Current Closed Loop Control for Increasing Virtual Stiffness in Haptic Interaction

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Abstract—Virtual stiffness is a critical performance measure for haptic devices. This paper proposes a new controller to increase the virtual stiffness. The design of the controller is based on theoretical analysis of the system stability condition, which indicates that by adding a current closed loop the stability can be increased. We implemented the current closed loop controller with a first order filter and a PID control law. When properly selected parameters of the filter and the PID controller, it is possible to increase the maximum achievable virtual stiffness. Experimental results demonstrate the effectiveness of the proposed method.

I. INTRODUCTION

In a haptic interaction system, operators manipulate the haptic device to explore the virtual environment to achieve a virtual sense of touch. Stability is a prerequisite for the haptic interaction system [1]. The increase of the stability contributes to more applications that the haptic device can be applied to. For haptic interaction, the stability can be represented by the range of the virtual stiffness that a haptic system can stably present.

J. Colgate et al [2] derived the stability condition which showed that there existed a maximum achievable virtual stiffness for given sampling period, device damping and virtual damping. It indicated that increasing the device damping or decreasing the sampling period can enlarge the maximum virtual stiffness. Blake Hannaford [1] proposed the PO/PC method to ensure stable contact by looking at the energy. J. Kim et al [3] also derived the same stability condition and proposed the energy bounding algorithm for stable haptic interaction. R. Adams et al [4] proposed an approach to ensure the stability of haptic devices through the virtual coupling network. K. Lee et al [5] applied multirate control to improve the stability. K. Akahane et al [6] introduced a 10 KHz high definition haptic rendering for both fidelity and stability. For the same purpose, J. Kim et al [7] proposed a multirate control framework with energy bounding algorithm. M. Kawai et al [8] added an additional mechanical damping to haptic device for improving the stability. However, the additional mechanical damping, which not only exists in the constraint space but also the free space, reduced the back driven ability. J. Mehling et al [9] proposed to enlarge the system damping by using electronic damping. This technique is limited by the driven motor and the parameters are difficult to choose.

On the other hand, some previous researchers proposed force feedback control, focusing on reducing the inertia and gravity of the haptic device to improve the fidelity. B. Bae et al. [10] used a current sensor instead of high-cost force/torque sensor for improving the resolution of torque reflection. D. Borro et al [11] proposed a control strategy to minimize the device inertia by estimating the operator’s intention with force sensors. M. Ueberle et al [12] proposed to compensate the inertia and gravity of haptic devices by force feedback control.

This paper proposes a current closed loop control strategy aiming at enlarging the range of virtual stiffness and improving the stability. Section 2 illustrates the current closed loop control model. Section 3 presents the stability condition under this control strategy. Section 4, in terms of experiments, validates that the current closed loop control is efficient and feasible in improving the stability. Section 5 is the conclusion of this paper and points out the future work.

II. CURRENT CLOSED LOOP CONTROL MODEL

The haptic interaction system is composed of operator, haptic device and virtual environment. The operator is assumed to be passive and constant while he applies active force and is in force self-control at the same time. The operator passive impedance is modeled as a linear time invariant 2nd order model \( H(s) \) with mass \( M_h \), damping \( B_h \) and spring \( K_h \) [13] [14]. The haptic device impedance is modeled as 2nd order model \( D(s) \) with mass \( M_d \) and damping \( B_d \). The virtual environment is modeled as spring model \( E(z) \) with stiffness \( K_e \).

\[
H(s) = M_h s^2 + B_h + \frac{K_h}{s} \quad (1)
\]

\[
D(s) = M_d s + B_d \quad (2)
\]

\[
E(z) = \frac{TK_{e} z}{z - 1} \quad (3)
\]

Fig 1 illustrates the traditional open loop impedance control model of the haptic interaction. In Fig 1, \( T, 1/\tau \), and \( ZOH \) respectively denote the sampling period, the computational time delay and the zero order holder with the sampling period \( T \); \( \dot{X} \), \( X \) and \( X_r \) denote the velocity, displacement and sampling displacement of haptic device; \( F_{active}, F_{fp}, F_{f} \) denote the operator’s active force, passive force and the composition of forces to the haptic device; \( F_{v} \) and \( F_{f} \) denote the virtual force computed by virtual environment and the composition of forces on the end of
haptic device.

The overall system transfer function is,

\[
\frac{\dot{X}(s)}{F_{\text{active}}(s)} = (M_d + M_s + 0.5K_bT^2)s^2 + (B_d + B_s - 1.5K_bT)s + K_s + K_e
\]  

Therefore, the stability condition is as follows:

\[
K_e < \frac{B_d + B_s}{1.5T}
\]  

Compared with the traditional open loop impedance control, the current closed loop control is implemented by adding a current sensor in the motor circuit for the current feedback. In this way, the output torque of the motor can be indirectly obtained as well as the force imposed on haptic device. Fig. 2 illustrates the current closed loop control model. In Fig. 2, the models of the operator and device are the same as those in Fig. 1.

Fig. 1. System block diagram of the open loop impedance control

Fig. 2. Discrete system block diagram of the current closed loop control

Firstly, the current of the motor is measured by the sensor. The current sampling period is \(T\). Then the imposed force on haptic device \(F_m\) by driven motor is calculated as equation (6). \(J\) is the velocity Jacobi matrix, \(K_e\) is the torque constant of the driven motor.

\[
F_m = J^{-1}K_eI
\]  

Secondly, the first order filter is utilized to smooth the imposed force and eliminate its high-frequency element, especially when the device becomes unstable. \(F(z)\) is the first order filter.

Finally, the error feedback control with a PID controller is implemented. The error between the virtual force \(F_v\) and the filtered force signal is as the input of the PID controller, the output of the PID controller is added to the virtual force \(F_v\) as the control signal to the driven motor. \(G(z)\) denotes the PID controller.

In Fig. 2, considering the servo frequency of the PID controller is much higher than the 1 KHz servo frequency of haptic rendering, the PID controller can be regarded as time continuous, as shown in Fig 3. Compared with Fig 1, \(F(s)\) and \(G(s)\) denote the transfer functions of the first order filter and the PID controller.

\[
F(s) = \frac{1}{\tau s + 1}
\]  

\[
G(s) = \frac{K_p + K_i}{s + K_d s}
\]  

III. STABILITY CONDITION

In terms of the current closed loop control model, as shown in Fig. 3, the relation between \(F_m(s)\) and \(F_v(s)\) is shown as equation (9)

\[
F_m(s) = \frac{1 + G(s)}{1 + G(s)F(s)}F_v(s)
\]  

Substituting the transfer functions of the filter and the PID controller into equation (9),

\[
F_m(s) = \text{coef} \cdot F_v(s) + \frac{\tau K_d}{\tau + K_d} sF_v(s) + g_i(s)F_v(s)
\]  

Where:

\[
\text{coef} = \frac{\tau^2(1 + K_p)(\tau + K_d)}{(\tau + K_d)^2}
\]

\[
g_i(s) = \frac{(1 + K_p)(1 - \text{coef})s + \frac{\tau^2 K_i}{\tau + K_d}s + (1 - \text{coef})K_i}{(\tau + K_d)s^2 + (1 + K_p)s + K_i}
\]

Given equation (10), in order to make the operator feel that the virtual stiffness is the same as that without the current closed loop control, let \(\text{coef} = 1\), then,

\[
K_p = \frac{K_d}{\tau}
\]  

At the same time, assuming \(K_i = 0\), then the equation (10) becomes

\[
F_m(s) = F_v(s) + \frac{\tau K_d}{\tau + K_d} sF_v(s)
\]  

Because the virtual environment model is spring model, \(E(z) = \frac{TK_e z}{z-1}\) (equation (3)), when the haptic device is in the constraint space, the virtual force should be presented as follows:

\[
F_v(s) = E(z) \cdot \frac{1}{z} \cdot \text{ZOH}(z) \cdot \text{SAMPLE}(z) \left. \frac{\dot{X}(s)}{z^{\omega s^T}} \right|_{z = e^{sT}} = \frac{K_e (2 - sT)(1 - sT)}{2s} \dot{X}(s)
\]  

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Substituting the equation (13) into equation (12),

\[
F_m(s) = \frac{K}{s} \ddot{x}(s) - \left( \frac{3T}{2} - \frac{\tau K_d}{\tau + K_d} \right) K \dot{x}(s)
\]

\[
+ \left( \frac{T^2 K_d - 3\tau K_d K_T}{2(\tau + K_d)} \right) s \ddx + \frac{\tau K_d K_T T^2}{2(\tau + K_d)} s^2 \ddx
\] (14)

In terms of the equation (4),

\[
F_{active}(s) = (H(s) + \frac{1}{D(s)}) \dot{x}(s) + F_m(s)
\] (15)

Consequently, the overall system transfer function becomes,

\[
\dot{x}(s) = \frac{s}{F_{active}(s)} = a_0 s^3 + a_4 s^2 + a_5 s + a_6
\] (16)

Where:

\[
a_0 = 0.5 \tilde{K} K_s T^3, a_1 = M + K_s T^2 (0.5 - 1.5 \tilde{K})
\]

\[
a_2 = B + K_s T (\tilde{K} - 1.5), a_3 = K_h + K_e
\]

\[
\tilde{K} = \tau K_d f((\tau + K_d) T)
\]

\[
M = M_h + M_d, B = B_h + B_d
\]

Obviously, when \( \tilde{K} = 0 \), the equation (16) is the same as equation (4). When \( \tilde{K} > 0 \) and \( a_0 > 0 \), in terms of Hurwitz Criterion, the overall system is stable if and only if:

\[
\begin{aligned}
& a_1 > 0, a_2 > 0, a_3 > 0 \\
& a_4 a_2 - a_0 a_3 > 0
\end{aligned}
\] (17)

Because \( a_5 > 0 \) is always satisfied, the stability conditions become as shown as follows:

\[
M + K_s T^2 (0.5 - 1.5 \tilde{K}) > 0
\] (18)

\[
B + K_s T (\tilde{K} - 1.5) > 0
\] (19)

\[
K_s [MT(\tilde{K} - 1.5) + 0.5 BT^2 (1 - 3 \tilde{K}) - 0.5 \tilde{K} K_s T^3]
\]

\[
+ MB \frac{2}{4} K e T^4 (2 \tilde{K}^2 - 3 \tilde{K} + 1) > 0
\] (20)

The first step is to simplify the inequality (18) and (19). The necessary and sufficient condition for the inequality (18) and (19) is,

\[
\begin{aligned}
K_s < \frac{B}{(1.5 - \tilde{K}) T} & \quad \text{when } \tilde{K} \in [0, 1.5 M + 0.5 BT] \\
K_s < \frac{M}{(1.5K - 0.5) T^2} & \quad \text{when } \tilde{K} \in [1.5 M + 0.5 BT, \infty)
\end{aligned}
\] (21)

Generally \( M_h \approx 3.36 \text{Kg} \), \( B_h \approx 5.2 \text{N} \cdot \text{s} / \text{m} \), \( K_h \approx 343.4 \text{N} / \text{m} \) [15]. According to the dynamics model of the haptic device, \( M_d \approx 0.1 \text{kg} \), \( B_d \approx 0.3 \text{N} \cdot \text{s} / \text{m} \).

Considering \( T = 10^{-3} \), then \( BT << M \). Therefore,

\[
\frac{1.5 M + 0.5 BT}{M + 1.5 BT} \approx 1.5
\] (22)

The second step focuses on the analysis and simplification of the inequality (20).

When \( \tilde{K} \in [0.5, 1] \), it can be inferred,

\[
[0.5 \tilde{K} K_s T^3] \ll [MT(\tilde{K} - 1.5)]
\]

\[
[0.5 BT^2 (1 - 3 \tilde{K})] \ll [MT(\tilde{K} - 1.5)]
\] (23)

When \( \tilde{K} \in [0, 0.5] \), the inequality (25) is satisfied,

\[
\frac{3MBT^3 (2 \tilde{K}^2 - 3 \tilde{K} + 1)}{(MT(1.5 - \tilde{K}))^2} \ll 1
\] (25)

So the necessary and sufficient condition for inequality (20) is,

\[
\begin{aligned}
K_s < \frac{B}{(1.5 - \tilde{K}) T} & \quad \text{when } \tilde{K} \in [0, 1] \\
K_s < \frac{MT(\tilde{K} - 1.5) + \sqrt{(MT(\tilde{K} - 1.5))^2 + 3MBT^3 (2 \tilde{K}^2 - 3 \tilde{K} + 1)}}{1.5T^2 (2 \tilde{K}^2 - 3 \tilde{K} + 1)} & \quad \text{when } \tilde{K} \in (1, \infty)
\end{aligned}
\] (26)

In short, when \( \tilde{K} \in [0, 1] \), the system stability condition is,

\[
K_s < \frac{B}{(1.5 - \tilde{K}) T}
\] (27)

When \( \tilde{K} \in (1, 1.5) \), the system stability condition is,

\[
K_s < \min \left\{ \frac{MT(\tilde{K} - 1.5) + \sqrt{(MT(\tilde{K} - 1.5)^2 + 3MBT^3 (2 \tilde{K}^2 - 3 \tilde{K} + 1)}}{1.5T^2 (2 \tilde{K}^2 - 3 \tilde{K} + 1)} \right\}
\] (28)

When \( \tilde{K} \in [1.5, \infty) \), the system stability condition is,

\[
K_s < \min \left\{ \frac{MT(\tilde{K} - 1.5) + \sqrt{(MT(\tilde{K} - 1.5)^2 + 3MBT^3 (2 \tilde{K}^2 - 3 \tilde{K} + 1)}}{M} \right\}
\] (29)

Compared with the system stability condition under traditional open loop impedance control (Inequality (5)), it is certainly that when \( \tilde{K} \in [0, 1] \), the maximum achievable stiffness would increase.

IV. EXPERIMENT

These experiments are conducted to derive the maximum virtual stiffness under the open loop impedance control and current closed loop control to validate that current closed loop control can enlarge the virtual stiffness. The experiments are based on a 3-DOF haptic device as shown in Fig. 4(a), the definition of coordinate system and mechanism dimension are illustrated in Fig. 4(b). The position resolution of the haptic device end point \( P \) is 0.05mm. However, in the following experiments, only the axis 1 is utilized and the axis 2 and 3 are fixed, \( \theta_1 \in [30^\circ, 150^\circ] \), \( \theta_2 \) and \( \theta_3 \) are approximately set to \(-12^\circ\) and \(18^\circ\), as shown in Fig. 4(c). The maximum output force at the end point \( P \) is set to 2.5N because of the measurement range of the current sensor. Based on the kinematics analysis, the position \((x, y, z)\) of the end point \( P \) in coordinate \( O \) can be easily obtained. The virtual environment is a virtual wall of 1-DOF spring and its model is demonstrated as equation (30). That is, only the \( x \) coordinate projection of end point’s motion trajectory is considered. The operator operates the end point \( P \) to contact the virtual wall from the free space.
\[ F_e = \begin{cases} 0 & x \geq 0 \\ -K_e x & x < 0 \end{cases} \quad (30) \]

The virtual environment is rendered in PC and the current closed loop control algorithm is implemented in the DSP board which communicates with PC by USB2.0. The sampling frequency of haptic rendering is 1 KHz \[16\]. The sampling frequency of the PID controller is 20 KHz. The displacement and velocity of the endpoint \( P \) in \( X \) coordinate as well as the motor current are recorded in the experiments.

In the experiments of open loop control, the operator can feel the vibration when the virtual stiffness is greater than 2.8N/mm, as shown in Fig. 5. So the maximum achievable virtual stiffness is approximately 2.8N/mm under open loop control.

Similar experiments under another two conditions are also conducted, where \( \tau = 0.05 \), \( K_p = 0.0068 \), \( K_i = 0 \), \( K_d = 0.00034 \), \( \bar{K} = 0.383 \) and \( \tau = 0.05 \), \( K_p = 0.0034 \), \( K_i = 0 \), \( K_d = 0.00017 \), \( \bar{K} = 0.169 \). In terms of the system stability condition, the maximum achievable virtual stiffness would increase to 3.6N/mm and 3.2N/mm respectively under these two conditions. The experimental results are that the maximum achievable virtual stiffness approximately increases to 4.2N/mm and 3.6N/mm. These almost accord with the theoretical results.

In the experiments of the current closed loop control, with the given parameters \( \tau = 0.05 \), \( K_p = 0.01 \), \( K_i = 0 \), \( K_d = 0.0005 \), \( \bar{K} = 0.495 \), the device becomes unstable when the virtual stiffness exceeds 5.1N/mm, as shown in Fig. 6. So the maximum achievable virtual stiffness is 5.1N/mm, increasing to 1.82 times comparing with that under the open loop control. According to the system stability condition, the maximum achievable virtual stiffness would increase to 1.49 times under this condition. This is approximately consistent with the experimental results.

Fig. 7 illustrates the comparison of the maximum achievable virtual stiffness between theory and experiment under different conditions. These experimental results are consistent with the theoretical analysis, which shows that the
theoretical stability condition under current closed loop control is effective.

According to the theoretical analysis, appropriate parameters of the filter and the PID controller would ensure the presented stiffness by haptic device. The following analysis and experiments are to testify this conclusion.

Based on the dynamic model of the device, when the device friction is neglected, equation (31) is satisfied.

\[ F_m - F_h = M \alpha + B \gamma \]  

(31)

Fig. 5 and 6 indicate that when the haptic device stably contacts the virtual wall, the velocity and acceleration of the haptic device are nearly zero, so

\[ F_m = F_h \]  

(32)

Therefore, when the device is stable in haptic interaction, the gradient of \( F_m \) v.s the device end point displacement in \( X \) coordinate demonstrates the virtual stiffness, as shown in Fig. 8. This figure shows that the stiffness that the device stably presents under the current closed loop control is similar to that under the open loop control when the virtual stiffness is set to 2.4N/mm in virtual environment. Also, when the virtual stiffness is set to 5.0N/mm in virtual environment, the presented stiffness is approximately 5.0N/mm. Thereby these experiments validate the current closed loop control can ensure the presented stiffness when parameters of the filter and PID controller are selected by equation (11).

\[ F_m = \frac{1}{m} \left( -F_h - B \gamma \right) \]  

(33)

\[ \frac{1}{m} = k_v \]  

(34)

\[ K_e = k_v \]  

(35)

\[ K_e = \frac{1}{m} \]  

(36)

\[ K_e = \frac{F_h}{\gamma} \]  

(37)

\[ K_e = \frac{F_m}{\gamma} \]  

(38)

Fig. 8. \( F_m \) v.s the \( X \) coordinate displacement of end point \( P \).

V. Conclusion

Stability is a critical consideration in haptic simulation. It is necessary to ensure the realistic haptic feeling and prevents physical harm to the operator and device. We have proposed a method to enlarge the range of virtual stiffness and improve the system stability. The effectiveness of the method has been proved theoretically. With the experimental testing on the one axis of the 3-DOF haptic device, we have shown that it is possible to increase the virtual stiffness with the current closed loop control. In the future, the proposed method will be further tested on multi-axis of the haptic device.

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