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Low-Complexity Near-Optimal Iterative Signal Detection Based on MSD-CG Method for Uplink Massive MIMO Systems

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Abstract
Massive multiple-input multiple-output (MIMO) wireless system is increasingly becoming a vital factor in fifth-generation (5G) communication systems. It is attracting considerable interest due to improve range, spectral efficiency, and coverage as compared to the conventional MIMO systems. In massive MIMO systems, the maximum likelihood detector achieve the optimum performance but it has exponential complexity for realistic antenna configurations systems, Moreover, Linear detectors commonly suffer from a matrix inversion which is not hardware-friendly. There is an increase in the computational complexity associated with the unique benefits of the massive MIMO systems. The system might be classified as an ill-conditioned problem and hence, the signal cannot be detected. To reduce the data detection complexity, we investigate a linear detector based on the multiple search direction conjugate gradient (MSD-CG) in the massive MIMO uplink systems. Several theoretical iterative techniques that can be used to balance complexity and performance for massive MIMO detection have been proposed in the literature. These methods whose convergence rate for common applications is slow where there is a decrease in the base station to user antenna ratio. In this paper, the performance of the CG method has been advanced by a projection method that necessitates a search direction in each sub-domain instead of making all search directions conjugate to each other. In this regard, our results show that the proposed algorithm with realistic antenna configurations is superior to the existing methods in terms of computational complexity for large-scale MIMO systems.

Keywords Multiple-input multiple-output (MIMO) · Massive MIMO · Ill-conditioned problem · Multiple search direction conjugate gradient (MSD-CG) · Conjugate gradient (CG)

1 Introduction

The massive MIMO system has not only been proven to be energy efficient and spectrum but also, it is robust and secure. Therefore, due to these positive attributes, the massive MIMO system has been forecast to enable the high date rate systems [1]. To achieve the

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goals of the fifth-generation wireless communication, the large-scale MIMO system is preferred since it serves multiple users simultaneously due to the hundreds of antennas at the BS. The large-scale MIMO system has increased dimensions which help to lower the complexity detection signal algorithm in the uplink which in turn enables the system to achieve orders of magnitude increases in energy efficiency and spectrum [1, 2]. Researchers are planning to employ the massive MIMO system within the latest industrial standards which is more superior as compared to the conventional MIMO systems. This is because the massive MIMO can serve tens of terminals from one base station (BS) that is equipped with a few hundred antennae at the same time-frequency resource [2]. However, researchers have agreed that the massive MIMO detection system poses some challenge due to the prohibitive computational complexity that is attributed to the significantly increased number of antennae [3].

For assuring attractive level of benefits from the use of large scale MIMO in practice, it is imperative to resolve some challenging issues and problems. The foremost issue is the detection of practical signal algorithm in the uplink [4]. The complexity of the optimal detector i.e. the maximum likelihood detector increased up to a greater extent with the number of transmit antennas increases; eventually it no longer remain practical for the large scale MIMO systems. For achieving optimal ML detection performance while keeping the complexities at lower level, it is proposed to choose from several non-linear signal detection algorithms. One of such algorithm is fixed complexity sphere detecting algorithm (FSD) that is based on the sphere decoding (SD) algorithm. In this type of algorithm, the underlying lattice structure of the received signals is used. It is proved to be the most capable and promising option that helps achieving the ML detection performance with minimum level of complexity. This algorithm is significant for the typical small scale MIMO system however, when the dimension of MIMO becomes larger and the modulation is also hiked (e.g., 128 antennas at the BS with 64 QAM modulation) then the level of complexities resulted from this algorithm also became unaffordable.

There are several linear detection approaches that necessitate a better tradeoff between the complexity and performance for massive MIMO uplink systems. Some of these methods include the minimum square error (MMSE) and the zero-forcing (ZF) [4]. It should be noted that an increase in the number of antennae results in the subsequence increase in the complexity of the involved matrix inversion. Specifically, the complexity of matrix inversion is $O(N^3)$ for a corresponding $N \times N$ dimensional channel matrix $\mathbf{H}$ for a massive MIMO system.

The complexity of the maximum likelihood algorithm which is the optimal signal detection algorithm increases exponentially following an increase in the transmit antennas which hinders the working of the large-scale MIMO systems [3, 4]. Apparently, several algorithms that have reduced complexity, that is, the tabu search [5] and sphere decoding [7] algorithms; however, it remains a challenge to avoid the complexity involved when the modulation order is high in large-scale MIMO system. In this regard, the low-complexity linear detection approaches have been studied [6–8], but the high complexity remains to be impossible in massive MIMO system since the algorithms use an unfavorable matrix inversion. In the light of the past literature, the approximation methods can only achieve a marginal complexity reduction of the matrix inversion as evident from the presented methods in the literature [9–15, 19–28].

To this far, we have considered the Neumann series expansion [12], the Richardson [13] and the Gauss-Siedel iterative (GS) [14] methods which are the three approaches that are free from matrix invasion. It should be noted that the method based on the latter offers matrix H higher flexibility as compared to the method based on the former approach which utilized the advantage brought about by the diagonal-property of the matrix in this case. Nevertheless, it is essential to note that the CG method advocated by [10–16] is considered to be superior on large-size matrices from the linear systems of equations. As aforementioned, massive MIMO systems have various problems, but the most prominent is attributed to the variations in the BS-user antenna ratio due to the realistic placement of the antenna. The rate of convergence of CG method in an antenna configuration involving small BS-to-user antenna ratio reduces rapidly due to the steady increase in the value of k of the corresponding matrix [10]. However, from a mathematical perspective, when a system does not converge fast it implies that a method must perform a lot of iterations to get the desired result which introduces the complexity in the system. In this regard, it is fundamental to advance the CG method so that it can handle higher convergences with the realistic BS-to-user antenna ratio.

Another method is advocated based on the use of Cholesky incomplete factorization technique abbreviated as ICCG to act as a pre-conditioner to changing a linear system into a simple system that can be solved easily to achieve a better convergence. However, when this ICCG approach is used in the massive MIMO system, it results in just like other conventional Cholesky algorithms [16] which are based on decomposition. Recently, a new detection algorithm was advocated by authors in [17]. This method was able to detect soft-output data with low difficulty compared to the other approaches. To perform box-constrained equalization and estimate MMSE, the authors exploited the OCD.

To that end, there are two important factors upon which the convergence rate depends upon due to its inherent properties. First, the loading factor of the system (α) is the indication that BS would not be able to serve fast growing number of users simultaneously when it is equipped with fixed number of antennas. In literature, the antenna configuration widely adopted is 128×8 or 16 (BS v.s. users) due to which many-user cases are not considered by the corresponding detection methods. Secondly, in correlation channels where there is high coefficient of channel correlation (ζ), the iterative linear detectors performance turned to be unstable. Thirdly, there is max-log-LLR computation involved in the process that is either linked with large scale matric inversion or iterative process causes high level of complexity as well as long latency. Hence, it is crucially required to have an iterative linear detector for 5G applications that not only assures stable convergence rate but also provides low complexity level, even when applied to varied types of scenarios and cases. Complexity level is the most important and attention gaining issue that could be understood from two varied aspects. First, in order to get comparable performance level like typical linear detectors including MMSE, the pace of convergence is quite slow due to which there are more iterations and eventually the graph of complexity and latency touched high levels. Secondly, when attempt is made to speed up the convergence, there are some common preconditions adopted that merely reply upon the Cholesky decomposition that is incomplete [18]. However, in case of massive MU-MIMO, the threshold has a pre-processing phase, that again, causes the problem of excessive complexity which is unaffordable. View the problem from both of these aspects, it is unveiled that these situations offset and even completely eliminates the complexity advantage attached with the iterative linear detector and it became quite a difficult and challenging task to maintain a balance point.

In this paper, We encompasses substantially extended as well as thoroughly revised versions of the conference publication in [33]. We avoided the complicated matrix invasion by exploiting the MSD-CG method [18] which offers detection signal with low-complexity near-optimal signal. This was achieved after enhancing the convergence of the CG method by using a search direction in each sub-domain instead of making all search directions conjugate to each other that provided an alternative or projection mean of avoiding the matrix invasion. In additional, we use a diagonal-approximate initial solution to the presented method. The ability of the advocated algorithm to efficiently compute the solution of the problem of matrix inversion iteratively to attain the desired accurate results is verified in the simulation results. It should be noted that the MSD-CG method has never been applied for the signal detection. Therefore, this work provides the first attempt to employ the approach in the large-scale MIMO uplink systems.

The remaining part of this paper is divided into five sections analyzing various concepts of this study. Section 2 briefly describes the introductions of the MMSE and the massive MIMO. Section 3 examines the presented algorithm based on the MSD-CG method. On the other hand, Sect. 4 presents the complexity analysis and numerical simulation. Finally, Sect. 5 concludes the study.

The vectors and matrices are represented by the lowercase and uppercase boldface letters respectively. Besides, for a matrix $\mathbf{A}$, we use $a_{k,l}$ for the entry in the lth column and the kth row of the matrix $\mathbf{A}$, the kth entry of a column vector $\mathbf{a}$ is defined as $a_k = [\mathbf{a}]_k$, $\mathbf{A}^H$ denoted the Hermitian transpose and $\|\mathbf{a}\|_2 = \sqrt{\sum_k |a_k|^2}$ defines the $l_2 - norm$. Moreover, $\mathbf{E}$ represents the expectation operator, $\mathrm{Re}(a)$ and $\mathrm{Im}(a)$ represents the real and imaginary part of a complex number a, respectively.

2 System Model and Data Detection

This section provides a summary of the efficient techniques for the detection of linear MMSE and introduces a model that is regarded as the OFDM-based uplink massive MIMO system.

2.1 System Model

First, an uplink system of MU-MIMO-OFDM is considered where data is simultaneously sent to the BS with N antennae from U single−antenna users over W subcarriers, where with $N >> U$.The individual users, that is, $i = 1, ..., U$ maps their own encoded bit stream onto a finite set O of constellation points, with bits per constellation point, and a unite mean transmitted power. Therefore, the inverse Discrete Fourier Transform (DFT) is used in the transformation of the obtained W frequency-domain symbols to time-domain (TD) [24]. Apparently, all the users transmitted their time domain signals at the same time through the frequency-selective channel after the prepending the cycle prefix. However, it should be noted that the frequency domain (FD) signals that reach the BS antenna are changed or transformed from the time signals after removing the cyclic prefixes. For simplicity purposes, we assume the perfect channel-state information (CSI), a perfect synchronization, and a sufficiently long prefix has been gotten via pilot-based training. In this regard, we model the FD input-output relation concerning to the assumptions mentioned above on the w^{th} sub-carrier as [23],

$$\mathbf{y}_w = \mathbf{H}_w \mathbf{s}_w + \mathbf{n}_w \tag{1}$$

where

- $s_w \in O^U$ has the transmitted symbols by all U users,
- $H_w \in C^{N \times U}$ represents the complex channel matrix.
- $y_w \in C^N$ represents the received associated vector.

- $n_w \in C^N$, is the additive, white, and Gaussian with variance of σ^2.

2.2 Data Detection

Solving the maximum likelihood (ML) problem helps in the minimization of the symbol error-rate for optimal data detection as per the model in equation (1) [25].

$$\hat{\mathbf{s}}_w^{ML} = \underset{\mathbf{z}\in O^U}{\arg\min} \left\|\mathbf{y}_w - \mathbf{H}_w\mathbf{z}\right\|_2^2 \tag{2}$$

It should be noted that even with the best-known decoding sphere algorithm, the solution of equation (2) quickly leads to prohibitive complexity regarding massive MU-MMO system [3, 4]. Therefore, at low computational complexity, an individual can determine the approximate solution to the ML problem. Virtually calculating of the s that is near the solution of the ML is possible for nonlinear and linear equalization methods that are meant to ease the finite-alphabet $\mathbf{z} \in O^U$ in equation (2). The equation below shows how the estimated $\hat{\mathbf{s}}$ near constellation point can be sliced element-wise;

$$\hat{\mathbf{s}}_w = \underset{\mathbf{z}\in O^U}{\arg\min} \left|[\hat{\mathbf{s}}]_i - \mathbf{z}\right| \forall i = 1, 2, ..., U \tag{3}$$

The equation above is referred as the hard-output data detection. On the other hand, the soft-output data detection can be employed in the calculation of the reliability data for every bit broadcasted inform of LLR value [26].

2.3 Linear MMSE Equalization

The linear MMSE data detection is the most abundant equalization-based data algorithm [17, 24, 26]. This approach was proved to necessitate the ASIC and FPGA designs in massive MU-MIMO system. Furthermore, linear detectors can attain a near-ML error-rate performance [8, 9] for large BS-to-user ration which is represented by $p = \frac{N}{U}$. Besides, these systems should have a ratio of a value equal or greater than two.

The fundamental principle of the MMSE data detection is to incorporate a quadratic penalty function and relax the constraints to the U-dimensional complex space $\mathbf{z} \in C^U$, from the $\mathbf{z} \in O^U$ in the ML problem in equation (2). In particular, the least-square problem is primarily solved by the MMSE equalization [4, 18]:

$$\hat{\mathbf{s}}_w^{MMSE} = \underset{\mathbf{z}\in C^U}{\arg\min} \left\|\mathbf{y}_w - \mathbf{H}_w\mathbf{z}\right\|_2^2 + \sigma^2\|\mathbf{z}\|_2^2 \tag{4}$$

The MMSE equalization possesses a closed-form solution because the objective function in equation (4) is quadratic in $\mathbf{z}$. If an individual wants to find the explicit solution to equation (4) then he or she should calculate the regularized Gram matrix as shown below:

$$\mathbf{A}_w = \mathbf{G}_w + N_0\mathbf{I}_U \tag{5}$$

where $\mathbf{G}_w = \mathbf{H}_w^H\mathbf{H}_w$, $N_0 = \sigma^2$ and the equation given below represents the matched filter vector,

$$\hat{\mathbf{s}}_w^{MF} = \mathbf{H}_w^H\mathbf{y}_w \tag{6}$$

Then, the estimated MMSE in equation (4) is calculated as;

$$\hat{\mathbf{s}}_w^{MMSE} = \mathbf{A}_w^{-1}\hat{\mathbf{s}}_w^{MF} \tag{7}$$

For systems having hundreds of BS antennas such as the massive MU-MIMO, it is a challenge to solve the regularized Gram inverse $\mathbf{A}_w^{-1}$ and its matrix $\mathbf{A}_w$ since it leads to prohibitive complexity in a closed form approach [3]. However, the closed form approach favors the small-scale MIMO system since it has been found to be more efficient when there are fewer antennas. In section III, an efficient and iterative technique of solving equation (4) is presented. Besides, the equalization algorithm avoids the expansive computations; for instance, the problem resented by solving the regularized Gram matrix and its inverse.

3 The Proposed Algorithm Based on the MSD-CG Method

For large-scale uplink MIMO systems, it is guaranteed that the matrix $\mathbf{A}_w$ which is referred to as MMSE filtering will always positively definite Hermitian, if and only if the channel matrix $\mathbf{H}_w$ is column asymptotically and column full-rank orthogonal. In this regard, equation 2 is solved iteratively without the inversion of the matrix by exploiting MSD-CG method [18]. The MSD-CG is less complex than the traditional method of solving a matrix equation. Therefore, the filtering MMSE's matrix can be decomposed as follows;

$$\mathbf{A}_w = \mathbf{D} + \mathbf{L} + \mathbf{L}^H \tag{8}$$

where $\mathbf{L}$ represents the lower triangular component $\mathbf{A}_w$ and $\mathbf{D}$ represents diagonal components of the same matrix. Then we can apply the MSD-CG technique to approximate the signal from the transmitted vector. It should be noted that the MSD-CG can solve the systems of the N-dimensional linear equations $\mathbf{Bx} = \mathbf{b}$ is solved using the MSD-CG method, where $\mathbf{b}$ and $\mathbf{x}$ are $N \times 1$ are measurement and solution vectors, respectively; while $\mathbf{B}$ represents the $N \times N$ square Hermitian matrix. Therefore, this method can be used to solve equation (4) by solving the equation of the form (7) without the use of matrix inversion as follows.

$$\hat{\mathbf{s}} = \arg\min_{\hat{\mathbf{s}} \in C^U} \left\| \mathbf{H}_w^H \mathbf{b} - \mathbf{A}\hat{\mathbf{s}} \right\|_2^2 \tag{9}$$

where $\mathbf{A} = \mathbf{H}_w^H \mathbf{H}_w + N_0 \mathbf{I}_U$ is a positive definite matrix, representing Gram matrix. The proposed method can be used to approximate the ith iteration signal vector s as shown below;

$$\hat{\mathbf{s}} = \hat{\mathbf{s}}_{i-1} + \mathbf{P}_i \boldsymbol{\alpha}_i \tag{10}$$

where $\mathbf{P}_i$ is an $U \times t$ projection matrix and $\boldsymbol{\alpha}_i$ is a vector of size t.

Algorithm 1 summarizes our MSD-CG-based approach for soft output data detection and the computed flops per iteration. The MMSE estimate has been iteratively realized without matrix inversion that has an algorithm with low complexity signal with the help of MSD-CG method. Moreover, the proposed MSD-CG-based algorithm is more advantageous as provided by its complexity analysis algorithm. On line 1, we obtain the Gram Matrix and the match vector as in (5) and (6), respectively. Besides, on line 2, we first set a diagonal approximate initial solution to the method above to reduce complexity and accelerate the rate of convergence. Finally, we proceed to compute the noise-plus-interference (NPI) variance and the channel gain for the log-likelihood ratio (LLR) computation using an appropriate method that does not need to calculate the exact value of the inverted matrix as follows.

LLR Approximation for the proposed method : To figure out the LLR values for the proposed method using MSD-CG method, we estimate an approximation based on the method proposed in [8, 17] for single-carrier frequency-division multiple access-based (SC-FDMA) and orthogonal frequency-division multiplexing-based (OFDM) systems. This method approximates the channel gains by

$$\bar{\mu}_{w,i} = \mathbf{h}_{w,i}^{-1}\mathbf{g}_{w,i} \tag{11}$$

where $\mathbf{h}_{w,i}^{-1}$ represents the ith inverse squared column norm of the $\mathbf{H}_w$ and $\mathbf{g}_{w,i}$ indicates the entry in the ith main diagonal of the Gram matrix $\mathbf{A}_w$. Moreover, the SINR approximation $\bar{\rho}_{w,i}$ is given

$$\bar{\rho}_{w,i} = \frac{\bar{\mu}_{w,i}}{1 - \bar{\mu}_{w,i}} \tag{12}$$

Please refer to [8] for more details.

Computational complexity for the proposed method : The Computational complexity is analyzed as shown in Algorithm 1. We compute the required number of multiplications for each step in the proposed method. The total number of multiplications at each iteration is given by:

$$O = 12Ut + 6U - 3t \tag{13}$$

where t represents the number of search directions. Note that $Itr_{\max}t$ should be much smaller than U, so that the computational complexity of proposed algorithm is less than $O(U^3)$. To that end, we compare the computational complexity of the proposed method with other methods in the literature in the next section.

Algorithm 1: MSD-CG for soft-output MMSE detection	**Flops**
Input: H, **the N×** U **the channel matrix**y, **the** $N \times$ **1the received vector**x_0,**the initial guess**t, **Number of the Subdomains (search directions)**Itr_{max}, **the maximum allowed iterations Output:** x_{itr},**the approximate solution**	
1:$G_w = H_w{}^H H_w, A = G_w + N_0 I_U, b = H_w{}^H y_w$	
2: $r = b - Ax_0$, $x_0 = inv(D-L)b$, $Itr = 1$	$4U - 1$
3: $P_1 = \mathcal{T}(r_0)$, $W_1 = AP_1$	$2U + U(t-1)$
4: *While* $\left(Itr < Itr_{max}\right)$ *do*	
5: $\alpha = P_{itr}^t r$	$(2U-1)t$
6: $x_{itr} = x_{itr-1} + P_{itr}\alpha$	$(2t-1)U + U$
7: $r = r - W_{Itr}\alpha$	$(2t-1)U + U$
8: $\beta = -\ W_{itr}^t r$	$(2U-1)t$
9: $P_{itr+1} = \mathcal{T}(r) + P_{itr}diag(\beta)$	$2Ut$
10: $W_{Itr+1} = A\mathcal{T}(r) + W_{Itr}diag(\beta)$	$2U - U(t-1) + 2Ut$
11: Compute $\hat{\mu}_{w,i}$ **in (11)**	
12: Compute$\hat{\rho}_{w,i}$ **in (12)**	
13: $Itr = Itr + 1$	1
14: ***End While***	

Algorithm 1: MSD-CG for soft-output MMSE detection	Flops
* $\mathcal{T}(x)$ **represents the method that transforms the** $U \times 1$ vector x **in** t **vectors size** $U \times 1$ **that correspond to the projection of** x **onto the subdomains** δ_i **for** $i = 1, 2, \dots, t$.	

4 Simulation Results

To confirm the working of the signal detection approach proposed in this paper, The simulation results of BER performance are carried out to compare the proposed method with the CG method which is the recently proposed algorithm [10]. We also compare it with other benchmark algorithms such as Neumann method [12], SOR Method [11], Richardson method [13] and GS method [14]. For these methods, the default setting parameters are used according to their toolboxes and their publications. Besides, the MMSE algorithm is included in the study as a reference point in the comparison due to its BER performance although it involves an exact but complicated approach of matrix inversion. In this regard, we focus on several massive MIMO systems which have $U \times N = 8 \times 32$, $U \times N = 8 \times 64$, $U \times N = 16 \times 64$, $U \times N = 8 \times 128$, $U \times N = 16 \times 128$ and $U \times N = 24 \times 128$, respectively.

We also employ the 64 QAM modulation scheme and the rate-1/2 which is a convolutional code [15]. We also adopt flat Rayleigh channel which are always fading. The soft-information is then extracted after the detection of multi-user signal for channel decoding using the estimated signal vector (by calculating the LLRs which is the log-likelihood ratios). The experiments have been carried out using the MATLAB software on an Intel Core i7 CPU 2.4-GHz processor and 4G MB RAM.

First, we compare the performance of the proposed method based on MSD-CG method with $t = 2$ and all other methods; when $U \times N = 8 \times 128$. Figures. 1 and 2 shows the simulation results of BER vs. SNR of the presented detectors for number of iterations equals two and three, respectively. The parameters are set as: Number of antennae $N = 128$ and Number of users $U = 8$ with various values of SNR from -10 dB to 20 dB. Figures 1 and 2 demonstrates that the proposed algorithm, based on MSD-CG method, producing the lowest BER consistently in comparison to all others, and thus it outperforms the other iterative detectors at $Itr = 2$ and $Itr = 3$, respectively. We also observe that the proposed algorithms work even in cases which cause difficulties for other methods, as when the iteration number is fairly small ($Itr = 2$). It is obvious that the performance of the Iterative-based methods degrade as the number of iterations decreases.

To that end, Table 1 shows the corresponding comparison between the different algorithms in terms of the average computation times for the $U \times N = 8 \times 128$ case. Table 1 shows that the proposed method is comparable to the other algorithms even it improve the BER performance for iterative methods. Moreover, Table 1 also shows that the CG method and Richardson approaches is less complex than the others algorithm. However, with

Table 1 $U \times N = 8 \times 128$ Case: Average computational times for each method (in sec)

	MMSE	CG	SOR	Neurmann	Richardson	GS	MSD-CG
$Itr_max = 2$	9.04e–05	5.01e–05	1.09e–04	9.63e–05	4.91e–05	1.03e–04	6.75e–05
$Itr_max = 3$	1.02e–04	8.15e–05	1.31e–04	1.26e–04	5.41e–05	1.31e–04	9.15e–05

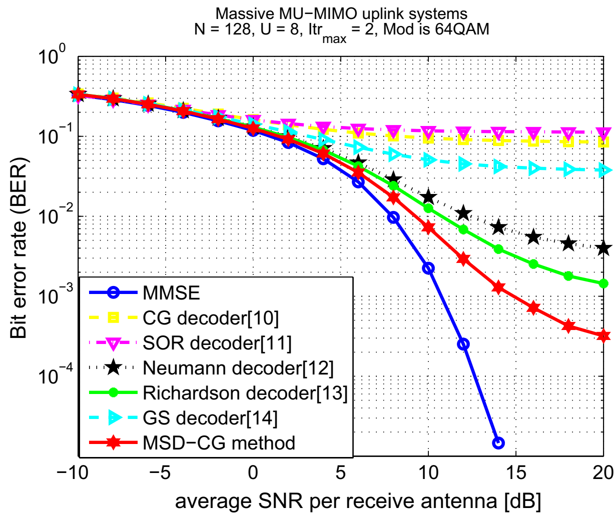

Fig. 1 BER performance comparison between the proposed approximated method and other methods to compute LLRs

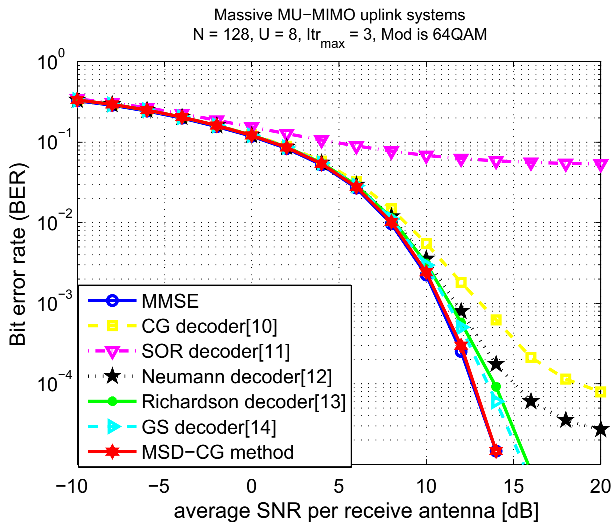

Fig. 2 BER performance comparison between the proposed approximated method and other methods to compute LLRs

the advent of more powerful computing platforms including Graphics Processing Units (GPUs) the performance accuracy holds more merit. Moreover, the proposed algorithm requires a 10 dB SNR to obtain the desired BER of 10−3 in Fig. 3. While, the GS-based and the Neumann-based methods require SNRs of 12 dB and 13 dB to obtain the same desired BER of $10^(- 3)$, respectively.

It is also worthwhile to compare the presented algorithm with a relatively large user set. Thus, Figs. 3 and 4. Present the performance of the proposed method based on MSD-CG method with $t = 2$ and others aforementioned methods; when $U \times N = 16 \times 128$. As shown in Figs. 3 and 4, the proposed method again obtains the performance of the

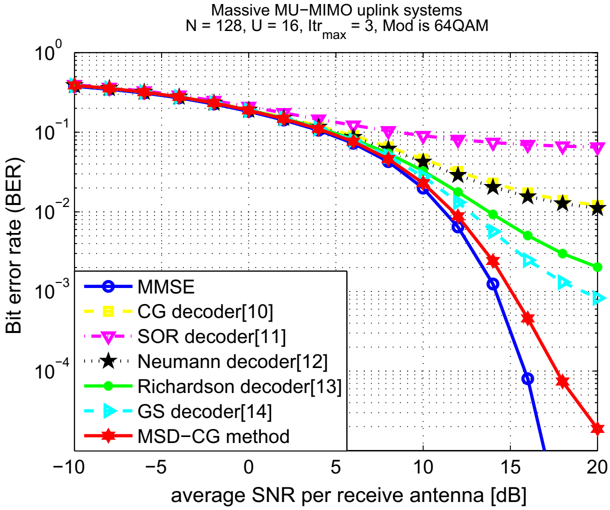

Fig. 3 BER performance comparison between the proposed approximated method and other methods to compute LLRs

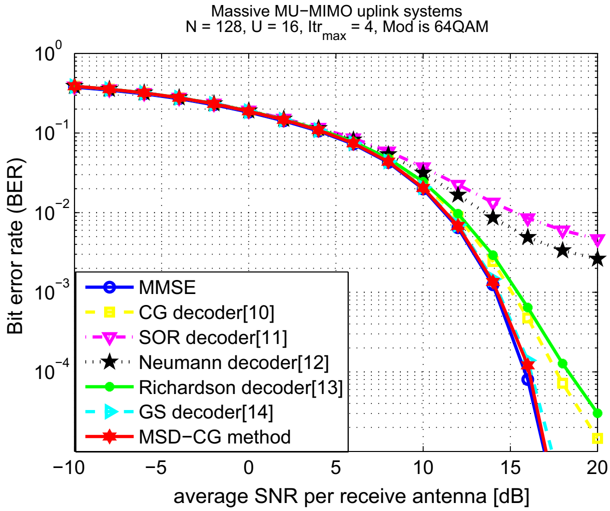

Fig. 4 BER performance comparison between the proposed approximated method and other methods to compute LLRs

MMSE algorithm and outperforms other presented algorithms in the literature. One can observe that the proposed method performs more consistently and exhibits improvement in the performance as number of iterations increases. Additionally, the comparison between the different approaches in terms of the average computation times presented in Table 2. The proposed method is comparable to the other algorithms even it improves the BER performance as shown in Table 2. Furthermore, a commonly used measure of the computational complexity of an algorithm is in terms of the number of real-valued multiplications. To study the performance of the proposed detector and to compare the detectors in terms of computational complexity, the number of real-valued multiplications performance index in [34] is used and averaged over 500 independent realizations of the received data. Figure 5 presents the comparison of computational complexity among the proposed algorithm, MMSE, Neumann , GS method and Richardson method,

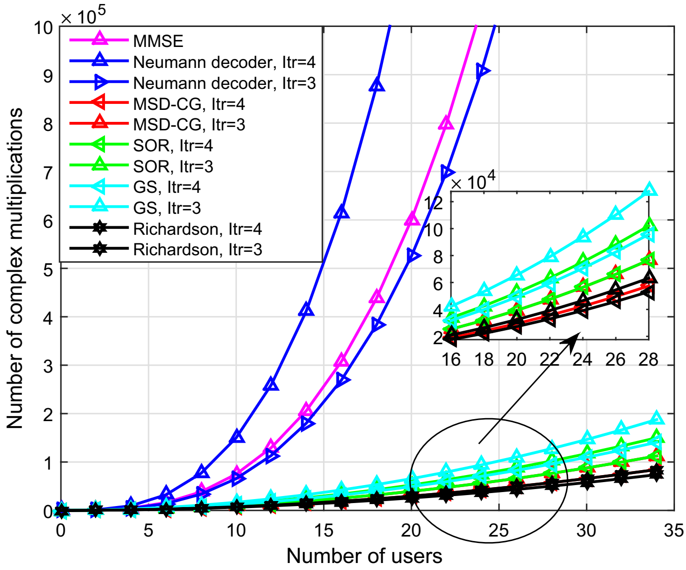

Fig. 5 The computational complexity

Table 2 $U \times N = 16 \times 128$ Case: Average computational times for each method (in sec)

	MMSE	CG	SOR	Neurmann	Richardson	GS	MSD-CG
$Itr_m ax = 3$	1.42e–05	9.32e–05	2.35e–04	3.25e–04	1.05e–04	2.27e–04	1.12e–04
$Itr_m ax = 4$	1.44e–04	1.23e–04	2.84e–04	4.12e–04	1.27e–04	2.83e–04	1.37e–04

where the proposed algorithm requires substantially less than the other two algorithms when the number of iterations is small. Although the Richardson algorithm requires a slightly lower complexity than the proposed algorithm when its number of iterations is three, the system BER performance of the MSD-CG method outperforms the Richardson algorithm as shown in Figs. 3 and 4.

It is also worthwhile to compare the proposed algorithms with various ($p = N/U$) ratio. Thus, the resulting BER performance is shown in Fig. 6a–d. Figure 6 presents comparison between the proposed algorithm with the method of CG, the GS-based method, the conventional method based on Neumann and others in the literature at different p ratio. We observed that the proposed algorithm performs well with different number of antenna and users. It is also shown that the BER performance of the MMSE algorithm becomes closer to that of all conventional approaches when there is an increased number of iteration. However, when a similar number of iteration is employed, then the proposed method becomes more superior as compared to the other approaches. Moreover, one can observe that as the value of N increases, there is an associated improvement in the performance of the MMSE approach. The performance of the all methods becomes better with an increase in the number of iteration although it still experiences a performance loss for the conventional ones. This provides a reason as to why the other conventional ones in the literature are less superior to the proposed

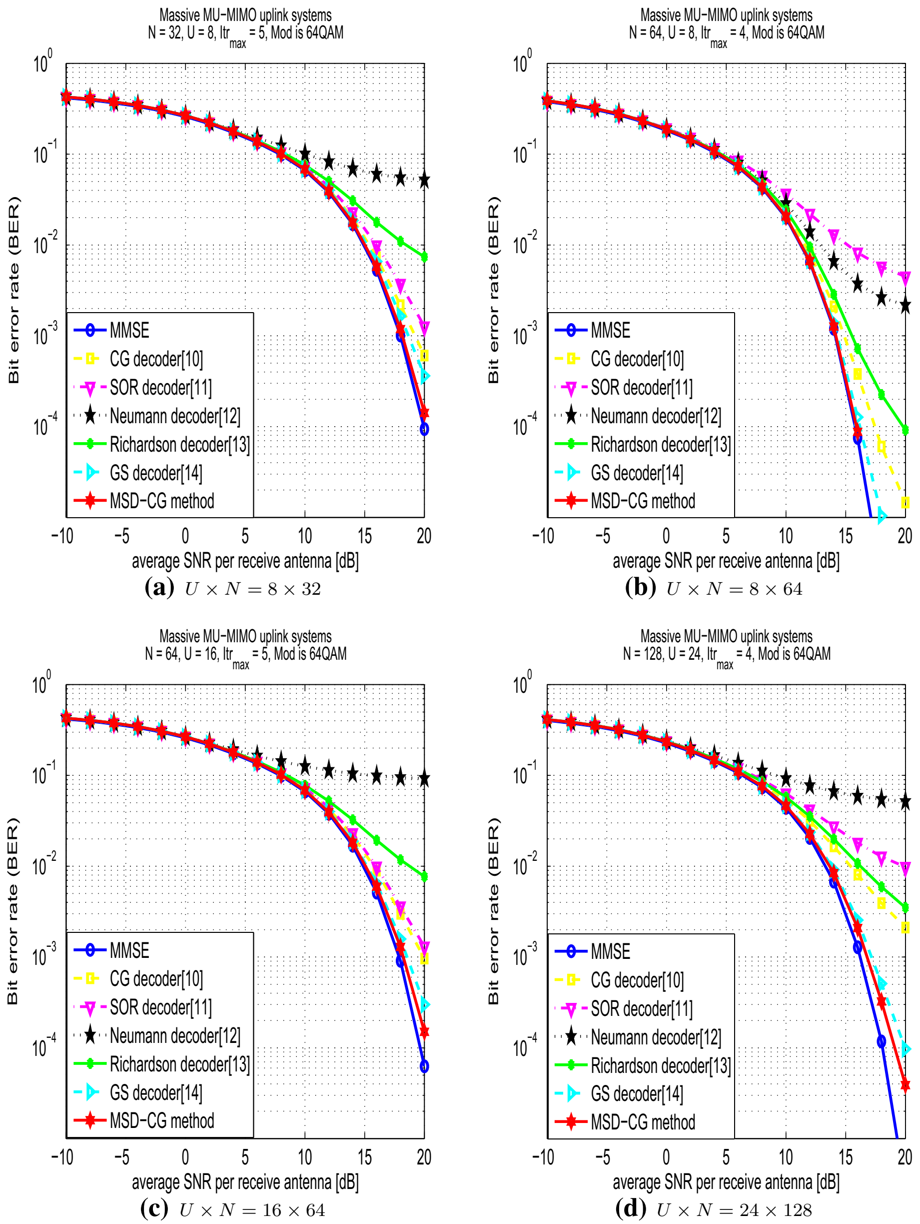

Fig. 6 BER performance comparison in the massive MIMO uplink for **a** $U \times N = 8 \times 32$, **b** $U \times N = 8 \times 64$, **c** $U \times N = 16 \times 64$, and **d** $U \times N = 24 \times 128$

algorithm. In additional, The Neumann series method performs well in the case of ($N \times U = 128 \times 8$), that emphasizes the perception in [12] that this method requires a large BS to user ratio ($p = N/U$).

5 Conclusions

In this study, a low complexity signal detection method based on the MSD-CG algorithm is proposed. The proposed algorithm based on the MSD-CG method estimates the transmitted signal by iteratively solving the linear equation and avoiding the matrix inverse operation in the MMSE approach. However, the complexity is substantially reduced from $O(U^3)$ to $O(U^2)$.The method is adapted for massive MIMO uplink system to bypass the high-dimensional matrix inversion problem demanded by the MMSE criterion. The adapted MSD-CG algorithm iteratively estimates the transmitted signal due to eliminating the need of matrix inverse operation. Also, the presented method reduces the system complexity and enhances overall system performance by employing an initial solution using the diagonal-approximate. This results in speeding up the rate of convergence of the detection process. However, with a small number of iterations, it was proved that the simulation results and the convergence of the MSD-CG method can attain the near-optimal achievement of the algorithm by the MMSE method. Moreover, the idea behind using the MSD-CG approach can also be applied to wireless communication such as precoding or other signal processing problems in massive MIMO system.

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