Abstract—Four-bar structured hard disk drive (HDD) potentially leads to higher achievable closed-loop bandwidth compared to the traditional single-arm structure, but also brings challenges in control design due to nonlinearity. This paper studies track-seeking control of four-bar structured HDD. First, interconnection and damping assignment passivity-based control (IDA-PBC) method is used to design a preliminary controller. Second, in order to improve the transient performance, a switching law is proposed to switch between two IDA-PBC controllers which are with small and large dampings respectively. Simulation results show that the proposed IDA-PBC based switching control can overcome the limitation of a single IDA-PBC controller and achieves better transient performance.

I. INTRODUCTION

Modern hard disk drives (HDD) adopt single-arm structured suspensions driven by voice coil motors (VCM). Experimental frequency response of a practical HDD suggests that the structure of HDD is anything but rigid. There are many flexible dynamics contributed from various bending modes, torsional modes and sway modes of the suspension [1]. These lightly damped resonant modes are subject to variation from drive to drive and as a result limit the achievable closed-loop bandwidth of the servomechanism [2]. Except for novel control algorithms, another solution to push higher the bandwidth is to improve the hardware by using stiffer but lighter materials or adopting novel structure suspensions.

Fig. 1 is a sketch map of the four-bar structured HDD considered in this paper whose design is currently patent pending. Different from the traditional single-arm structure, the suspension of this four-bar structure is driven by an actuator through a connection link. The main benefit of this structure is that it provides additional stiffness without degrading the mass requirement and thus achieves better tradeoff between mass and stiffness. Though on one hand the four-bar structure potentially leads to higher achievable closed-loop bandwidth, on the other hand it also brings high nonlinearities which challenge the control design. In [3], track-following control of four-bar structured HDD has been investigated via low-frequency nonlinearity pre-compensation. This paper focuses on track-seeking control of the four-bar structured HDD.

The main effort in this paper is on how to design a track-seeking controller which exhibits good transient performance and preserves the performance under different track-seeking situations. First, we use interconnection and damping assignment passivity based control (IDA-PBC) to design a preliminary controller to shape the potential energy and change the damping. The controller obtained in this way has a clear physical explanation, i.e., it can be explained as attaching a spring and a damper with nonlinear coefficients to the four-bar structure. By carefully choosing these coefficients, the error dynamics can be made to be independent of the start and the target track positions, thus the track-seeking performance preserves under different seeking situations. Second, we then improve the transient response by switching between two different IDA-PBC controllers which are with a small and a large dampings respectively. The proposed switching law is driven by tracking error so that the property of preserving the performance still holds in this case. Some stability results for error dynamics are also provided in this paper. Refer to [4], [5], [6] and the reference therein for more results regarding stability of switched systems and refer to [7], [8], [9], [10], [11] for Hamiltonian systems and IDA-PBC design.

This paper is arranged as follows. In Section II, we introduce the Lagrange model of four-bar structure as well as its Hamiltonian form. Section III is the main part of this paper where IDA-PBC based switching controller is designed and some stability results are presented. In Section IV, simulation results are provided to show the validity of the proposed design and some concluding remarks are made in Section V-A.

II. DYNAMICAL MODEL

A. Lagrange Model

Consider the rigid four-bar system depicted in Fig. 2. It consists of the driving bar $A_3A_4$, the driven bar $A_1A_2$, the connection bar $A_2A_3$ and the forth bar $A_1A_4$ which is fixed with the pedestal. The joint $A_4$ is the driving joint where a control torque $\tau$ is provided by an actuator and the R/W head is fixed somewhere on the driven bar $A_1A_2$. By choosing $\theta_4$
as the generalized coordinate, this system can be described as the following Lagrange model
\[
\mathcal{D}(\dot{\theta})\ddot{\theta} + \mathcal{C}(\dot{\theta}, \dot{\theta})\dot{\theta} + \frac{\partial F(\theta)}{\partial \theta} = \tau
\]
(1)
where \(\theta = (\theta_1, \theta_2, \theta_4)^T\). The inertia of moment
\[
\mathcal{D}(\theta) = J_{3c} + m_3 l_{3c}^2 + m_2 l_2^2 + m_1 (l_1 - l_{1c})^2 + J_{1c} l_3^2 \sin^2(\theta_1 + \theta_2 + \theta_4)
\]
\[
+ (m_2 l_{2c} + J_{2c}) l_3^2 \sin^2(\theta_1 + \theta_4)
\]
\[
+ 2m_1 l_{2c} \cos(\theta_1 + \theta_2 + \theta_4) \sin(\theta_1 + \theta_4)
\]
\[
l_2 \sin \theta_2
\]
where the definitions of the parameters are given in Table I. \(\mathcal{C}(\dot{\theta})\dot{\theta}\) is the Coriolis/centrifugal force with
\[
\mathcal{C}(\theta, \dot{\theta}) = \frac{1}{2} \left[ \frac{\partial \mathcal{D}}{\partial \dot{\theta}_1} \frac{\partial \dot{\theta}_1}{\partial \theta_1} + \frac{\partial \mathcal{D}}{\partial \dot{\theta}_2} \frac{\partial \dot{\theta}_2}{\partial \theta_2} + \frac{\partial \mathcal{D}}{\partial \dot{\theta}_4} \frac{\partial \dot{\theta}_4}{\partial \theta_4} \right] \dot{\theta}_4.
\]
\(F(\dot{\theta})\) is the dissipation term given by
\[
F(\dot{\theta}) = \frac{1}{2} (\sigma_1 \dot{\theta}_1^2 + \sigma_2 \dot{\theta}_2^2 + \sigma_3 \dot{\theta}_3^2 + \sigma_4 \dot{\theta}_4^2)
\]
\[
= \frac{1}{2} \mathcal{K}(\theta) \dot{\theta}_4^2
\]
with \(\sigma_i\) the dissipation coefficient of the \(i\)th joint and
\[
\mathcal{K}(\theta) = \begin{bmatrix}
\frac{\partial \theta_1}{\partial \theta_1} & \frac{\partial \theta_2}{\partial \theta_2} & \frac{\partial \theta_2}{\partial \theta_4} & 1
\end{bmatrix}
\begin{bmatrix}
\mathcal{D} & \mathcal{C} & \frac{\partial \mathcal{D}}{\partial \theta_4} & \frac{\partial \mathcal{C}}{\partial \theta_4} & \frac{\partial F(\theta)}{\partial \theta_4}
\end{bmatrix}.
\]
(2)
\[
N = \begin{bmatrix}
\sigma_1 + \sigma_3 & \sigma_3 & \sigma_3 & \sigma_3 & \sigma_3 & \sigma_3 & \sigma_3
\end{bmatrix}
\]
Note that \(\theta\) is subject to the geometric constraints \(\lambda_\alpha(\theta) = 0\) and \(\lambda_\beta(\theta) = 0\) where
\[
\lambda_\alpha(\theta) := l_1 \cos \theta_3 - l_2 \cos(\theta_1 + \theta_2) + l_3 \cos \theta_4 - c_0
\]
\[
\lambda_\beta(\theta) := l_1 \sin \theta_3 - l_2 \sin(\theta_1 + \theta_2) - l_3 \sin \theta_4
\]
which imply
\[
\frac{\partial \theta_1}{\partial \theta_4} = \frac{l_3 \sin(\theta_1 + \theta_2 + \theta_4)}{l_1 \sin \theta_2}
\]
\[
\frac{\partial \theta_2}{\partial \theta_4} = \frac{l_3 \sin(\theta_1 + \theta_4)}{l_2 \sin \theta_2}
\]
\[
\frac{\partial \theta_3}{\partial \theta_4} = \frac{l_3 \sin(\theta_1 + \theta_2 + \theta_4)}{l_1 \sin \theta_2}.
\]
The parameter \(c_0\) represents the distance between \(A_1\) and \(A_4\).

B. Hamiltonian Form

Define \(q = \theta_4\) and the Hamilton
\[
\mathcal{H} = \frac{1}{2} p^T D^{-1}(q) p.
\]
Then system (1) is equivalent to the controlled Hamiltonian system
\[
\begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} = (\mathcal{J} - \mathcal{R}) \nabla \mathcal{H} + \mathcal{G} \tau
\]
(4)
where \(\nabla\) represents the gradient of a smooth function, i.e.,
\[
\nabla \mathcal{H} = \left[ \frac{\partial \mathcal{H}}{\partial q}, \frac{\partial \mathcal{H}}{\partial p} \right]^T,
\]
\[
\mathcal{J} = \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix}, \quad \mathcal{R} = \begin{bmatrix}
0 & 0 \\
0 & \mathcal{K}(q)
\end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix}
0 \\
1
\end{bmatrix}.
\]

III. TRACK-SEEKING CONTROL DESIGN

A. IDA-PBC Design [7]

Denoted by \((\bar{q}, 0)^T\) the point to be stabilized.

Proposition 1: For any positive function \(\alpha(q, p) > -\mathcal{K}(q)\) and any smooth function \(\mathcal{V}(q, \bar{q})\) satisfying
\[
\left. \frac{\partial \mathcal{V}(q, \bar{q})}{\partial q} \right|_{q = \bar{q}} = 0, \quad \left. \frac{\partial^2 \mathcal{V}(q, \bar{q})}{\partial q^2} \right|_{q = \bar{q}} > 0
\]
and
\[
\left. q \frac{\partial \mathcal{V}}{\partial q} = 0 \right\} = \{\bar{q}\},
\]
the controller
\[
\tau(q, p) = -\alpha(q, p) \frac{\partial \mathcal{H}}{\partial p} - \frac{\partial \mathcal{V}}{\partial q}
\]
(7)
asymptotically stabilizes system (4).

One sees that the first part of the controller (7) injects additional damping to the system, while the second part shapes the total energy by adding potential energy \(\mathcal{V}(q, \bar{q})\).

A typical choice of the potential energy is
\[
\mathcal{V}(q, \bar{q}) = \int_{\bar{q}}^q \beta(\mu) (\mu - \bar{q}) d\mu
\]
(8)
where $\beta(\mu) > 0$ is a positive function. It is easy to check that
\[
\begin{align*}
\frac{\partial V(q, \dot{q})}{\partial q} & = \beta(q)(q - \dot{q})|_{q = \ddot{q}} = 0, \\
\frac{\partial^2 V(q, \dot{q})}{\partial q^2} & = \frac{\partial \beta(q)}{\partial q} (q - \dot{q})|_{q = \ddot{q}} + \beta(\ddot{q}) = \beta(\ddot{q}) > 0,
\end{align*}
\]
i.e., the conditions (5) and (6) naturally hold. With this choice, the controller (7) becomes
\[
\tau(q, p) = -\alpha(q, p)D^{-1}(q)p - \beta(q)(q - \dot{q}). \quad (9)
\]
A physical explanation of this controller is that it is equivalent to attaching a spring and a damper to the structure with nonlinear coefficients $\beta(q)$ and $\alpha(q, p)$ respectively, as depicted in Fig. 3.

B. Choice of $\alpha$ and $\beta$

Choose
\[
\begin{align*}
\alpha &= D(q)\bar{\alpha}(e, \dot{e}) - \frac{1}{2} \frac{\partial D(q)}{\partial q} \ddot{q} + K(q) \\
\beta &= D(q)\bar{\beta}(e)
\end{align*}
\] (10)
where $\bar{\alpha} > 0$ and $\bar{\beta} > 0$ only depend on the tracking error $e = q - \ddot{q}$ and the velocity $\dot{e}$. Note that $D^{-1}(q)p = \dot{q}$, thus the controller (9) can be equivalently represented as the nonlinear PD controller
\[
\tau(q, p) = -\alpha(q, p)\dot{e} - \beta(q)e. \quad (11)
\]
Applying this controller to the original system (1), we get
\[
D(q)\ddot{e} + \bar{\alpha}(e, \dot{e})\dot{e} + \bar{\beta}(e)e = 0. \quad (12)
\]
Note that $D(q) > 0$ within the whole operation range, thus the error dynamics of the system is
\[
\ddot{e} + \bar{\alpha}(e, \dot{e})\dot{e} + \bar{\beta}(e)e = 0. \quad (13)
\]
One sees that with the above choice of $\alpha$ and $\beta$, the error dynamics depends only on the tracking error and is independent of the start and target track positions.

C. IDA-PBC Based Switching Control

1) Switching Law Design: In the error dynamics (13), if $\bar{\alpha} > 0$ and $\bar{\beta} > 0$ are positive constants, then the error dynamics is naturally asymptotically stable. However, it is difficult to achieve good transient performance with constant coefficients. Note that for given $\bar{\beta}$, the coefficient $\bar{\alpha}$ characterizes the damping of the error dynamics. This motivates the following switching control.

For given $\bar{\beta} > 0$, we assign two constant values for $\bar{\alpha}$ which are corresponding to small and large dampings, denoted by $\bar{\alpha}_S$ and $\bar{\alpha}_L$, respectively. When $\bar{\alpha} = \bar{\alpha}_S$, the corresponding controller is with small damping and leads to fast response but might be with large overshoot. On the other hand, the controller with $\bar{\alpha} = \bar{\alpha}_L$ is overdamped and leads to slow response without overshoot. In order to achieve fast response without overshoot, we let $\bar{\alpha}$ switch between $\bar{\alpha}_S$ and $\bar{\alpha}_L$ according to the switching law
\[
\bar{\alpha}(e, \omega) = \begin{cases}
\bar{\alpha}_S, & e + \omega T > 0 \\
\bar{\alpha}_L, & e + \omega T \leq 0
\end{cases} \quad (14)
\]
where $\omega := \dot{e}$ is the velocity and $T \geq 0$ is an adjustable parameter. An explanation of this switching law is given in Fig. 4. Note that $\dot{e} := e + \omega T$ is actually a simple linear predictor of the tracking error $T$ moments later. Thus the proposed switching law means that when the predicted tracking error would not cross zero in time $T$, the small damping mode is activated and the output to the system approaches the target position fast. Whenever the predicted tracking error would cross zero in time $T$, the controller switches to large damping mode to avoid overshoot. The parameter $T$ roughly characterizes how much time the switching is ahead of the zero-crossing of the tracking error. The structure of the whole IDA-PBC based switching control system is depicted in Fig. 11.

The reset law given in (14) has a clear intuitive explanation, but may be sensitive to measurement noise and uncertainties. See simulation results in Fig. 10 and Fig. 11. In order to assure certain robustness, the actual reset law used in this paper is modified to
\[
\bar{\alpha}(e, \omega) = \begin{cases}
\bar{\alpha}_S, & (e - \varepsilon \omega)(e + \omega T) > 0 \\
\bar{\alpha}_L, & (e - \varepsilon \omega)(e + \omega T) \leq 0
\end{cases} \quad (15)
\]
where $\varepsilon > 0$ is positive constant. The reset law (14) is the special case of (15) with $\varepsilon = 0$.

2) Stability Analysis: The error dynamic can be alternatively represented by the switched system
\[
\begin{bmatrix}
\dot{e} \\
\dot{\omega}
\end{bmatrix} = A_{\bar{\alpha}(e, \omega)} \begin{bmatrix} e \\ \omega \end{bmatrix} \quad (16)
\]
where
\[
\sigma(e, \omega) = \begin{cases} 
1, & (e - \varepsilon \omega)(e + \omega T) > 0 \\
2, & (e - \varepsilon \omega)(e + \omega T) \leq 0
\end{cases}
\]
\[
A_1 = \begin{bmatrix} 0 & 1 \\ -\bar{\beta} & -\bar{\alpha}_S \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -\bar{\beta} & -\bar{\alpha}_L \end{bmatrix}.
\]

The following common Lyapunov function based result is obviously true.

**Proposition 2:** The error dynamics (16) is asymptotically stable for any \( T \geq 0 \) if there is a positive definite matrix \( P > 0 \) such that \( A_i^T P + PA_i < 0, i = 1, 2 \).

If the conditions of Proposition 2 hold, then we can adjust \( T \) freely without destroying the stability. When common Lyapunov function is not available, we can resort to piecewise quadratic Lyapunov function method to test the stability. Note that the lines \( e - \varepsilon \omega = 0 \) and \( e + T \omega = 0 \) partition the state space into two parts \( M_S \) and \( M_L \) where
\[
M_S = \{(e, \omega)^T \in \mathbb{R}^2 \mid (e - \varepsilon \omega)(e + T \omega) > 0\},
\]
\[
M_L = \{(e, \omega)^T \in \mathbb{R}^2 \mid (e - \varepsilon \omega)(e + T \omega) \leq 0\}.
\]

See Fig. 6. \( M_S \) and \( M_L \) correspond to the areas where \( \bar{\alpha} \) takes values \( \bar{\alpha}_S \) and \( \bar{\alpha}_L \) respectively. On the straight line \( h_e(e, \omega) := \varepsilon \omega - e = 0 \), it is easy to check that
\[
\dot{h}_e(e, \omega) = -(\varepsilon^2 \bar{\beta} + \varepsilon \bar{\alpha} + 1)\omega.
\]

Thus \( \dot{h}_e < 0 \) when \( \omega > 0 \) and \( \dot{h}_e > 0 \) when \( \omega < 0 \). This implies that near the boundary \( \varepsilon \omega - e = 0 \), the trajectory of the switched system always goes from \( M_L \) into \( M_S \). This is indicated by the small arrows on the straight line \( \varepsilon \omega - e = 0 \) in Fig. 6. On the straight line \( h_T(e, \omega) := e + T \omega = 0 \),
\[
\dot{h}_T(e, \omega) = (T^2 \bar{\beta} + T \bar{\alpha} + 1)\omega.
\]

We always assume that \( \bar{\alpha}_S < \bar{\alpha}_L \) and there are three cases:

(a) If \( \bar{\alpha}_S < \bar{\alpha}_L \leq T \bar{\beta} + 1/T \), then near the boundary \( e + T \omega = 0 \), the trajectory goes from \( M_S \) into \( M_L \). This situation is indicated in Fig. 6 by small arrows on the boundary.

(b) If \( \bar{\alpha}_L > \bar{\alpha}_S > T \bar{\beta} + 1/T \), then on the boundary \( e + T \omega = 0 \), the trajectory goes from \( M_L \) into \( M_S \).

(c) If \( \bar{\alpha}_S < T \bar{\beta} + 1/T < \bar{\alpha}_L \), then chattering will occur on the boundary \( e + T \omega = 0 \).

In the practical design, \( \bar{\alpha}_S \) should be far less than \( T \bar{\beta} + 1/T \) so that the system has fast response when the output is far from the target track position, thus Case (b) above would not happen. Case (c) leads to infinitely fast switching and should be avoided. Thus we only focus on Case (a).

**Proposition 3:** Suppose that \( \bar{\alpha}_S < \bar{\alpha}_L \leq T \bar{\beta} + 1/T \) and denote
\[
Q = \begin{bmatrix} 1 & \frac{T \varepsilon}{T \bar{\beta} - \varepsilon T} \\ \frac{T \varepsilon}{T \bar{\beta} - \varepsilon T} & -\varepsilon T \end{bmatrix}.
\]

Then the error dynamics (16) is asymptotically stable. \( \Box \)

**Remark 1:** Less conservative results can be obtained by using multi-Lyapunov functions and piecewise quadratic Lyapunov functions with finer partition of the state space. Refer to [12], [13].

3) Choice of \( \bar{\alpha}_S, T \) and \( \bar{\alpha}_L \): The following steps provide a heuristic method to obtain a set of preliminary parameters which can be used as a start point of parameter tuning.

(i) First choose \( \bar{\alpha}_S \) to meet the rise time specification. For instance, suppose that the 5% settling time is required to be less than \( T_r \), then we choose a \( \bar{\alpha}_S \) such that the time it takes for the tracking error to move from \( e(0) \) to 95%\( e(0) \) is less than \( T_r \).

(ii) Choose a \( T > 0 \) such that the line \( e + T \omega = 0 \) intersects with the phase trajectory of the system with \( \bar{\alpha}_S \) at a point where the tracking error is less than 5% of the initial error \( e(0) \). See Fig. 7 for an intuitive explanation.

(iii) Choose \( \bar{\alpha}_L \) to be approximately equal to (slightly less than) \( T \bar{\beta} + 1/T \). Note that when \( \bar{\alpha}_L = T \bar{\beta} + 1/T \), then on the boundary \( e + T \omega = 0 \), there holds \( \dot{h}_T = 0 \) which means that this boundary is actually an invariant set of the switched system and the phase trajectory slides to the origin along the line \( e + T \omega = 0 \) (if without disturbance and uncertainty). See Fig. 7.

**IV. Simulation**

In the simulation, the sampling period is chosen as \( T_s = 1.5 \times 10^{-5}s \) and all the parameters of the system are given in Table I. With these parameters, the range of \( q = \theta_4 \) is from about \(-0.7325 \) rad to about 0.4386 rad.

First, we fix \( \beta = 1 \times 10^7 \) and choose \( \bar{\alpha}_S = 3 \times 10^3 \) to meet the rise time specification. Choose \( T = 2.5 \times 10^{-5}s \) and \( \bar{\alpha}_L = T \bar{\beta} + 1/T = 4.025 \times 10^4 \). Fig. 8 gives the output

![Fig. 6: Partition of the state space](image-url)
responses to the switching control and the single IDA-PBC control with \( \bar{\alpha} = \bar{\alpha}_S \) and \( \bar{\alpha}_L \), respectively, with the start position \( q_0 = 0.1 \text{rad} \), target position \( \bar{q} = 0.15 \text{rad} \) and \( \varepsilon = 10 \). Fig. 9 gives the corresponding phase trajectories of the error dynamics for the switching control and the single IDA-PBC control with \( \alpha = \bar{\alpha}_S \) respectively, which explains how the switching law works.

In Fig. 8, the output response for the switching control exhibits a bit of undershoot. This can be eliminated by making \( \bar{\alpha}_L \) smaller than \( T/3 + 1/T \). In the following simulations, by choosing \( \bar{\alpha}_L = 3 \times 10^4 \), the undershoot almost disappears as depicted in Fig. 10 (solid curve).

The next simulation compares the switching laws (14) and (15). The output responses and the phase trajectories for these two switching laws are given in Fig. 10 and Fig. 11 respectively. With switching law (14), when the tracking error approaches zero, the controller wrongly switches to the small damping mode and push the phase trajectory away from the origin. This situation is well overcome by introducing a positive \( \varepsilon \).

The following simulations show output responses under different track-seeking situations. Fig. 12 shows the track-seeking performance for different start track positions with the same seeking span. For comparison, the start position are moved to the same point and thus what Fig. 12 shows is actually the responses of the tracking error \( q - \bar{q} \). Fig. 13 gives the corresponding control inputs. Fig. 14 gives the output responses for different seeking spans with the same start track positions. These simulations show that the proposed switching controller exhibits perfect transient performance and preserves the performance under different situations.

A. Conclusion

This paper studied track-seeking control of four-bar structured hard disk drive systems. First, a preliminary track-seeking controller has been designed based on the IDA-PBC method. Then, based on the IDA-PBC controller, a switching law has been proposed to switch the damping of the IDA-PBC controller to improve the transient performance. Simulation results showed that the proposed switching control leads to perfect track-seeking performance and the nonlinearity is well compensated for.

REFERENCES

Fig. 10: Output Responses for switching laws (14) and (15)

Fig. 11: Phase trajectories for switching laws (14) and (15)

Fig. 12: Tracking error at different seeking situations ($q_0$: start track position; $\bar{q}$: target track position)

Fig. 13: Control input for different seeking situations ($q_0$: start track position; $\bar{q}$: target track position)

Fig. 14: Output responses for different seeking spans (start position $q_0 = -0.6$ rad)


