ORIGINAL ARTICLE

Numerical simulation of peristaltic flow of a Carreau nanofluid in an asymmetric channel

Noreen Sher Akbar a,*, S. Nadeem b, Zafar Hayat Khan c

a DBS&H, CEME, National University of Sciences and Technology, Islamabad, Pakistan
b Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan
c School of Mathematical Sciences, Peking University, Beijing 100871, PR China

Received 22 August 2013; revised 12 October 2013; accepted 19 October 2013

KEYWORDS
Peristaltic flow; Nanofluid; Carreau fluid model; Asymmetric channel; Fourth and fifth order Runge–Kutta–Fehlberg method

Abstract In this article, we studied MHD peristaltic flow of a Carreau nanofluid in an asymmetric channel. The flow development is carried out in a wave frame of reference moving with velocity of the wave \( c_1 \). The governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and then tackled numerically using the fourth and fifth order Runge–Kutta–Fehlberg. Numerical results are obtained for dimensionless velocity, stream function, pressure rise, temperature and nanoparticle volume fraction. It is found that the pressure rise increases with increase in Hartmann Number and thermophoresis parameter.

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1. Introduction

Nanofluids are moderately new category of fluids which consist of a base fluid with nano-sized particles (1–100 nm) suspended within them. The term ”nanofluid” was first proposed by Choi [1]on Argonne National Laboratory at 1995. He investigated that nanofluids reduced pumping power as compared to pure liquid to achieve equivalent heat transfer intensification and particle clogging as compared to conventional slurries, thus promoting system miniaturization. Xuan and Roetzel [2], analyzed theoretically the flow of a nanofluid inside a tube using a dispersion model. Heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids is presented by Khanaf et al. [3] for various pertinent parameters. They developed model to analyze heat transfer performance of nanofluids inside an enclosure taking into account the solid particle dispersion. Review of convective heat transfer enhancement with nanofluids is given by Sadik and Pramuanjaroenkij [4]. The literature survey of Sadik and Pramuanjaroenkij [4] shows that nanofluids significantly improve the heat transfer capability of conventional heat transfer fluids such as oil or water by suspending nanoparticles in these base liquids. In the peristaltic literature nanofluid was first introduced by Nadeem and Akbar [5]. They discussed endoscopic effects on the peristaltic flow of a nanofluid. They made the analysis in two concentric tubes and discussed the applications of endoscope. Recent articles on the nanofluids are cited in Refs. [6–11].

* Corresponding author. Tel.: +92 05190642182.
E-mail address: noreensher@yahoo.com (N.S. Akbar).
Peer review under responsibility of Faculty of Engineering, Alexandria University.
Peristalsis is a mechanism of fluid flowing by means of moving contraction on the tubes/channels walls. This analysis was first discussed by Latham [12]. He presented fluid motion in peristaltic pump. He also discussed the characteristic of pressure rise versus flow rate. After the pioneering work of Latham [12], Jaffrin and Shapiro [13] investigated the peristaltic pumping. They made the analysis under the assumption of long wave length and low Reynolds number approximations. Later on a number of analytical, numerical and experimental studies of peristaltic flows of different fluids have been reported under different conditions with reference to physiological and mechanical situations. Peristaltic pumping by a sinusoidal traveling wave in the porous walls of a two-dimensional channel filled with a viscous incompressible conducting fluid under the effect of transverse magnetic field is investigated theoretically and graphically by El-Shehawey and Husseny [14]. Non-linear peristaltic transport of a Newtonian fluid in an inclined asymmetric channel of width 1 + d, 1 + b, 1 + d, 1 + d, and 1 + d, and 1 + d, is the infinite shear rate viscosity, 1 + d, is the extra stress tensor, and 1 + d, is the wave length, 1 + d, has a sinusoidal wave propagating with constant speed 1 + d, and streamlines have been presented in section three. Conclusion, the aim of the current study is to discuss the peristaltic flow of Carreau nanofluid in an asymmetric channel. The study of Carreau nanomodel for peristaltic flow problems is not explored so far. Therefore, to fill this gap in the present analysis we have discussed the peristaltic flow of Carreau fluid model with nanoparticle phenomena in an asymmetric channel. The article presentation is carried out as follows. Next section describes the mathematical formulation of the problem. Numerical solution graphically and in tabulated form for velocity, pressure rise, temperature, nanoparticle phenomena and streamlines have been presented in section three. Conclusion, the article presentation is written in the last section of the article.

2. Mathematical formulation

Let us discussed an incompressible Carreau fluid with nanoparticle in an asymmetric channel of width 1 + d, 1 + d, and 1 + d, 1 + d, 1 + d, and nanoparticle concentrations 1 + d, 1 + d, are given to the upper and lower wall of the channel. The wall surfaces are selected to satisfy the following expressions

\[
Y = H_1 = d_1 + a_1 \cos \left( \frac{2\pi}{\lambda}(X - ct) \right),
\]

\[
Y = H_2 = -d_2 - b_1 \cos \left( \frac{2\pi}{\lambda}(X - ct) + \phi \right).
\] (1)

In the above equations 1 + d, 1 + d, and 1 + d, are the waves amplitudes, 1 + d, is the wave length, 1 + d, 1 + d, is the channel width, 1 + d, is the wave speed, 1 + d, is the time, 1 + d, is the direction of wave propagation and 1 + d, is perpendicular to 1 + d. The phase difference 1 + d, varies in the range \(0 \leq \phi \leq \pi\). When 1 + d, = 0 then symmetric channel with waves out of phase can be described and for 1 + d, = \(\pi\), the waves are in phase. Moreover, 1 + d, 1 + d, 1 + d, and 1 + d, satisfies the following relation

\[
d_1^2 + b_1^2 + 2a_1b_1 \cos \phi \leq (d_1 + d_2)^2.
\]

The coordinates, velocity components and pressure between fixed and wave frames are related by the following transformations:

\[
\begin{align*}
\tilde{x} &= X - c_1 t, \quad \tilde{y} = Y, \quad \tilde{u} = U - c_1, \quad \tilde{v} = V, \quad \tilde{p}(\tilde{x}, \tilde{t}) = P(X, t).
\end{align*}
\] (2)

In which 1 + d, 1 + d, 1 + d, and 1 + d, are the coordinates, velocity components and pressure in the wave frame.

The constitutive equation for Carreau fluid is given by [19]

\[
\begin{align*}
\frac{(\eta - \eta_w)}{(\eta_0 - \eta_w)} &= \left( 1 + \frac{(\Gamma_g)^2}{2} \right)^{-2}, \\
\tilde{\tau}_g &= \eta_0 \left[ 1 + \frac{(n - 1)}{2} \left( \frac{G_s}{\mu} \right)^2 \right] \tilde{\gamma},
\end{align*}
\] (3a, b)

in which 1 + d, is the extra stress tensor, 1 + d, is the infinite shear rate viscosity, 1 + d, is the zero shear rate viscosity, 1 + d, is the time constant, 1 + d, is the Power law index and 1 + d, is defined as

\[
\tilde{\gamma} = \sqrt{\frac{1}{2} \sum \tilde{\epsilon}_i \tilde{\epsilon}_j} = \sqrt{\frac{1}{2} \Pi}.
\] (3c)

Here 1 + d, is the second invariant strain tensor.

Velocity stream function relation and nondimensional quantities are

\[
u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x},
\] (4)

\[
\begin{align*}
x &= \frac{2\pi \tilde{x}}{\lambda}, \quad y = \frac{\tilde{y}}{c_1}, \quad u = \frac{\tilde{u}}{c_1}, \quad v = \frac{\tilde{v}}{c_1}, \quad t = \frac{2\pi t}{\lambda}, \quad \delta = \frac{2\pi d_1}{\lambda},
\end{align*}
\] (5)

\[
\begin{align*}
d &= \frac{d_1}{d_2}, \quad P = \frac{2n \tilde{P}}{\mu c_1 \lambda}, \quad h_1 = \frac{h_1}{d_2}, \quad h_2 = \frac{h_2}{d_2}, \quad Re = \frac{\rho c_1 d_1}{\mu}, \quad a = \frac{a_1}{d_2},
\end{align*}
\] (6)

\[
\begin{align*}
b &= \frac{a_2}{d_2}, \quad d &= \frac{d_2}{d_1}, \quad S = \frac{3d_1}{a_1}, \quad We = \frac{c_1}{Re}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \frac{\sigma}{c_1} = \frac{p_e D_n(T_1 - T_0)}{Q},
\end{align*}
\] (7)

\[
N_r = \frac{(pc_1 D_n(T_1 - T_0))^2}{Q}, \quad G_r = \frac{g \omega d_1^2(T_1 - T_0)}{Q c_1},
\] (8)

Using Eqs. 2, 3, 4, 5, 6 after using the long-wavelength and low-Reynold’s number approximation, we finally obtain the following system of equations for Carreau nanofluid.

\[
\begin{align*}
\frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial x^2} + \frac{(n - 1)}{2} We^2 (\frac{\partial^2 \Psi}{\partial y^2})^3 - M^2 \Psi + G_s \frac{\partial \theta}{\partial y} + B_s \frac{\partial \sigma}{\partial y} &= 0, \quad (6)
\end{align*}
\] (9)

\[
\begin{align*}
\frac{dP}{dx} &= \frac{\partial \Psi}{\partial y} + \frac{(n - 1)}{2} We^2 (\frac{\partial^2 \Psi}{\partial y^2})^3 - M^2 \Psi + G_s \frac{\partial \theta}{\partial y} + B_s \sigma, \quad (7)
\end{align*}
\] (10)

\[
\begin{align*}
0 &= \frac{\partial^2 \theta}{\partial y^2} + Pr N_r \frac{\partial \sigma}{\partial y} + Pr N_r \frac{\partial \Psi}{\partial y}, \quad (8)
\end{align*}
\] (11)
where $Pr$, the Prandtl number, $We$ is the Weissenberg number, $Nb$, the Brownian motion parameter, $Nt$ the thermophoresis parameter, $Gr$, the local temperature Grashof number, $M$ the Hartmann number, $Br$ local nanoparticle Grashof number.

Corresponding boundary conditions for asymmetric channel in nondimensional form take the following form

\[
\Psi = \frac{F}{2} \frac{\partial \Psi}{\partial y} = -1, \text{ at } y = h_1 = 1 + a \cos x, \quad (10a)
\]

\[
\Psi = -\frac{F}{2} \frac{\partial \Psi}{\partial y} = -1, \text{ at } y = h_2 = -d - b \cos(x + \phi), \quad (10b)
\]

\[
\theta = 0, \sigma = 0, \quad v = h_1 = 1 + a \cos x, \quad (10c)
\]

\[
\theta = 1, \sigma = 1, \quad v = h_2 = -d - b \cos(x + \phi) \quad (10d)
\]

The time mean $Q$ (in the wave frame) are defined as

\[
Q = F + 1 + d. \quad (11)
\]

The dimensionless pressure rise $\Delta P$ is defined as

\[
\Delta P = \int_0^1 \left( \frac{dP}{dx} \right) dx. \quad (12)
\]

### 3. Results and discussion

We have examined the pressure rise, pressure gradient, velocity, temperature, nanoparticle phenomena and streamlines for Carreau nanofluid model numerically. In order to analyze the pressure rise per wavelength the pressure rise against volume flow rate is portrayed in Fig. 1(a–f). It is observed that the

![Figure 1](http://dx.doi.org/10.1016/j.aej.2013.10.003)

**Figure 1** (a–f) Pressure rise versus flow rate.
Figure 2  (a–f) Velocity profile.

Figure 3  (a–c) Temperature profile.
Figure 4  (a–c) Nano particle profile.

Figure 5  Streamlines for $a = 0.5, b = 0.5, d = 1, \phi = 0.4, G_{\text{r}} = 2, B_{\text{r}} = 2, Nb = 0.5, Nt = 0.5, K = 1, Pr = 1$. 
**Table 1** Numerical values of velocity profile for \( x = 1, a = 0.1, d = 1, Nb = 0.5, Nt = 0.5, Br = 1, Gr = 5, M = 2, Q = 2, \phi = 0.4, We = 0.2, b = 0.5, n = 2 \).

<table>
<thead>
<tr>
<th>( y )</th>
<th>( u(x, y) )</th>
<th>( y )</th>
<th>( u(x, y) )</th>
<th>( y )</th>
<th>( u(x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1.10</td>
<td>−1.00000</td>
<td>−0.39</td>
<td>0.41064</td>
<td>0.39</td>
<td>0.16781</td>
</tr>
<tr>
<td>−1.00</td>
<td>−0.70803</td>
<td>−0.29</td>
<td>0.46499</td>
<td>0.49</td>
<td>0.04238</td>
</tr>
<tr>
<td>−0.89</td>
<td>−0.38724</td>
<td>−0.19</td>
<td>0.49067</td>
<td>0.59</td>
<td>−0.10135</td>
</tr>
<tr>
<td>−0.79</td>
<td>−0.14664</td>
<td>−0.09</td>
<td>0.48981</td>
<td>0.69</td>
<td>−0.26296</td>
</tr>
<tr>
<td>−0.69</td>
<td>0.04980</td>
<td>0.09</td>
<td>0.42715</td>
<td>0.89</td>
<td>−0.63894</td>
</tr>
<tr>
<td>−0.59</td>
<td>0.20594</td>
<td>0.19</td>
<td>0.36109</td>
<td>0.99</td>
<td>−0.85334</td>
</tr>
<tr>
<td>−0.49</td>
<td>0.32519</td>
<td>0.29</td>
<td>0.27432</td>
<td>1.10</td>
<td>−1.00000</td>
</tr>
</tbody>
</table>

**Table 2** Numerical values of pressure rise \( \Delta P \) for \( a = 0.5, Pr = 1, d = 1, Nb = 0.5, Nt = 0.5, Br = 2, Gr = 2, M = 2, \phi = 0.4, b = 0.5, n = 2, We = 1.5 \).

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( \Delta P )</th>
<th>( Q )</th>
<th>( \Delta P )</th>
<th>( Q )</th>
<th>( \Delta P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−3.0</td>
<td>11.59503</td>
<td>−1.0</td>
<td>6.13636</td>
<td>1.0</td>
<td>0.597007</td>
</tr>
<tr>
<td>−2.5</td>
<td>9.87075</td>
<td>−0.5</td>
<td>4.97221</td>
<td>1.5</td>
<td>−1.46174</td>
</tr>
<tr>
<td>−2.0</td>
<td>8.45405</td>
<td>0.0</td>
<td>3.74410</td>
<td>2.5</td>
<td>−7.02264</td>
</tr>
<tr>
<td>−1.5</td>
<td>7.26583</td>
<td>0.5</td>
<td>2.30386</td>
<td>3.0</td>
<td>−10.7076</td>
</tr>
</tbody>
</table>

The trapping for different values of \( We, n \) and \( M \) is shown in Fig. 5(a–f). It is seen from Fig. 5(a and b) that the size of the trapping bolus decreases with an decrease in \( We \) (in the upper and lower part of the channel). Fig. 5(c and d) shows that the size of trapping bolus increases with an increase in \( n \) in the upper and lower parts of the channel. Influence of \( M \) on streamlines is shown through Fig. 5(e and f), it is observed that with the increase in \( M \) size of trapping bolus decreases but number of bolus increases. Tables 1 and 2 gives the numerical values for velocity profile and pressure rise.

4. Concluding remarks

The present study discussed the MHD peristaltic flow of a Carreau nanofluid in an asymmetric channel. The main points of the current analysis are as follows:

1. The qualitative behaviors of Hartmann number \( M \), Power law index \( n \), Weissenberg number \( We \), temperature Grashof number \( Gr \) and amplitude ratio \( a \) on pressure rise are same.
2. The pressure rise increases with the increase of Hartmann number \( M \), Power law index \( n \), Weissenberg number \( We \), the thermophoresis parameter \( N_t \), temperature Grashof number \( Gr \) and amplitude ratio \( a \) on the pressure rise are same.
3. The behavior of velocity near the channel walls and at center is not similar in view of the Hartmann number \( M \) and Power law index \( n \).
4. The velocity field increases with the increase in \( M \) and \( n \) near the channel walls however it decreases in the center of the channel.
5. Velocity field increases with an increase in flow rate \( Q \).
6. The temperature profile increases when thermophoresis parameter \( N_t \), Prandtl parameter \( P_r \) and Brownian motion parameter \( N_b \) are increased.

Please cite this article in press as: N.S. Akbar et al., Numerical simulation of peristaltic flow of a Carreau nanofluid in an asymmetric channel, Alexandria Eng. J. (2013), http://dx.doi.org/10.1016/j.aej.2013.10.003
Numerical simulation of peristaltic flow of a Carreau nano fluid in an asymmetric channel

7. Nanoparticle phenomena decreases when there is an increase in the values of thermophoresis parameter $N_t$, prandtl parameter $P_r$, and increases with an increase in Brownian motion parameter $N_b$.
8. Temperature profile increases with an increase in $N_t$ and $N_b$.
9. Nanoparticle phenomena decreases with in $N_t$ and increases with in $N_b$.
10. The size of the trapping bolus decreases with the decrease in $We$ (in the upper and lower part of the channel).
11. The size of trapping bolus increases with the decrease in $n$ in the upper and lower parts of the channel.
12. It is observed that with the increase in $M$ size of trapping bolus decreases but number of bolus increases.

References


