\mathcal{F}I} \) is the number of deployed \( \mathcal{F} \)Is within \( \vec{G} \)
- \( M_{cov} \) is the rate of miss-covered edges among the whole graph (i.e., \( \sum_{e \in E(\vec{G})} D(e) \))
- \( O_{cov} \) is the rate of K-covered edges among the whole graph (i.e., \( \sum_{e \in E(\vec{G})} D(e) \)). That means the rate of segments covered at least by two \( \mathcal{F} \)Is.

#### C. Base-FIDA: baseline approach

For comparison purposes, we present here a simple deployment strategy named Base-FIDA. It recursively deploys \( \mathcal{F} \)Is each \( D_{th} \) starting from the sink node of the river (i.e., sea and/or ocean). The process will be stopped until all the sources nodes are covered. In fact, the deployment is performed following the reverse direction of the current river. The position of the first \( \mathcal{F} \)I will be deployed at the sink node.

It is worth pointing out that Base-FIDA will guarantee 0\% of over and miss detected of segments in the rivers. However, once Base-FIDA arrives to the source nodes, many \( \mathcal{F} \)Is will be deployed but cover a short segments compared to the coverage range of any \( \mathcal{F} \)I (i.e., \( D_{th} \)).

#### D. Evaluation Results

As mentioned earlier, our proposal BT-FIDA will exploit the prominent positions generated by Min-FIDA, Max-FIDA and Avg-FIDA. In fact, we can see that our proposal is a hybrid approach, which selects at each level the best positions obtained by the latter three heuristics. In Table II, we present the results obtained by our proposal compared with the baseline approach (i.e., Base-FIDA) and the aforementioned heuristics: Min-FIDA, Max-FIDA and Avg-FIDA.

**Table II: Cost Performance**

<table>
<thead>
<tr>
<th>Strategy</th>
<th>( N_{\mathcal{F}I} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BT-FIDA</td>
<td>23</td>
</tr>
<tr>
<td>Base-FIDA</td>
<td>54</td>
</tr>
<tr>
<td>Min-FIDA</td>
<td>19</td>
</tr>
<tr>
<td>Max-FIDA</td>
<td>34</td>
</tr>
<tr>
<td>Avg-FIDA</td>
<td>25</td>
</tr>
</tbody>
</table>

As we can see, our proposal BT-FIDA outperforms all the heuristics and achieves the desired goal of minimising the deployment cost (i.e., \( N_{\mathcal{F}I} \)), while guaranteeing the full-river coverage. In other words, the rate of lack-covered areas \( M_{\mathcal{F}I} = 0\% \).

Specifically, as illustrated in Table II, our proposal reduces the cost of deployment by 32.35\% compared to Max-FIDA, while guaranteeing the full-coverage of the whole river as depicted in Fig. 4. On the other hand, BT-FIDA needs only more 4 field installations than Min-FIDA in order to remove the hole detection. We notice that the Avg-FIDA heuristic presents both hole and over detected areas within a network, which may miss the propagation of the pollutant. Besides, our
In literature, the classical deployment issue of terrestrial sensor networks has been studied in depth. In fact, we can classify them in three groups. First, the random strategies [11] in which the sensors are randomly deployed with respect to a predefined random distribution. The second is a regular deployment [12] [13] algorithms, which make use of basic structures such as: triangle, square, polygon, hexagon, etc. Finally, virtual force approaches [14] in which sensors move by attractive and/or repulsive forces of neighboring sensors and obstacles until the convergence to the steady state (i.e., best topology). Unfortunately, the above proposals cannot be applied. In fact, the coverage definition and the assumptions (e.g. target propagation model, topology of deployment area, etc.) are different than those of field installation deployment problem in rivers.

The underwater 2-dimensional and 3-dimensional deployment algorithms [15] [16] proposed in literature cannot be applied within a water monitoring of rivers context. Indeed, all the sensor nodes are deployed at the same depth with two-dimensional strategies and sensor nodes may be floating at an arbitrary one with three-dimensional approaches. We notice that the above methods focused on the event detection within a predefined small zone and the monitored event moves only in this latter. The proposed strategies do not deal with different geographical zone (i.e., field installations), which are correlated in term of sensing coverage range in a river. Noting that the above methods can be exploited within one field installation in the river to guarantee full barrier coverage and reduce the needed number of sensors.

The second research filed similar to our problem is the urban traffic surveillance based on wireless sensor networks [17] [18] [19]. In fact, we can say that a river is comparable to road map and the river current to the traffic flow. Nevertheless, the road traffic is bidirectional and the pollutant target substance spreading has a unique sense. This considerably affects the way of sensor placement. Moreover, a car cannot disappear in the road. It means that even if roads present a large hole of coverage areas, the system will be able to detect the cars by focusing the deployment in the road connection points. It is not the case for pollutant substance monitoring in which the detection strongly depends on its molarity.

VI. RELATED WORK

To the best of our knowledge, there is no research paper tackling the geographical filed installation deployment problem in rivers. In fact, i) the stream river direction, ii) river topology and iii) the propagation model of pollutant substances in rivers makes the underwater sensor network deployment problem atypical and more challenging. Indeed, the classical coverage definition of an area is not valid and it has been evolved in our paper. However, hereafter we will describe the main network deployment research fields, which are very close to our problematic but unfortunately the related strategies cannot be used.

In [9] [10], the authors aimed to develop novel remote real-time monitoring technologies that can continuously collect water quality parameters in lakes, rivers and reservoirs. The system can be used to investigate various water quality parameters for real time surveillance by remote users via Internet. Unfortunately, the deployment problem of sensors has not been addressed.

In Fig. 5, we illustrate the impact of the size of the best partial solution at each level of the solution tree. We notice that even $K_{\text{limit}}$ our proposal deploys the same number of FIs, which is equal to 23 and guarantees a full-coverage. On the other hand, we notice that the reduction of the rate of over-covered segment is tiny. Consequently, we conclude that setting $K_{\text{limit}}$ to 15 is sufficient and the performance are steady.

VII. CONCLUSION

In this paper, we studied the underwater sensor network deployment problem for river monitoring. To the best of our knowledge, we are the the first to tackle this problem. We proposed a novel deployment algorithm called BT-FIDA based on the backtracking and beam search heuristics. The proposal is validated with a real river in Amazon rainforest. Through extensive simulation results, we have shown that the deployment cost is minimised and the full-coverage is guaranteed.

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REFERENCES


Algorithm 2: Min-FIDA heuristic

1. Inputs: $\mathcal{G} = (V(\mathcal{G}), E(\mathcal{G}))$, $D_{th}$

2. Output: $\mathcal{PS}_{Min}$

3. $\mathcal{PS}_{Min} \leftarrow \emptyset$

4. $\mathcal{G}' \leftarrow \mathcal{G}$

5. while $V(\mathcal{G}') \neq \emptyset$ do

6. $W_0 \leftarrow \{w \in V(\mathcal{G}') : L_w = 0\}$

7. $W_1 \leftarrow \{w \in V(\mathcal{G}') : L_w = 1\}$

8. foreach $w_0^i \in W_0$ do

9. $w_1^i \leftarrow w_0^i + (\text{successor of } w_0^i)$

10. $S_{w_1^i} \leftarrow \emptyset$

11. $Pos \leftarrow \text{position}(w_0^i)$

12. $k \leftarrow 0$

13. while $w_1^i$ is not overtaken do

14. Deploy a $\mathcal{FL}_k$ which is $D_{th}$ away from $Pos$

15. if $\mathcal{FL}_k$ overtakes $w_1^i$ then

16. $S_{w_1^i} \leftarrow S_{w_1^i} \cup \{FI_k\}$

17. else

18. $\mathcal{PS}_{Min} \leftarrow \mathcal{PS}_{Min} \cup \{FI_k\}$

19. $Pos \leftarrow \text{position}(\mathcal{FL}_k)$

20. $k \leftarrow k + 1$

21. $V(\mathcal{G}') \leftarrow V(\mathcal{G}') \setminus W_0$ (remove source nodes)

22. $E(\mathcal{G}') \leftarrow E(\mathcal{G}') \cup \{w \in W_1 : \text{remove incoming links}\}$

23. foreach $w_1^i \in W_1$ do

24. Locate $\mathcal{FL}_f$ satisfying:

25. $d(w_1^i, \mathcal{FL}_f) = \max_{\mathcal{FL}_k \in S_{w_1^i}} \{d(w_1^i, \mathcal{FL}_k)\}$

26. $\mathcal{PS}_{Min} \leftarrow \mathcal{PS}_{Min} \cup \{\mathcal{FL}_f\}$

27. if $\mathcal{FL}_f$ is deployed within $e_{w_1^i}$ then

28. $\text{Position of } w_1^i \text{ is moved to } \mathcal{FL}_f$

29. else

30. $V(\mathcal{G}') \leftarrow V(\mathcal{G}') \cup \{mn_{w_1^i}\}$ (add meta-node)

31. $E(\mathcal{G}') \leftarrow E(\mathcal{G}') \cup \{em_{w_1^i}\}$ (add meta-link between $mn_{w_1^i}$ and $w_1^i$)

32. $D(em_{w_1^i}) \leftarrow d(w_1^i, w_1^i) + (D_{th} - d(w_1^i, \mathcal{FL}_f))$

33. $V(\mathcal{G}') \leftarrow V(\mathcal{G}') \setminus \{w_1^i\}$

34. $E(\mathcal{G}') \leftarrow E(\mathcal{G}') \setminus \{e_{w_1^i}\}$