Moving Average Hybrid FIR Filter in Ultrasound Image Processing

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Abstract. — In this work, we present a novel technique for ultrasound image processing (2D signals). The Moving Average Hybrid FIR filter (MAH-FIR) which it work with two different FIR filter, the Moving Average (MA) and Median Hybrid (FMH) filters. We experimented with continuous linear regression function and Lagrange multiplier to obtain the approximation of the MA-FIR and FMH-FIR filters, respectively. Furthermore, we showed both filters composed into a block diagram. Finally, we present evaluations qualitative and quantitative of ultrasound image processing.

Keywords—MAH, MA and Hybrid FIR Filters

I. Introduction

In the area of the digital image processing, the filter design is a powerful tool for the enhanced, reconstruction, restoration and denoising images. Furthermore, the requirements of systems in some particular cases are unknowns due to the inherent blurred, to the moving of general system or the speckle in the data. Thus, here we present the development a new digital filter to the reducing noise with edges enhancing applies a Finite Impulse Response (FIR) filter.

The design to new filter involve two currents techniques to the estimation and predictive of signals. The MA-FIR filter which proposed to first time by Shmaliy in [1] to the noisy removal in signals noised of the Global Position Systems (GPS). By other hand, the FMH-FIR filter was proposed in [2] by Heinonen et. al to the enhanced and attenuating of noise in root signals, e.g. the triangular waves. Finally, in the linear combination of the both Fir filters we use some mathematical tools [3] to the obtained of the MAH-FIR filter.

By the other hand, the bigger variety of the filters in Biomedical image processing, specifically in ultrasound [4], are nowadays a tool very powerful to the monitoring of Fetus, study of the Heart, Kidney, Liver, Cysts, Tissue and human carotid artery disease and so fort. Thus, here we proposed of the new design of the MAH-FIR filter to the analysis of data in ultrasound. The enhancing and denoising of images, is the central problem due to the poor resolution that offers the ultrasound images currently (e.g. the model AV3018).

II. The Observation Equation

In all real applications, measurements are perturbed by noise. In the course of acquiring, or processing digital images, for example, the noise-induced degradation may be dependent or independent data. The noise is usually described by its probabilistic model, e.g. Gaussian noise is characterized by two moments; mean and variance. The observation equation in discrete time is defines as

$$y_n = x_n + v_n \qquad n = 0, 1, 2, \dots \tag{1}$$

where each image is form in lexicographic order (e.g. in rows or columns). The observed image y_n, includes the original image x_n and the independent and identically distributed noise process v_n. The model defined in (1) knows as free-model due to the removal the Point Spread Function (PSF) in the system, in other words, the system unknown the model.

III. From Linear Regression to the MA FIR Filter

In this section, first briefly review the continuous linear regression function to the reducing of noise in signals or images (lexicographic formed). The linear regression is a method that models the relationship between a dependent variable y_n and the independent variable x_n. Where the continuous linear regression function is defined in [3] as

$$\hat{x}_n = a_n + b_n\left(t_n - c_n\right) \tag{2}$$

where, t_n is the discrete time and each coefficient a_n, b_n, and c_n are defined as follow

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$$a_n = \frac{1}{N}\sum_{i=0}^{N-1} y_{n-i} \ ; \ b_n = \frac{\mathrm{cov}(y_n, t_n)}{\mathrm{var}(t_n)} \ ; \ c_n = \frac{1}{N}\sum_{i=0}^{N-1} t_{n-i} \qquad (3)$$

where, N is the number of the points in the average, cov(.,.) and var(.) are the covariance operation between the perturbed signal and discrete time, and the variance respectively.

From [1], can view to the transformation each of coefficients in the continuous linear regression function as follow

$$c_n = \Delta\left(n - \frac{N-1}{2}\right) \qquad (4)$$

where, $n = 0,1, \ldots$, and the Δ is the sample in discrete time (i.e. $\Delta = t_n - t_{n-1}$). To transform the b_n coefficient in (3), we applied the follow definitions

$$\mathrm{var}(t_n) = \frac{1}{N}\sum_{i=1}^{N-1} (t_{n-i} - c_n)^2 \qquad (5)$$

$$\mathrm{cov}(y_n, t_n) = \frac{1}{N}\sum_{i=0}^{N-1} (y_{n-i} - a_n)(t_{n-i} - c_n) \qquad (6)$$

next, substitute (4)–(6) for (2), make the transformation, and get the desired unbiased estimate, we obtained the follow equation

$$\hat{x}_n = \sum_{i=0}^{N-1} \frac{2(2N-1) - 6i}{N(N+1)} y_{n-i} \qquad (7)$$

where, we can view that the weighting function, $h(.)$, of this filter can be defined by

$$h_1(i) = \begin{cases} \dfrac{2(2N-1) - 6i}{N(N+1)} & 0 \le i \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

which evidently is nonzero for the averaging interval only. Finally, the model of MA FIR filter is defined as follow

$$\hat{x}_n = \sum_{i=0}^{N-1} h_1(i) y_{n-i} \ . \qquad (9)$$

In the Figure 1 we can see the operation of (8) and (9) with some samples of the noisy signal (y_n) to the prediction of $\hat{x}_n$.

IV. From FIR Filter Design to the FMH-FIR Filter

By the other hand, also we present a briefly review the Median Hybrid FIR filter to the prediction of the root signals. First, in the Fig. 1 we show the block diagram of this filter. Next, the hybrid FIR filter is characterized by follow procedure

$$\hat{x}_n^{FW} = E^{FW}\left[x(n-1), x(n-2), \ldots, (n-N)\right] \qquad (10a)$$

$$\hat{x}_n^{BW} = E^{BF}\left[x(n+1), x(n+2), \ldots, (n+N)\right] \qquad (10b)$$

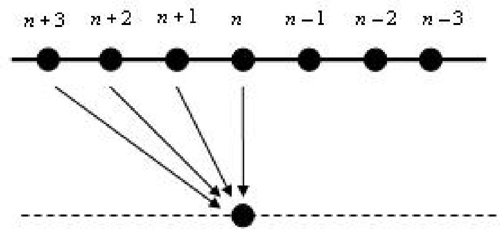

Fig. 1 Prediction of the new points apply the MA FIR filter

$$y_n = \mathrm{MED}\left[\hat{x}_n^{FW}, x_n, \hat{x}_n^{BW}\right] \qquad (10c)$$

where, E^{FW} and E^{BW} are the forward and backward extrapolating operators, respectively. The development of each FIR filter of the hybrid filter is exposed below.

To begin the design of hybrid FIR filter assuming that the additive noise has flat power spectrum, the noise power gain is given by

$$F = \sum_{i=0}^{N-1} h^2(i) \qquad (11)$$

the overall optimization problem is minimize the quantity F with respect to the tap coefficients subject to the following two constraints

$$g_0 = \sum_{i=0}^{N-1} h(i) = 1 \quad ; \qquad g_1 = \sum_{i=0}^{N-1} i h(i) = 0 \ . \qquad (12)$$

The constraint (12-left) mean the filter produces a unity response in the case of the input ramp signal, and the constraint (12-right) mean that filter response at the zero frequency is equal a zero. The optimization of the above constrained problem can be conveniently carried out using the Lagrange multipliers [3]. This is a technique which has been often used to solve for FIR filter coefficients in close form. When applying this technique, the problem is to minimize the following function

$$L(h(0), h(1), \ldots, h(N-1), \lambda_0, \lambda_1) = F + \lambda_0 g_0 + \lambda_1 g_1 \qquad (13)$$

where, λ_0 and λ_1 are the Lagrange coefficients. The minimum of this function is found by setting the partial derivatives with respect to all of the arguments equal to zero. As the first step, we have

$$\frac{\partial L}{\partial h(i)} = 2h(i) + \lambda_0 + \lambda_1 i = 0 \qquad (14)$$

this is solved for $h(i)$, which is substituted into (12) to obtain the Lagrange coefficients for the forward FIR filter as

$$\lambda_0 = \frac{4(2N+1)}{N-N^2} \quad ; \quad \lambda_1 = \frac{12}{N^2-N} \tag{15}$$

next, introducing the (15) into (14) and we obtained the weighting function, $h(.)$, as below

$$h_2(i) = \begin{cases} \dfrac{2(2N+1)-6i}{N(N-1)} & 0 \le i \le N-1 \\ 0 & \text{otherwise} \end{cases} \quad . \tag{16}$$

Finally, the model of median hybrid FIR filter is defined as

$$\hat{x}_n = \sum_{i=0}^{N-1} h_2(i) y_{n-i} \quad . \tag{17}$$

V. The Moving Average Hybrid FIR Filter

In this section, we present the design of the new filter to the enhancing and denoising ultrasound images. Here we recalled the characteristic function of the MA-FIR filter defined in (8) and (9) which generate an unbiased estimate of the signal. Also, recalled the block diagram of the median hybrid FIR filter showed in Figure 2. To realize the linear combination of the both functional, first we can see in the Figure 1 and the Figure 3 the prediction of the points with both filters.

The new generation of the points can be enhanced if substituted the sub-structure of left hand of the general structure of the FMH-FIR filter (Figure 2) by the MA-FIR filter due at that this filter apportion statistical information of the original signal in the estimation of the new points.

In the Figure 4, we present the new block diagram of the Moving Average Hybrid FIR filter. Here, the estimation is carried out trough the both filters in parallel form. Finally, the output of each filter is introduced in the median operator to the selection of the minor error. The estimation of the new points is showed in the Figure 5.

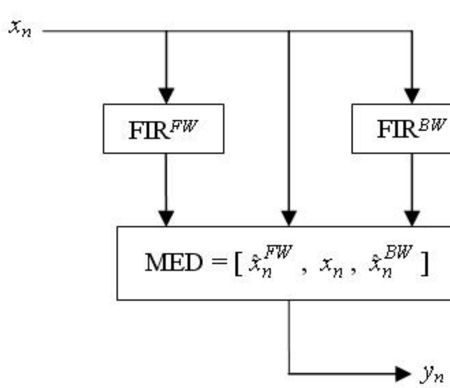

Figure 2 General structure of FMH-FIR Filter.

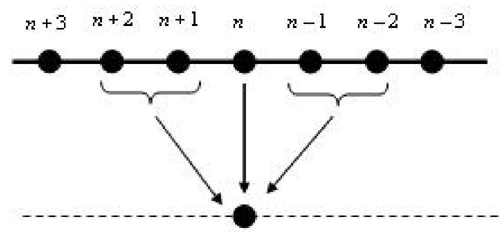

Figure 3 Prediction of the new points apply the FMH-FIR filter

VI. Simulation

Here we present some simulation and numerical evaluation of the new filter design. The initial conditions to be simulation are follows: the ultrasound image is show in Figure 6, was used a test image in the set of experiments based on the FIR filters proposes in the sections III, IV and V. The blurred and noisy image is shown in Figure 7. This image was extrapolate from 598 × 789 to 256 × 256 pixels to the producing a blurred into image (Figure 6), also, the image was contaminate by additive Gaussian noise with a density power of 0.01. By other hand, the quantitative evolution metrics apply to the numerical experimentation are tree; SNR, IOSNR and RMSE.

The Signal-to-Noise Ratio (SNR) is the classical metric for quantitative evaluation so much in signal processing as the image processing. The SNR metric is defined as

$$SNR_{dB} = 10\log_{10}\left(\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}[x(i,j)]^2}{\sum_{i=1}^{n}\sum_{j=1}^{n}[\hat{x}(i,j)-x(i,j)]^2} \right) \tag{18}$$

The Improvement Output Signal-to-Noise Ratio (IOSNR) metric was proposed to first time by Katsaggelos et al. in [5] to try the quality of restoration in the images processing by iterative algorithms, and is defined as

$$IOSNR_{dB} = 10\log_{10}\left(\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}[y(i,j)-x(i,j)]^2}{\sum_{i=1}^{n}\sum_{j=1}^{n}[\hat{x}(i,j)-x(i,j)]^2} \right) \tag{19}$$

where, $x(i, j)$, $y(i, j)$ and $\hat{x}(i, j)$ are the original data, degraded data and estimate data respectively. This metric can only be used for simulation cases when the original image is available. While the IOSNR metric do not always reflect the perceptual properties of the human visual system, this subject provide an objective standard by which to compare different techniques.

The Root Mean Square Error (RMSE) can be used for implementation and control of quality in the image processing. The RMSE is defined as

$$RMSE = \frac{1}{n}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\hat{x}(i,\, j) - y(i,\, j)\right]^2} \tag{20}$$

VII. CONCLUDING REMARKS

The development of new filter to the biomedical image processing is a topic that involve, in sometime, the combined of classical and current methodologies to the produced a new computational structures with subject reduced the blur in zoon ultrasonic images applications in future ultrasonic machine.

In the Figure 8 showed the enhanced the zoon ultrasound image with the FMH-FIR filter, we can see that exist a general saturation of pixels due at the instability of weigh function defined in (17) from the filter. Even trough the qualitative evaluation is poor, the noise level is reduced. By other hand, in the Figure 9 we present a enhanced zoon ultrasound image with the MA-FIR filter, we can see that the edges and homogeneous zones are detected and enhanced due a the statistical information of the signal, furthermore, the linear development of MA filter is the to delimit of performance of filter. Finally, in the Figure 10, we show the enhanced zoon ultrasound image with the MAH-FIR filter, here the balance of the statistical information and the block diagram of the FMH-FIR filter as carried out. The analysis of the image is follow; first, the pixels in the image are enhanced due a MA-FIR filter, and second, the general structure of MAH-FIR filter development a new approximation of the median hybrid filters.

By other hand, in the Table 1 we present some numerical evaluation of the image test with the tree filters showed in this work. We can see that the MAH-FIR filter enhance the performance of the MA- and FMH-FIR filters. The reducing of the noise and the enhanced of the edges in the image are showed in numerical form in the table.

Finally, the design and the computational implementation of the new filter is the first step of the new generation of hybrid FIR filters. The polynomial approximation of the filters to the increase of the kind is the next step of this investigation in biomedical ultrasound images processing.

TABLE I QUANTITATIVE EVALUATION OF THE FIR FILTERS

	SNR (dB)	RMSE	IOSNR (dB)
AM filter	13.6007	14.3689	4.8409
Hybrid filter	11.4303	18.4478	2.6672
AMH filter	14.1149	13.5429	5.3302

VIII. ACKNOWLEDGMENT

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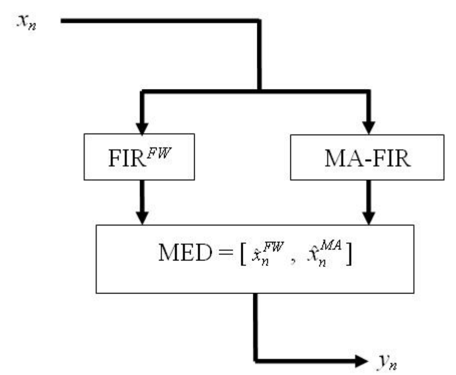

Figure 4 General structure of MAH FIR Filter

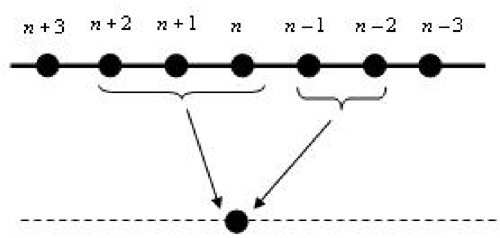

Figure 5 Prediction of the new points apply the MAH-FIR filter

Fig. 6 Original ultrasound image

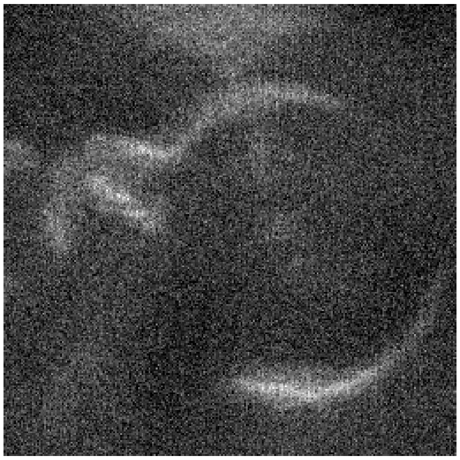

Fig. 7 Blurred and noisy Image

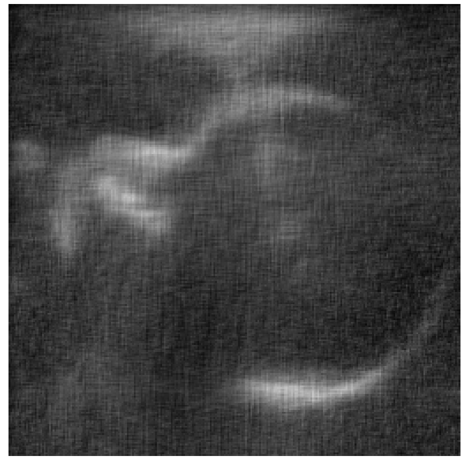

Fig. 10 Enhanced image with MAH filter

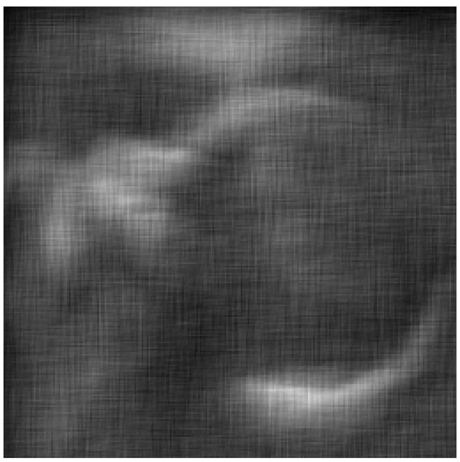

Fig. 8 Enhanced image with Hybrid FIR filter

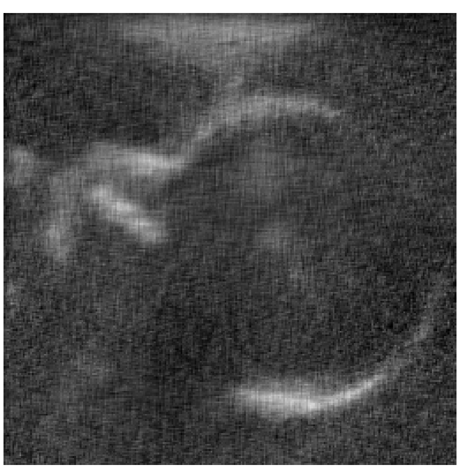

Fig. 9 Enhanced image with MA FIR filter