



A Comprehensive Overview of Image Enhancement Techniques

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Abstract

Image enhancement plays an important role in improving image quality in the field of image processing, which is achieved by highlighting useful information and suppressing redundant information in the image. In this paper, the development of image enhancement algorithms is surveyed. The purpose of our review is to provide relevant researchers with a comprehensive and systematic analysis on image enhancement techniques and give them a valuable reference. Various image enhancement algorithms were mentioned and underlying difficulties, limitations, merits and disadvantages were discussed in applying these techniques in the past two decades with three aspects: supervised algorithm, unsupervised algorithm and quality evaluation, respectively. Further, we summarize some existing problems and analyze the future development trend of existing enhanced algorithms.

1 Introduction

Image enhancement is one of the most important technologies in the field of image processing, and its purpose is to improve the quality of images for specific applications. In general, the basic principle of image enhancement is to modify the information contribution of an image so that it is more suitable for a specific application [1].

Traditional image enhancement methods are mainly based on spatial domain and frequency domain processing. The spatial image enhancement [2] is to directly process the pixels in the image, such as the classic modified histogram methods [3–5], the improved unsharp mask methods [6–8]. The frequency domain image enhancement is to convert the image to the frequency domain in a certain mathematical function such as Fourier transform (FT), discrete cosine transform (DCT) and discrete wavelet transform (DWT), then perform image processing based on the unique properties of the frequency domain, and finally convert it to the original image space. With the rapid development

of image enhancement technology, various novel methods have emerged, such as retinex model [9–11], fuzzy theory [12, 13], neural network [14, 15], etc. For example, as shown in Fig. 1, we investigated 162 literatures on image enhancement. The methods involved mainly include histogram equalization, Retinex model, visual cortex neural network and deep learning methods. Each image enhancement method has its own advantages and disadvantages.

The main advantages of spatial domain image enhancement are simple understanding, low complexity and real-time implementation. However, there are also some disadvantages in the spatial domain image enhancement method, such as the lack of sufficient robustness and imperceptibility requirements [16]. It is difficult to propose a method that has a good enhancement for all images. This is mainly caused by the following factors: the non-universality of image enhancement algorithm, the choice of evaluation index, the influence of noise and the choice of optimal parameters, etc.

In this paper, we comprehensively provide the progress of image enhancement technology in the past two decades. Based on the latest trends in image processing, we mainly introduce the image enhancement methods separately in three aspects: supervised methods, unsupervised methods and quality evaluation. The block diagram of the whole framework is shown in Fig. 2 in this paper. The rest of paper is organized as follows. Section 2 introduces the image enhancement techniques based on unsupervised algorithms. Section 3 elaborates the supervised algorithms for image enhancement. Section 4 describe in detail the quality

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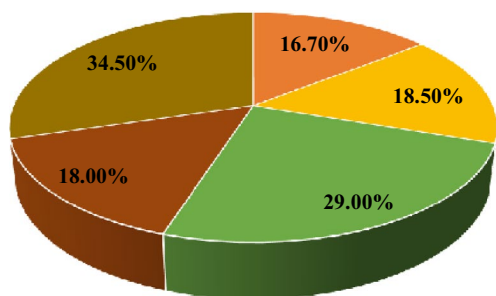


Fig. 1 The statistics of the number of papers with different image enhancement methods

- Histogram equalization
- Deep learning
- Retinex model
- Visual cortex neural network
- Others

evaluation content of image enhancement. Section 5 makes a conclusion for this paper.

2 Unsupervised Methods

The unsupervised algorithms do not require training samples or training labels, but directly model the data such as K-means [17], hierarchical clustering [18] and EM algorithm [19] etc. In the field of image enhancement, we survey several classic unsupervised algorithms: histogram specification, retinex model and visual cortex neural network. We will introduce in detail as follows:

2.1 Modifying the Histogram of Image

Histogram specification [20] is a method in the field of image processing that uses image histograms to adjust contrast. In this way, the brightness can be better distributed on the histogram. This can be used to enhance the local contrast without affecting the overall contrast. Histogram equalization achieves this function by effectively expanding the commonly used brightness. The histogram of a digital image with a gray level range of $[0, L - 1]$ is a discrete function [21]:

$$h(r_k) = n_k \tag{1}$$

where r_k is the k -th grayscale value, and n_k is the number of pixels with r_k in the image. The histogram of an image represents the grayscale distribution of the image.

Generally, we first normalize the histogram and then perform subsequent processing. Assuming that the dimension of the grayscale image is $M \times N$, and M, N represents the total number of pixels in the image, the normalized histogram can be expressed as:

$$p(r_k) = \frac{n_k}{MN}, k = 0, 1, \dots, L - 1 \tag{2}$$

where $p(r_k)$ represents the estimation of the probability of gray level r_k appearing in the image, and the sum of all components of the normalized histogram is equal to 1.

In this section, we mainly elaborate three typical image enhancement methods based on histogram modification: traditional histogram equalization, partial histogram equalization and histogram frequency weighting, respectively.

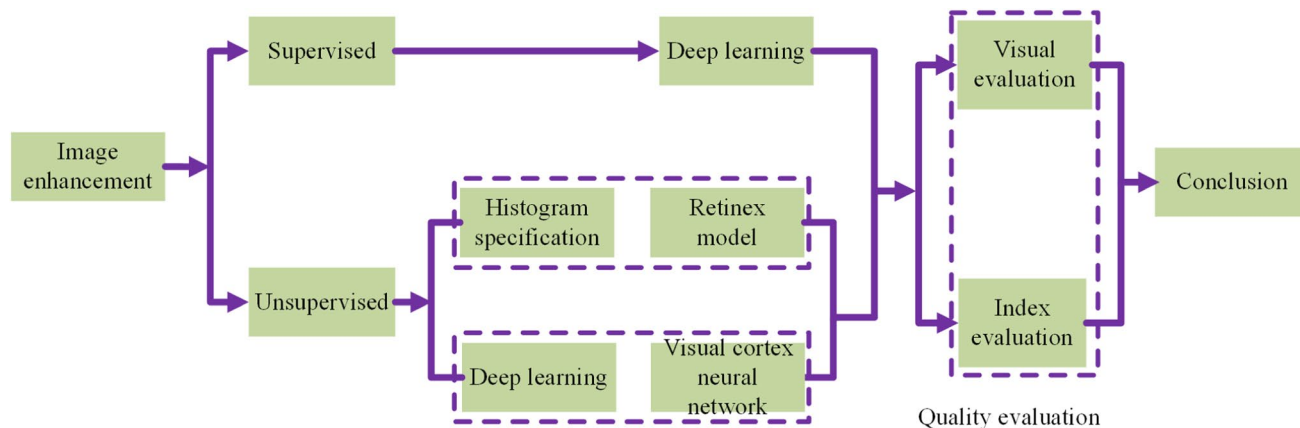


Fig. 2 The whole framework of image enhancement methods

2.1.1 Histogram Equalization

Earlier studies used to render the histograms of all gray levels averagely, but these methods usually overstretch gray levels with larger histogram boxes than others. Histogram equalization(HE) is a common method in image enhancement, which does not require parameter settings of external factors and can effectively enhance image contrast [22]. However, histogram equalization is a method to adjust the image globally, which cannot effectively improve the local contrast, and the effect is very poor in some situations.

Therefore, Karel [23] proposed an Adaptive Histogram Equalization (AHE) method, which calculates the local histogram of image and redistributes the brightness to change the image contrast. That is to say, AHE is more suitable for improving the local contrast of the image and obtaining more image details. An example of image enhancement

based on histogram equalization is shown in Fig. 3. Obviously, AHE is better than HE in enhancing the contrast of local details of the image, as shown by the red box in Fig. 3.

2.1.2 Local Histogram Equalization

Traditional histogram equalization(HE) techniques usually cause gray level overlap, local area detail reduction, obvious block effect, blurred background and target contours, etc. In order to preserve the brightness characteristics of the image and better enhance the local details of the image, many scholars have begun to devote themselves to the research of local histogram equalization(LHE) algorithms [4, 24–26]. A typical algorithm flow of local histogram equalization is shown in Fig. 4. Our detailed overview of local histogram equalization methods is as follows:

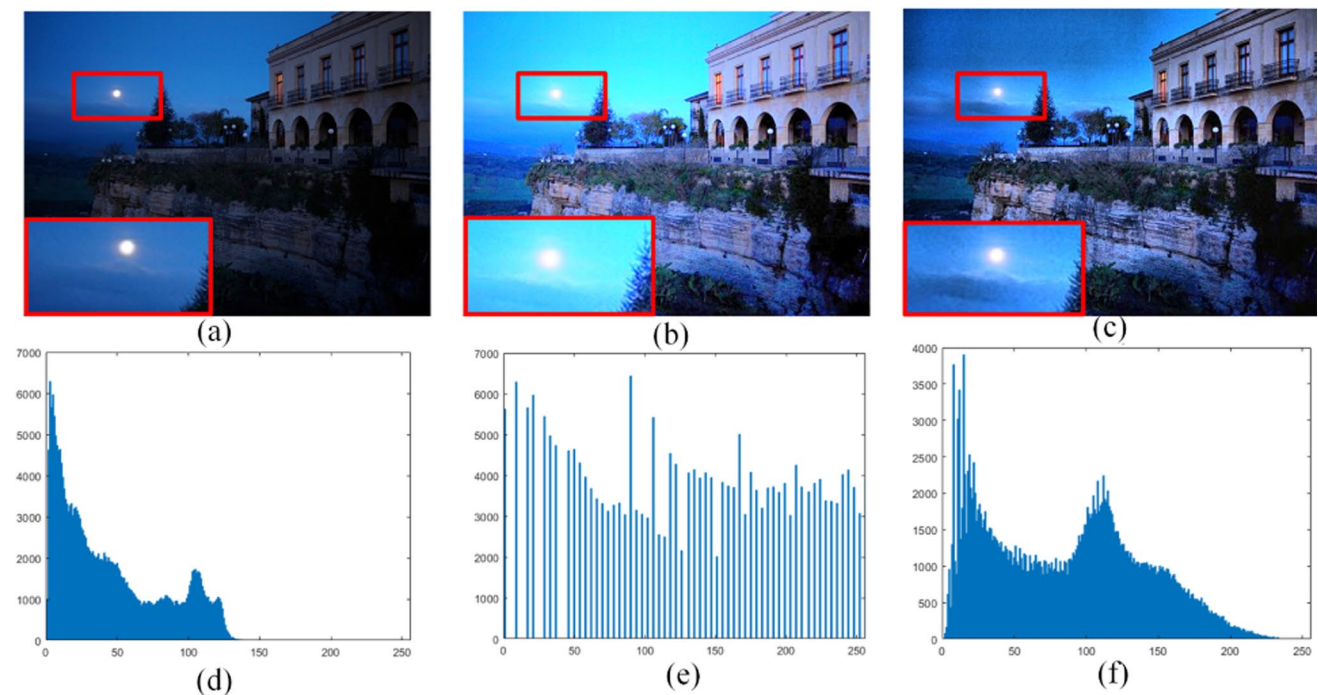
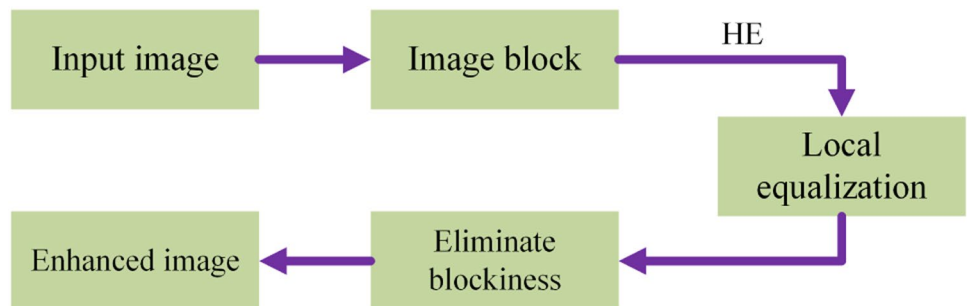


Fig. 3 Image enhancement based on histogram specification. **a** is original image, **b** and **c** are the enhancement results of histogram equalization (HE) and adaptive histogram equalization (AHE), respectively. **d–f** are the histograms of (**a–c**), respectively

Fig. 4 The algorithm flow of local histogram equalization



In 2001, Kim et al. [27] proposed a partially overlapped sub-block histogram equalization (POSHE), which is the most classic local histogram equalization algorithm. POSHE reduces the blocking effect caused by partial equalization and simplifies the number of equalizations. Lamberti et al. [25] proposed a cascaded multistep binomial filtering histogram equalization (CMBFHE) method based on the POSHE algorithm. They eliminate the blocking effect by constructing a cascaded filter. Specifically, the efficiency of the algorithm is much higher than POSHE algorithm, and the computational complexity is significantly reduced. In addition, a non-overlapped sub-blocks and local histogram projection (NOSHP) is presented by Liu et al. [26]. In their researches, the original image first is divided into lots of non-overlapping sub-blocks, and then histogram projection (HP) is performed respectively. Subsequently, each sub-block is related to its adjacent three sub-blocks with a certain weight, so that the overall image and local details can be enhanced. In recent years, Wang et al. [24] proposed adjacent-blocks-based modification for local histogram equalization (ABMHE), which divides the image into active area, inactive area and general area in advance according to the ratio of the value of the image gradient value, so that local areas in the image can be selected in a targeted manner. The detail enhancement effect of ABMHE algorithm is better than that of POSHE algorithm, but it needs to increase the prior calculation process of image area.

In summary, the local histogram method performs local equalization processing for different spatial location regions, and fully takes into account the local information of the image. Although the local details of the image enhancement method based on local histogram equalization will become richer, the average brightness will usually be greatly affected.

2.1.3 Histogram Frequency Weighting

The histogram frequency weighting technique considers the relationship between histogram equalization and image gray-scale frequency. Specifically, the desired enhancement effect is achieved by adjusting the original frequency of image. In recent years, classic image enhancement methods based on histogram frequency weighting [28–30] have emerged. Specifically, the details are as follows:

WTHE [29] is the most classic histogram frequency weighting method. It modifies the histogram by weighting and thresholding to achieve contrast enhancement. Specifically, WTHE sets two high and low thresholds. For high-frequency gray scales and low-frequency gray scales, the frequency is reduced, and for the gray scales in between, the frequency value is increased by gamma correction, and the total frequency is still 1. The enhanced result obtained by equalizing in this way is much better than the traditional HE

algorithm. However, it causes artifacts on some images with slope histogram peaks [31]. In 2011, Yun et al. [32] proposed a contrast enhancement method using a weighted histogram equalization (WHE), which weights the gray value of the HE result and the current gray value setting ratio to get the final result, so that the HE result only occupies a certain proportion. Similar to WHE, Wong et al. [30] performed histogram equalization with maximum intensity coverage. In recent years, researchers have employed the gamma function to modify the existing cumulative distribution function and implement the histogram frequency weighting technique [33, 34].

Finally, in order to show the image enhancement effects of different histogram modification methods, some examples of image enhancement are shown in Fig. 5.

2.1.4 Other Histogram Modification Based Methods

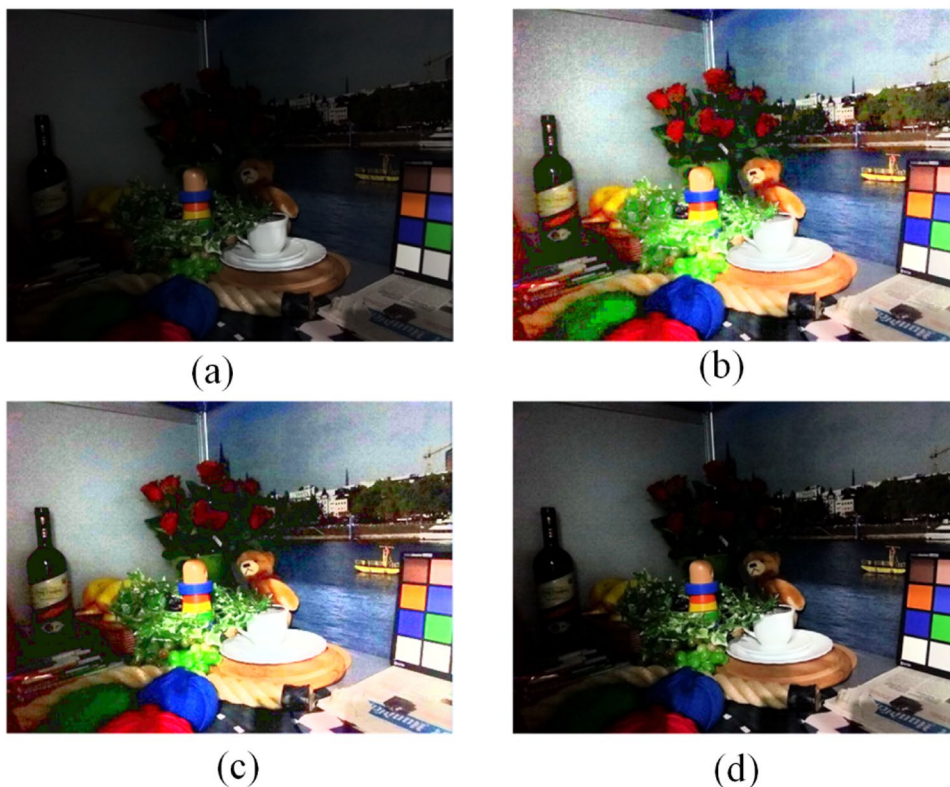
In this paper, considering the limitation of space, we only summarize a few main parts of the method of histogram modification. There are many other image enhancement methods based on histogram modification that have not been listed, such as double histogram equalization, dynamic sub-histogram equalization, transform domain equalization and histogram clipping. Here we briefly summarize.

Double histogram equalization is to decompose the original histogram into two histograms, and then equalize the two histograms separately. Typical methods are brightness preserving bi-histogram equalization [35] (BBHE), dualistic sub-image histogram equalization [4] (DSIHE), minimum mean brightness error bi-histogram equalization [36] (MMBEBHE). In recently years, the double histogram equalization technology combines the ideas of image analysis, such as the local equalization of the background area, target area, texture area, and smooth area [37, 38]. The dynamic histogram equalization takes into account the curve distribution of the histogram, and then selects the appropriate area for equalization. Typical methods are DHE [39] and BPDHE [40]. In addition, there are also transform domain histogram equalization [41–43] and histogram clipping [44–46], etc.

2.2 Retinex-Based Enhancement

The Retinex theory was proposed by Land et al. [47] in 1968, which is based on the principle of retinal imaging to simulate the human visual system (HVS). In this theory, it is assumed that image lightness depends on illumination and reflectance. The Retinex algorithm [48] was proposed based on the fact that when the human visual system judges the true color of an image, it is hardly disturbed by light. In this section, we elaborate the contents of the Retinex model as follows:

Fig. 5 Image enhancement based on histogram modification. **a** is the original image. **b**, **c** and **d** are the enhanced results of POSHE [27], WTHE [29] and AGCWD [33], respectively



2.2.1 Basic Retinex Algorithm

According to Land et al. [47–49], color vision is not determined by the intensity of visible light irradiated to the human eye, but the inherent reflectivity of the surface of the object. The human eye can filter out the influence of light in some way and directly obtain the reflectivity of the surface of the object to determine colour. Therefore, the formation of a low-light image can be described as follows:

$$L(x, y) = R(x, y) \cdot B(x, y) \tag{3}$$

where $L(x, y)$ is the original image, $R(x, y)$ is the reflection image, $B(x, y)$ is the illuminance image and (x, y) is the pixel coordinates.

Generally, Retinex performs image processing based on two methods: one is to divide the color image into three RGB channels and the Retinex algorithm is applied to each channel to solve the color shift problem. The other is to convert the image to a brightness channel in the color model (such as HSV color model, CIELuv color model) and its brightness channel is processed separately to solve the problem of uneven illumination.

2.2.2 Path-Based Retinex Algorithm

The path-based Retinex algorithm [50] is the most basic Retinex model. Its basic idea is to use the local brightest point

in the path-White Patch [51] (WP) to calculate the relative brightness of adjacent pixels in the path to obtain the reflection component. Given a normalized input image $L(x)$, a path can be selected from a certain point y to the target point x in the image. Assume that the brightness values of two adjacent pixels on the path are $S(x_k)$ and $S(x_{k+1})$ respectively. Then the relative brightness value at x can be obtained along this path:

$$R^\wedge(x) = \prod_{m=1}^{n-1} \delta_\epsilon \left(\frac{S_{m+1}}{S_m} \right) \tag{4}$$

where n is the number of pixels on the path and δ_ϵ is threshold function. In order to enhance the robustness of the algorithm, the relative pixel brightness of x at multiple paths can be expressed as:

$$R(x) = \frac{1}{N} \sum_{k=1}^N R_k^\wedge(x) = \frac{1}{N} \sum_{k=1}^N \prod_{m_k=1}^{n_k-1} \delta_\epsilon \left(\frac{S_{m_k+1}}{S_{m_k}} \right) \tag{5}$$

where N is the number of paths. In 2005, Provenzi et al. [52] proved that the threshold mechanism has very little contribution to the model algorithm, so (5) can be simplified as:

$$R(x) = \frac{1}{N} \sum_{k=1}^N \prod_{m_k=1}^{n_k-1} \left(\frac{S_{m_k+1}}{S_{m_k}} \right) \tag{6}$$

For the path-based Retinex algorithm, the researchers have focused on the selection of the pixel path, and lots of related work has gradually emerged. Marini et al. [53] used a random midpoint displacement method so that the path chosen is close to Brownian motion. Cooper et al. [54] used a double helix method to select the path. Their method has achieved good results in reducing noise. However, since the sampling method of the path is one-dimensional and depends on the direction information, rather than the neighborhood information, the processed image is prone to appear halo, artifacts, etc [55–57]. In order to solve these problems, therefore, Provenzi et al. [58] proposed a two-dimensional Random Spay Retinex (RSR). In the work of Provenzi et al. [58], the path method is designed to scatter points in a two-dimensional circle, and the density of the circular scatter points at the center x distance r is controlled by the density function. The algorithm is described as follows:

$$R(x) = \frac{1}{N} \sum_{k=1}^N \frac{L(x)}{\max \{L(i), i \in P_k(x)\}} \tag{7}$$

where $P_k(x)$ indicates the area of the k -th spot, $L(x)$ is the input image. Based on the work of Provenzi et al., the researchers have begun to improve the algorithm, such as the LRSR [59] (Light random sprays Retinex) model and SLRMSR [60] (Smart light random memory sprays Retinex) model proposed by Banic and Loncaric. In addition, Bertalmio et al. [61] proposed a kernel-based Retinex model, which replaces the two-dimensional spay process with a more efficient kernel.

Path-based Retinex algorithm can effectively improve the image contrast. However, due to the uncertainty of its initial position, end position and path selection, it is easy to introduce undesirable noise and affect the accuracy of illumination estimation. Besides, the computational complexity is high, so that it is inconvenient to apply in practice, and it is difficult to obtain high-quality pictures with high color fidelity and good visual effects.

2.2.3 Retinex Algorithm Based on Center/Surround

The Retinex center/surround model was first proposed by Land [62], and its core idea is to estimate the illuminance component with a Gaussian kernel function. According to Land [62], it is considered that the Retinex result of a certain point of the image should be the ratio of the luminous intensity of that point to the average luminous intensity of the adjacent positions. In addition, the Mach zone phenomenon is explained through experiments and the rationality of the method is established. Typical Retinex center/surround models include single-scale Retinex (SSR) [9] algorithm, multi-scale Retinex (MSR) [63] algorithm and multi-scale Retinex with color restoration factor (MSRCR) [11] algorithm.

According to [9], SSR model can be described as:

$$R_i(x, y) = \log I_i(x, y) - \log [F(x, y) * I_i(x, y)] \tag{8}$$

where I_i is the image distributed in the i -th color band, $R_i(x, y)$ is the enhancement result, $*$ means convolution operation, $F(x, y)$ represents the convolution kernel function and the formula is as follows:

$$F(x, y) = k \cdot \exp \left(-\frac{x^2 + y^2}{2\sigma^2} \right) \tag{9}$$

where k needs to satisfy $\iint F(x, y) dx dy = 1$. σ represents kernel parameter, which can determine the enhancement effect of image. Specially, the larger (smaller) σ is, the richer (weaker) the image detail information will be, and the lower (higher) the color fidelity will be.

The enhancement effect of the SSR model is closely related to the size of the Gaussian kernel. For a grayscale image, a small Gaussian kernel can increase the brightness of the dark parts of the image, but it may cause excessive compensation and make the dynamic range of the image too small, i.e., "halo" phenomenon. A large Gaussian kernel can increase the dynamic range of the image, but may make the model lose its local characteristics, and the shadow of the image cannot be compensated. Therefore, Rahman et al. [63] proposed the MSR algorithm. The basic idea of the algorithm is to sum multiple Gaussian kernel scale parameters by weight and its expression is as follows:

$$\mathfrak{R}_m(x, y) = \sum_{n=1}^N \omega_m [\log I_m(x, y) - \log (F_m(x, y) * I_m(x, y))] \tag{10}$$

where $F_m(x, y)$ is the filter kernel function, and the scale and the total number of scales are δ_m and N respectively. ω_m is the weighting factor corresponding to the m -th parameter, which satisfies $\sum \omega_m = 1$.

In order to reduce the color cast and restore the true color of the image, a multi-scale Retinex algorithm with color restoration factor (MSRCR) [10, 11, 64] was proposed. The model algorithm is expressed as follows:

$$\mathfrak{N} = c_i(x, y) \cdot \mathfrak{R}_m(x, y) \tag{11}$$

where \mathfrak{N} is the enhanced result of image, $c_i(x, y)$ is the color restoration factor and its expression is as follows:

$$c_i(x, y) = \mu \cdot \log \left(\eta \cdot \frac{I_i(x, y)}{\sum_i I_i(x, y)} \right) \tag{12}$$

where μ and η are the gain factor and offset that affect the color recovery of the image, and both are constant, I_i is the image distributed in the i -th color band.

Wang and Luo [65] improved Jobson’s [9] method and proposed a multi-layer image enhancement model. Although the Retinex algorithms based on center/surround have a good effect in terms of color fidelity, the time complexity is high, the halo phenomenon is obvious and image distortion is inevitable. Some examples of Retinex algorithm enhancement based on center/surround are shown in Fig. 6. From Fig. 6, we can see that the center/surround-based Retinex algorithm can effectively enhance the contrast of the image. However, the parameter settings in the algorithm have limitations, resulting in uncertainty in the contrast, chroma, and clarity of the enhanced image, so the enhancement effect is not ideal. Specially, the SSR algorithm can slightly improve the color of the sky, but the overall contrast is reduced, and the halo phenomenon is more serious. in Fig. 6f, the MSRCR algorithm has a color factor, and the color is obviously better than that of the MSR algorithm in Fig. 6e. With the in-depth research of the Retinex algorithm by scholars, especially for the two major defects of color distortion and halo in the traditional Retinex algorithm, a large number of improved algorithms have been proposed.

2.2.4 Partial Differential Equations(PDE) model

As a matter of fact, the basic Retinex algorithm is a calculation theory of color vision, and its details are not described mathematically [66]. In 1974, Horn et al. [67] simplified the Retinex model for the first time based on partial differential equations (PDE). For (3), they separated the illumination

component $B(x, y)$ and the reflection component $R(x, y)$ from $L(x, y)$ and constructed the partial differential equations. Take the logarithm of both sides of (3) to get:

$$\mathcal{L}(x, y) = \mathcal{R}(x, y) + \mathcal{B}(x, y) \tag{13}$$

where $\mathcal{L}(x, y) = \log L(x, y)$, $\mathcal{R}(x, y) = \log R(x, y)$, $\mathcal{B}(x, y) = \log B(x, y)$. Then, from (13) we can get:

$$\Delta \mathcal{L}(x, y) = \Delta \mathcal{R}(x, y) + \Delta \mathcal{B}(x, y) \tag{14}$$

where Δ is Laplacian. In addition, the following algorithm can be defined as:

$$\delta_\epsilon(\Delta \mathcal{L}(x, y)) = \begin{cases} \Delta \mathcal{L}(x, y), & |\Delta \mathcal{L}(x, y)| > \epsilon \\ 0, & \text{else} \end{cases} \tag{15}$$

According to Horn et al. [67], under the premise of uniform illumination, the illumination map $B(x, y)$ is smooth everywhere, and its partial derivative is a very small value. Thus, we get $\delta_\epsilon(\Delta \mathcal{L}(x, y)) \approx \Delta \mathcal{R}(x, y)$. Finally, its Poisson equation is as follows:

$$\Delta \hat{\mathcal{R}}(x, y) = \delta_\epsilon(\Delta \mathcal{L}(x, y)) \tag{16}$$

Horn first proposed to use an iterative strategy to solve it, but this is not the optimal method [56]. Blake et al. [68–70] improved Horn’s method by using a gradient method. Morel et al. [71, 72] derived a similar Poisson equation based on the path-based Retinex model. In addition, they proved that the Retinex model can be characterized as a discrete Poisson

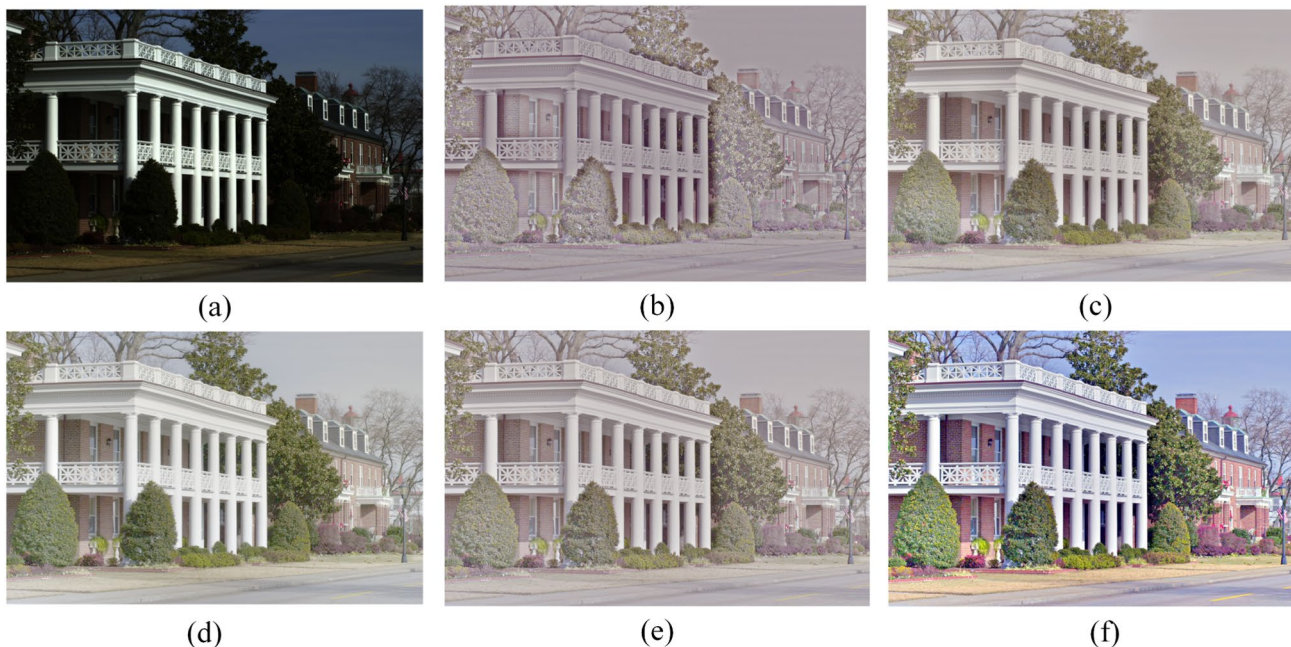


Fig. 6 Some examples of Retinex algorithm enhancement based on center/surround. **a** is the original image. **b–f** are the enhanced results of SSR with $\delta = 15$ [9], SSR with $\delta = 80$, SSR with $\delta = 250$, MSRCR($\delta_1 = 15, \delta_2 = 80, \delta_3 = 250$) [63] and MSRCR [11], respectively

equation with Neumann boundary conditions and expressed as follows:

$$\begin{cases} -\Delta_d \mathcal{R}(x, y) = \sum_{x_0 \in N_d(x, y)} \delta_\epsilon(\mathcal{L}(x, y) - \mathcal{L}(x_0, y_0)), x \in \Omega \\ \frac{\partial_d \mathcal{R}(x, y)}{\partial n} = 0, x \in \partial\Omega \end{cases} \quad (17)$$

where Δ_d and ∂_d represent the discrete Laplace operator and discrete partial differential symbols respectively. It can be seen that (17) and (16) are equivalent without considering the threshold function δ_ϵ . Ma et al. [73] analyzed Poisson equation (17) in detail and concluded that the solution can be regarded as an optimization problem of L2 norm, that is, L2-Retinex. Then,

$$\hat{\mathcal{R}} = \arg \min_R \left\{ \|\Delta \mathcal{R} - \delta_\epsilon(\Delta \mathcal{L}(x, y))\|_2^2 \right\} \quad (18)$$

Similarly, they also proposed a L1-Retinex model [74], which is expressed as follows:

$$\hat{\mathcal{R}} = \arg \min_R \left\{ \|\Delta \mathcal{R} - \delta_\epsilon(\Delta \mathcal{L}(x, y))\|_1 \right\} \quad (19)$$

In summary, the PDE model constructs partial differential equations based on some basic assumptions of Retinex theory, and its description is more accurate than the path-model. In addition, the PDE model has fewer parameters and the results obtained are more stable.

2.2.5 Retinex Models Based on Variational Methods

In image enhancement, in order to effectively eliminate the halo, scholars proposed the Retinex algorithm based on the variational framework, also known as the Retinex-like model. Classical variational models usually include two types: the HVS-based variational model and the Horn physical prior variational model.

A. The HVS-based Variational Model

Palmaamestoy et al. [75] first proposed the Retinex-like variational model based on HVS. In their method, the subject model is described as:

$$E_\omega(I) = D(I) + C_\omega(I) \quad (20)$$

where $E_\omega(I)$ is defined as the color correction energy functional, $D(I)$ is the dispersion term and $C_\omega(I)$ is the contrast term. The global item $D(I)$ is used to control the final generated image and satisfies the GW (Gray world) assumption [76]. Minimizing $C_\omega(I)$ can enhance the local contrast of the image, which has certain characteristics of the Retinex model. In particular, Palmaamestoy choose the dispersion energy as the global term $D(I)$.

The final optimized result is close to the original image I_0 and satisfies the GW assumption, that is, the final result is close to 1/2. The entropic dispersion term $D(I)$ is defined as follows:

$$\begin{aligned} D_{\alpha, \beta}^\epsilon(I) := & \alpha \sum_{x \in Y} \left(\frac{1}{2} \log \frac{1}{2I(x)} - \left(\frac{1}{2} - I(x) \right) \right) \\ & + \beta \sum_{x \in Y} \left(I_0(x) \log \frac{I_0(x)}{I(x)} - (I_0(x) - I(x)) \right) \end{aligned} \quad (21)$$

where $\alpha, \beta > 0$. In addition, the contrast term is described as follows:

$$C_\omega^f(I) := \frac{1}{4} \sum_{x \in Y} \sum_{y \in Y} \omega(x, y) f \left(\frac{\min(I(x), I(y))}{\max(I(x), I(y))} \right) \quad (22)$$

where $\omega(x, y)$ is the distance function, and its value decreases monotonously with the increase of the distance between pixels. $\omega(x, y)$ can be used to achieve "local" characteristics and f represents the gamma correction function. The Retinex-like model has the disadvantage of a large calculation. Therefore, Provenzi et al. [77] combined with wavelet based on Palmaamestoy et al.'s variational model to reduce the amount of calculation.

B. Horn's Physical Prior based Retinex

Kimmel et al. [78] believed that the spatial variation of the illumination component is smooth. According to the priori assumption, based on Horn's work [67], they proposed a variational form of Retinex and the penalty functional is as follows:

$$\begin{aligned} \text{Minimize } : F[\mathcal{B}] = & \iint_{\Omega} (|\nabla \mathcal{B}|^2 + \alpha(\mathcal{B} - \mathcal{L})^2 + \beta|\nabla(\mathcal{B} - \mathcal{L})|^2) dx dy \\ \text{Subjecto } : & \mathcal{B} \geq \mathcal{L} \text{ and } \langle \nabla \mathcal{B}, \vec{n} \rangle = 0 \text{ on } \partial\Omega, \end{aligned} \quad (23)$$

where Ω is the support of the image, $\partial\Omega$ is the partial differential of the image, etc., the boundary, and \vec{n} is the normal to the boundary. α, β are non-negative real parameters and are used to control the importance of the corresponding items. The first penalty term $|\nabla \mathcal{B}|^2$ ensures that the final illumination component is smooth, and the second term $(\mathcal{B} - \mathcal{L})^2$ is a fidelity term, which ensure that the difference between \mathcal{B} and \mathcal{L} is not too large. The third item $|\nabla(\mathcal{B} - \mathcal{L})|^2$ is equivalent to $|\nabla(\mathcal{R})|^2$, and its function is to obtain the spatial distribution of the reflected components as smooth as possible, so as to obtain a better visual effect. Since then, some novel models have been constructed based on the Kimmel et al.'s model by modifying or adding some constraints [79–81], which reflects the flexibility of the variational method. Ma and Osher [81] introduced

a total variation (TV) noise suppression model [82] to estimate the image illumination component. Their model is described as follows:

$$\text{Minimize} : F[\mathcal{B}] = \iint_{\Omega} \frac{1}{2} (\|\nabla \mathcal{B}\|_2^2 + t \|\nabla \mathcal{R}\|_2) dx dy \quad (24)$$

where $\|\nabla \mathcal{B}\|_2$ is the TV term, which is used to obtain a piecewise smooth reflection map, t is the weight coefficient. Ng and Wang [80] added fidelity items based on Ma and Osher’s works and obtain:

$$\text{Minimize} : F[\mathcal{B}] = \iint_{\Omega} \left(\frac{\alpha}{2} \|\nabla \mathcal{B}\|_2^2 + \|\nabla \mathcal{R}\|_2 + \frac{\beta}{2} \|\mathcal{B} - \mathcal{R} - \mathcal{L}\|^2 \right) dx dy \quad (25)$$

where $\|\mathcal{B} - \mathcal{R} - \mathcal{L}\|^2$ is the fidelity term, α and β are the weight coefficients. The variational model based on physical prior conditions is relatively flexible, and different constraints can be easily added to obtain better results.

C. Other Retinex-like Models

In recent years, a large number of Retinex-like models have been also emerging. Fu et al. [83] proposed a probabilistic model for image enhancement (PIE). A Maximum a Posteriori (MAP) was used in Fu et al.’s method to estimate reflectance and illumination in the linear domain. Finally, to effectively decompose the illumination and reflectivity, the MAP problem is transformed into an energy minimization problem. The Alternating Direction Multiplier Method (ADMM) [84] is used to simultaneously estimate the illumination and reflectance components. In particular, PIE is a processing algorithm based on linear space. Compared with the logarithmic domain processing algorithm, PIE has the advantages of preserving details well and avoiding over-smooth reflection components. In 2016, related work of [85] proposed a weighted variational model for simultaneous reflectance and illumination estimation (SRIE) with more details. In their method, noise can be suppressed to a certain extent. However, the unconstrained assumption of isotropy may destroy the illumination component.

Guo et al. [86] proposed a low-light image enhancement (LIME) method via illumination map estimation. Specially, a structure prior is imposed to refine the illumination map. On the basis of Max-RGB [47], they considered the defect that it can only boost global illumination, and introduced the following initial estimation:

$$\hat{B}(x, y) \leftarrow \max_{c \in \{R, G, B\}} L^c(x, y) \quad (26)$$

According to Guo et al.’s researches, $\hat{B}(x, y)$ in (26) guarantees that the recovery will not reach saturation. Although LIME has a good effect on low-light image enhancement, it can’t solve the problem of over-enhancement of illumination, because that it adds to the noise amplification. In recent years, related works in large numbers have taken noise as an important factor of Retinex-like, and proposed some low-light image enhancement models. Li et al. [87] proposed a Retinex model with a definite injected noise term, and for the first time tried to estimate the noise map based on the model. Ren et al. [88] proposed a Low-Rank Regularized Retinex Model (LR3M). They injected low-rank priors into the Retinex decomposition process for the first time and suppressed noise in the reflection map. Unlike the method of Li et al. [87], LR3M is full aware of noise and performs adaptive processing throughout the enhancement process. Recently, Xu et al. [89] proposed a structure and texture aware Retinex (STAR) model. In their method, STAR model is solved based on an alternating optimization algorithm. In addition, each sub-problem is transformed into a vectorized least squares regression, with closed-form solutions. Tang et al. [90] proposed a new prior constraint, ie., local flatness. Different from the previous works, they selectively add local flatness to the illumination by calculating the deviation between the estimated illumination map and the reference map.

Finally, some examples of Retinex algorithm enhancement based on variational methods for low-light image enhancement are shown in Fig. 7. Intuitively, the overall contrast enhancement of PIE [83] and SRIE [85] is not satisfactory. LIME [86] does not preserve the details of image well.

2.3 Visual Cortex Neural Network Based Enhancement

The visual cortex neural networks were proposed to simulate the mammalian visual cortex system. In recent years, a large number of visual cortex neural network models have been developed. Among them, Eckhorn model [91, 92], Parodi model [93] and Rybak model [94] are the most representative visual cortex models. Further, the pulse coupled neural network (PCNN) evolved from the Eckhorn model has been extensively and deeply studied by researchers. In this section, we mainly focus on two classic visual cortex neural networks: PCNN and Rybak model, to review image enhancement methods.

2.3.1 Basic PCNN Model and Image Enhancement

PCNN, the third generation neural network, was inspired by the visual cortex of cats and evolved from the Eckhorn model. As a single layer neural network that does not require any training [95], PCNN has been widely applied in image processing [96], based on its biological background mechanism. The basic PCNN is elaborated in Ranganath et al.'s work [96]. Furthermore, Lindblad et al [97] gave the discrete expression of the basic PCNN model as follows:

$$F_{ij}[n] = e^{-\alpha_f} F_{ij}[n - 1] + V_F \sum_{kl} M_{ijkl} Y_{kl}[n - 1] + S_{ij} \tag{27}$$

$$L_{ij}[n] = e^{-\alpha_l} L_{ij}[n - 1] + V_L \sum_{kl} W_{ijkl} Y_{kl}[n - 1] \tag{28}$$

$$U_{ij}[n] = F_{ij}[n](1 + \beta L_{ij}[n]) \tag{29}$$

$$Y_{ij}[n] = \begin{cases} 1, & \text{if } U_{ij}[n] > E_{ij}[n] \\ 0, & \text{else} \end{cases} \tag{30}$$

$$E_{ij}[n] = e^{-\alpha_e} E_{ij}[n - 1] + V_E Y_{ij}[n] \tag{31}$$

Obviously, a basic PCNN neuron $N(i, j)$ embedded in a 2-D array is mainly composed of two important parts: feedback input F_{ij} and linking input L_{ij} . Neighboring neurons communicate with each other through the two synaptic weights M_{ijkl} and W_{ijkl} respectively. In addition, the previous state altered by the attenuation factors $e^{-\alpha_f}$ and $e^{-\alpha_l}$ are also contained in the current feedback input F_{ij} and linking input L_{ij} . The input S_{ij} is only included in the feedback input F_{ij} . The feedback input F_{ij} and linking input L_{ij} are combined with a link factor β to obtain an internal activity U_{ij} . Then, the output Y_{ij} is obtained through the comparison of internal activity U_{ij} and dynamic threshold E_{ij} . Finally, the activation state of the neuron is determined, that is, the output Y_{ij} is 1 (activated) or 0 (not activated). The threshold E_{ij} is dynamic, because when the neuron is activated, the threshold would increase by the amplification item V_E . Otherwise, the dynamic threshold would decay by the factor $e^{-\alpha_e}$ until the neuron activates again. The parameters n is the discrete

iteration time of the neuron, V_F and V_L are the amplitudes of feedback input F_{ij} and linking input L_{ij} respectively, α_f , α_l and α_e are the exponential decay coefficients of feedback input, linking input and dynamic threshold, respectively.

In 2003, Zhang and Lu [98] studied the relationship between PCNN and human visual characteristics and applied it to image enhancement. Early literature [99] shows that the brightness perceived by the human visual system is different from the brightness of the light source, and it has a logarithmic relationship with the illuminance obtained by the human eye. In addition, the brightness perceived by human is not a simple function of light intensity, such as the Mach band effect [99, 100]. The logarithmic transformation characteristic of the image is shown in Fig. 8. The horizontal axis x is the pixel intensity of the image, where *Dark* is the darkest gray scale value of the image and *Bri* is the brightest gray scale value. In Zhang and Lu's work, the enhanced image is defined as the excited grayscale image perceived by PCNN as follows:

$$EnI_{ij} = ln(Bri) - (n - 1)\alpha_e \tag{32}$$

where EnI_{ij} is the perceptual excitation value of the (i, j) -th pixel in the enhanced image, $ln(*)$ is logarithmic transformation and n is the activated times ($n=1, 2, 3, \dots$) of PCNN. For the basic PCNN model, the parameters are set as follows:

$$M_{ijkl} = W_{ijkl} = \frac{1}{(i - k)^2 + (j - l)^2} \tag{33}$$

That is, the synapse weight between neurons is defined as the inverse of the square of the Euclidean distance [96, 101, 102]. The threshold exponential decay coefficient α_e is a small value and is set to 0.001. In addition, (27) is usually simplified to $F_{ij}[n] = S_{ij}$. $V_L = 1$.

Based on this theory, many scholars have proposed image enhancement algorithms for specific applications based on PCNN [103–106]. In summary, the image enhancement algorithm based on PCNN can be described as follows:

Algorithm 1: Enhancement by PCNN model.

- Input** : The gray image S_{ij}
Output: The enhancement result Y for each image.
- 1: Initial the parameters of PCNN.
 Calculate W_{ijkl} and M_{ijkl} according to (33),
 calculate F_{ij} and L_{ij} according to (27) and (28),
 respectively.
 - 2: $n = 0$
 - 3: **while** $n < N$ **do**
 - 4: $n = n + 1$
 - 5: update E_{ij} by (31).
 - 6: reapt.
 - 7: update U_{ij}, Y_{ij} according to (29), (30).
 - 8: calculate EnI_{ij} .
 - 9: $Y_0 = Y_{ij} \cdot EnI_{ij}$
 - 10: **if** *any* ($Y(:) == 0$)
 - 11: **break**;
 - 12: **end**
 - 13: **end while**
 - 14: Normalize linearly Y_0 to obtain Y .
-

Based on the capture characteristics and synchronous oscillation characteristics of PCNN, Li et al. [107–109] combined genetic algorithm and Otsu theory, etc., proposing many image enhancement algorithms with PCNN noise

reduction characteristics. In addition, Qi et al. [110] proposed a feedback pulse-coupled neural network (FPCNN) and applied it to image enhancement. In their work, the input image is first denoised by PCNN, and then segmented by PCNN, and the continuous output of multiple binary images is logically "OR" and "exclusive or (EOR)" operations to obtain image texture, edge and other information. Finally, these information is combined with the original image to form an enhanced image. In 2019, Nie et al. [111] proposed to employ the classical visual receptive field (CVRF), improving pulse coupled neural network (PCNN) and conducted a contrast enhancement of MRI images.

2.3.2 Time Matrix of PCNN

In addition to the human eye's method of perceiving brightness, namely (32), the time matrix is also one of the most important tools for image enhancement. The time matrix is defined to record the first firing time of each PCNN neuron [112]. In 2009, Zhan et al. [113] found that the sensitivity of the time matrix is inversely related to the intensity of the image pixels.

A neuron fires when its membrane potential is greater than the threshold. Then, the threshold would decay exponentially from the first predetermined $E_{ij}(0)$ by (31). That is, the neuron (i, j) is firing at the time T_{ij} when

$$U_{ij}(T_{ij}) > E_{ij}(T_{ij}) = e^{T_{ij}} E_{ij}(0) \tag{34}$$

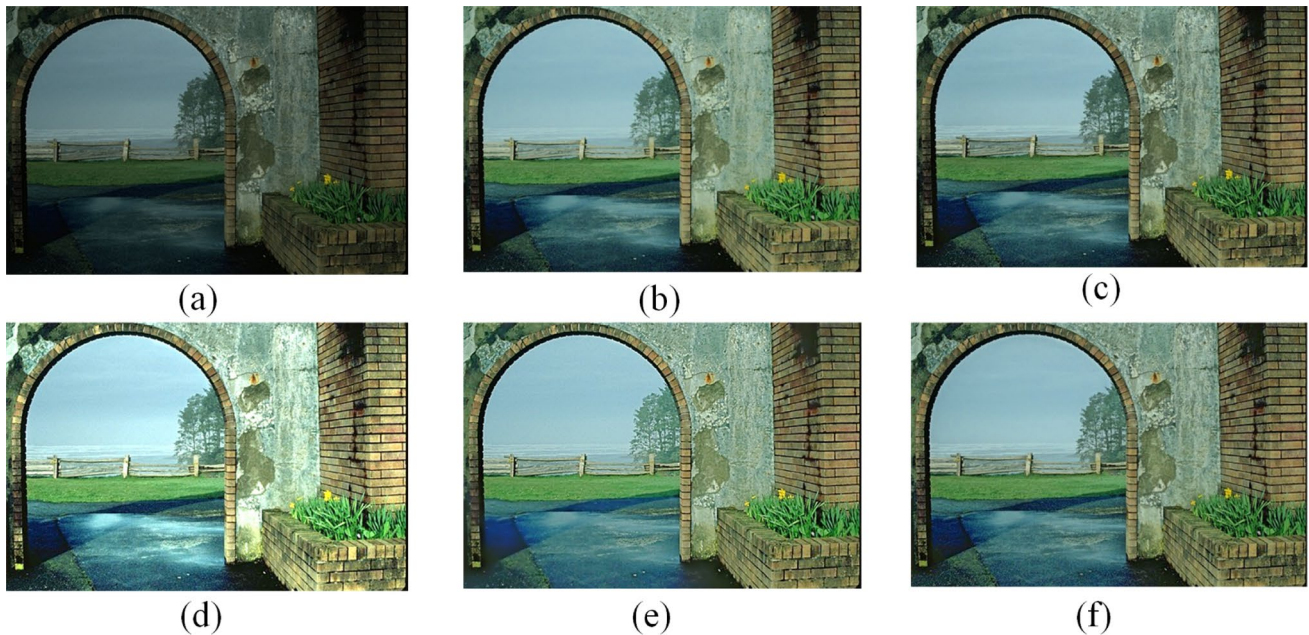


Fig. 7 Some examples of Retinex algorithm enhancement based on variational methods. **a** is the original image. **b–f** are the enhanced results of PIE [83], SRIE [85], LIME [86], Li et al. [87] and Tang et al. [90] respectively

Therefore, the first firing time T_{ij} can be obtained from (34),

$$T_{ij} = \ln \frac{U_{ij}(T_{ij})}{E_{ij}(0)} \tag{35}$$

From (35) we get the analytical solution of the time matrix T_{ij} , but T_{ij} still cannot be obtained because (35) is an implicit function. In implementation, the time matrix can be obtained according to the following formula [114]:

$$T_{ij}(n) = T_{ij}(n - 1) + nY_{ij}(n) \tag{36}$$

The premise that the time matrix can be obtained is that the threshold amplitude factor V_E must be large enough to ensure that the neuron fires only once. Therefore, the time matrix of PCNN can be obtained by the following Algorithm 2.

Algorithm 2: The time matrix of PCNN.

Input : The gray image S_{ij}
Output: The time matrix T_{ij} for each image.

- 1: Initial the parameters of PCNN.
 Calculate W_{ijkl} and M_{ijkl} according to (33),
 calculate F_{ij} and L_{ij} according to (27) and (28),
 respectively.
- 2: $firednum = 0, n = 0$.
- 3: **while** $n < N$ **do**
- 4: $n = n + 1$
- 5: update E_{ij} by (31).
- 6: reapt.
- 7: update U_{ij}, Y_{ij} according to (29), (30).
- 8: $firednum = firednum + sum(Y)$
- 9: $T = T + nY$
- 10: **end while**

2.3.3 Spiking Cortical Model (SCM)

The spiking cortical model (SCM) is a variant model of PCNN, which was proposed by Zhan et al. [113] in 2009. It consists of a leakage integrator, linking input and feedback input as follows

$$U_{ij}[n] = fU_{ij}[n - 1] + S_{ij} \sum_{kl} W_{ijkl} Y_{kl}[n - 1] + S_{ij} \tag{37}$$

The dynamic threshold item directly inherits the settings of the basic PCNN model and is expressed as:

$$E_{ij}[n] = gE_{ij}[n - 1] + hY_{ij}[n - 1] \tag{38}$$

Furthermore, the sigmoid activation function is employed as the ignition condition of SCM neurons and is described as follows,

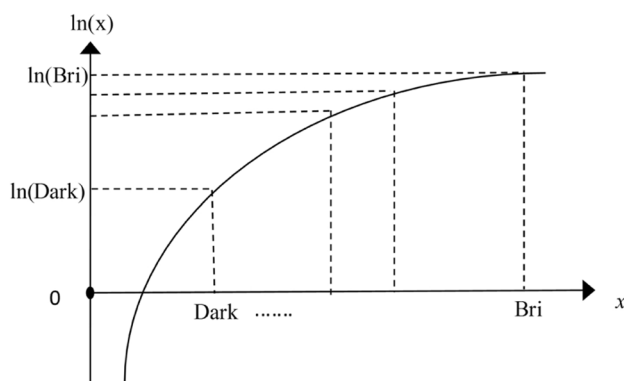


Fig. 8 Logarithmic transformation properties of image

$$Y_{ij}[n] = \begin{cases} 1, & 1/(1 + \exp(-\gamma(U_{ij}[n] - E_{ij}[n]))) > 0.5 \\ 0, & \text{else} \end{cases} \tag{39}$$

As we can see, SCM directly combines the linking term and the feedback term into a single equation to form an internal activity item U_{ij} .

2.3.4 Feature-Linking Model (FLM)

Compared with SCM, the feature-linking model (FLM) retains feeding wave and linking wave, while reducing parameters. In addition, an inhibition-linking item is contained in FLM. According to [115], FLM can be described as follows,

$$U_{ij}[n] = fU_{ij}[n - 1] + \left(\sum_{kl} M_{kl} Y_{kl}[n - 1] + S_{ij} \right) \times \left(1 + \beta \left(\sum_{kl} M_{kl} Y_{kl}[n - 1] - d \right) \right) \tag{40}$$

$$E_{ij}[n] = gE_{ij}[n - 1] + V_E Y_{ij}[n - 1]$$

$$Y_{ij}[n] = \begin{cases} 1, & \text{if } U_{ij}[n] > E_{ij}[n] \\ 0, & \text{else} \end{cases}$$

where f, g denote the attenuation time constant and d is a positive constant for the globally inhibitory.

2.3.5 Improved PCNN Based Enhancement

As mentioned above, based on the mechanism of human visual characteristics, the time matrix of PCNN can be used as an image enhancement tool. In implementation, the enhancement image can be obtained by the negative of the time matrix. Generally, image enhancement algorithms based on time matrix can be summarized as follows:

Step 1: Normalize the image intensity I by:

$$S = \frac{I - \min(I)}{\max(I) - \min(I)} + \epsilon \tag{41}$$

where $\min(I)$ is the minimum value of I and $\max(I)$ is the maximum value of I . ϵ is a small positive value, which is just to avoid zero value in the image.

Step 2: Obtain the time matrix T_{ij} by Algorithm 2.

Step 3: According to the definition of the time matrix, it can be found that the neuron corresponding to the pixels with higher intensity fires earlier than the neuron with a low stimulation. In other words, the neural network receives the higher intensity pixel input in the image and returns a smaller value in the time matrix. Thus, the values in the time matrix need to be reversed. Zhan et al. [115] gave the transformation function as follows:

$$T_{ij} = \max(T_{ij}) + 1 - T_{ij} \tag{42}$$

Step 4: Normalize T_{ij} and round to the nearest integer, obtaining the enhanced image \mathcal{J} .

$$\mathcal{J} = \left\lfloor 255 \frac{T - \min(T)}{\max(T) - \min(T)} + 0.5 \right\rfloor \tag{43}$$

In addition to the above enhancement algorithm, Zhan et al. [116] proposed a linking synaptic computation network (LSCN) and applied it to image enhancement. They considered the gamma band oscillations in visual cortical neurons as the key role to improve the model.

2.4 Rybak Model

Rybak model was derived from the visual cortex system of guinea pigs, and its core structure is the iso-orientation domain model. According to [94, 117], basic Rybak model is proposed based on the screen-type neuron-like structure (SNS) and is described as follows:

$$X_{ij}^S = F^S \otimes \|S_{ij}\| \tag{44}$$

$$X_{ij}^I = F^I \otimes \|Z_{ij}\| \tag{45}$$

$$Z_{ij} = f \left\{ \sum X_{ij}^S - \left(\frac{1}{\phi}\right) X_{ij}^I - h \right\} \tag{46}$$

where \otimes is the convolution operator. ϕ denotes the time constants of inertial block and h represents the threshold of summarizing element. Obviously, the neuron (i, j) receives stimulus input S_{ij} to form a linking input block X_{ij}^S . F^S is a local connection of "on-center/off-surround" and F^I is a local oriented connection, which exist in the visual cortex

neural networks with a large number of different resolutions. In (46), f is a nonlinear threshold function. It can be seen from (44) to (46), Rybak model mainly consists of two parts: feeding input X_{ij} and output Z_{ij} . The algorithmic structure of SNS is shown in Fig. 9.

In Rybak's theory, F^S is defined as a local connection. The input receptive field F^S can enhance the sharp edge information of images [118] and connection receptive field F^I with different forms of orientation connection has a key role in the orientation tuning response of image processing.

2.4.1 Rybak Neural Network (RYNN)

Based on Rybak's theory, Qi et al. [119] proposed a RYNN model by introducing the redefined threshold segmentation module and a nonlinear generator. RYNN's discrete expressions as follows:

$$\begin{cases} E_{ij}[n] = e^{-\alpha_e} E_{ij}[n-1] + V_E Z_{ij}[n-1] \\ X_{Iij}[n] = \alpha \sum_{kl} F_{ijkl}^I Z[n-1] + S_{ij} \\ X_{Sij}[n] = V_S \sum_{kl} F_{ijkl}^S S_{ij} \\ P_{ij}[n] = X_{Sij}[n] - \frac{1}{\phi} X_{Iij}[n] + h \\ Z_{ij}(n) = f(P, E) \end{cases} \tag{47}$$

where E_{ij} is dynamic threshold, which is similar as PCNN. f is a nonlinear generator and n is the number of iterations. Other parameters have the same meaning as the basic Rybak model. In Qi et al.'s work, input receptive field F_{ijkl}^S is refined as follows:

$$F^S(x, y) = a_1 \sqrt{x^2 + y^2} \exp\left(-\frac{x^2 + y^2}{c_1}\right) - a_2 \sqrt{x^2 + y^2} \exp\left(-\frac{x^2 + y^2}{c_2}\right) \tag{48}$$

where a_1 and a_2 are the distribution sensitivity. c_1 and c_2 are the spatial distribution of the excitement intensity and inhibitory intensity of the receptive field respectively. The receptive field $F^S(x, y)$ has been proved to have multi-scale adjustable characteristic, which is sensitive to the detailed texture information of the image.

2.4.2 Heterogeneous Rybak neural network (HRYNN)

As we know, Rybak neural networks are analogous to the components of actual neurons. However, lots of researches [120, 121] have shown that the visual cortex system of mammals is heterogeneous. Motivated by this fact, Qi et al. [119] proposed a heterogeneous Rybak neural network (HRYNN) as shown in Fig. 10.

In Qi et al.'s work, HRYNN is described as follows:

$$X_{Sij}[n] = V_S \sum_{kl} F_{ijkl}^S S_{ij} \tag{49}$$

$$X_m^I[n] = \alpha \sum_{kl} F_{m,kl}^I Z_m[n-1] + S_{ij} \quad (m = 1, 2, 3) \tag{50}$$

$$E_{m,ij}[n] = e^{-\alpha_e} E_{m,ij}[n-1] + V_E Z_{m,ij}[n-1] \quad (m = 1, 2, 3) \tag{51}$$

$$P_1[n] = X_{ij}^S[n] - \frac{1}{\phi} X_1^I[n] + h \tag{52}$$

$$P_2[n] = X_{ij}^S[n] - \frac{1}{\phi} X_2^I[n] + h + L_{12} \tag{53}$$

$$P_3[n] = X_{ij}^S[n] - \frac{1}{\phi} X_3^I[n] + h + L_{23} \tag{54}$$

$$Z_{m,ij}[n] = \begin{cases} k_m, & \text{if } P_{m,ij}[n] > E_{m,ij}[n-1] \\ 0, & \text{else} \end{cases} \quad (m = 1, 2, 3) \tag{55}$$

$$Z = Z_1 + Z_2 + Z_3 \tag{56}$$

where m denotes m -th RYNN subcell. It can be seen that the output of HRYNN represents the multi-region processing of the image. Furthermore, the setting of parameters of HRYNN is described as follows [119]:

As shown in Fig. 10, L_{12} and L_{23} are weak connections among different sub-cells of HRYNN. In the work of Qi et al., the linking item L_{12} of RYNN1 and RYNN2 is set as follows:

$$L_{12} = \beta_{12} \sum W_{ijkl}(1, 2) Z_{1,kl}(n-1) \tag{57}$$

where β_{12} is the local linking item between RYNN1 and RYNN2: $\beta_{12} = \max(I)/\text{mean}(I)$. Similarly, L_{23} is set to:

$$L_{23} = \beta_{23} \sum W_{ijkl}(2, 3) Z_{2,kl}(n-1) \tag{58}$$

Where β_{23} is the local linking item between RYNN2 and RYNN3: $\beta_{23} = \text{mean}(I)/\max(I)$. I is the input image. In addition, $W(1, 2)$ and $W(2, 3)$ are weak linking synaptic weight and set as follows:

$$W_{ijkl}(1, 2) = W_{ijkl}(2, 3) = \frac{1}{(i-k)^2 + (j-l)^2} \tag{59}$$

The parameter α_e is the exponential decay coefficient of dynamic threshold E . F_m^I ($m = 1, 2, 3$) represent the orientation connection of $30^\circ, 90^\circ$ and 120° , respectively, which detect input stimuli in different directions. In addition, $k_1 \neq k_2 \neq k_3$ and $k_1, k_2, k_3 \in (0, 1)$.

Algorithm 3: Enhancement by HRYNN model.

- Input** : The gray image S_{ij}
Output: The enhancement result Y for each image.
- 1: Initial the parameters of the HRYNN model.
 Calculate F_{ijkl}^S according to (47),
 calculate L_{12} and L_{23} according to (56), (57), respectively.
 - 2: $n = 0$.
 - 3: **while** $n < N$ **do**
 - 4: $n = n + 1$
 - 5: update E_{ij} by (31).
 - 6: reapt.
 - 7: update P_m, Z_m according to (50)-(53), (54)
 - 8: calculate Z by (54), calculate EnI_{ij} by (32).
 - 9: $Y_0 = EnI_{ij} \cdot Z$
 - 10: **if** any($Z(\cdot)$) = 0
 - 11: break;
 - 12: **end while**
 - 13: **end while**
 - 14: Normalize linearly to obtain Y .
-

Finally, the main steps of HRYNN algorithm for image enhancement is shown in Algorithm 3. In addition, some examples of image enhancement based on PCNN, SCM, FLM, LSCN and HRYNN are presented in Fig. 11. We can see that the edge detail enhancement effect of HRYNN is obviously better than other methods. This is because its receptive field has been specially set. In particular, FLM causes a bad effect on image details preservation.

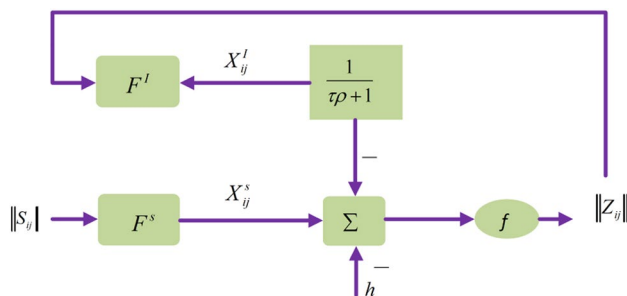


Fig. 9 (i, j) -th Rybak neuron-like element of the SNS

2.5 Deep Learning Based Image Enhancement

In recent years, with the development of artificial intelligence (AI) technology, deep learning has been widely applied in the field of image processing [122–124]. The supervised deep learning based image enhancement methods have great limitations. For each training set, a model must be retrained, which is not universal. Therefore, some researchers began to work on weakly supervised or unsupervised deep learning methods for low-light image enhancement. In this section, for low-light image enhancement, semi-supervised or unsupervised deep learning methods are surveyed in detail.

2.5.1 GAN-Like Based Image Enhancement

Generative Adversarial Network (GAN) is a deep learning model and one of the most promising methods for unsupervised learning on complex distributions in recent years. Here we conduct a review for GAN-like based image enhancement.

In 2018, Ignatov et al. [125] proposed a Weakly Supervised Photo Enhancer (WESPE) for low-light image enhancement based on Generative Adversarial Network (GAN). In their work, the input data and output data are low-quality images and high-quality images, respectively. However, they do not need to correspond in content. In addition, a transitive CNN-GAN structure is used to learn the mapping relationship between them. Two components: generator G and inverse generator F are the main modules of the entire WEPSE model. x is the input low-end image and the improved image $y = G(x)$ is generated through G . A reconstructed low-end image is obtained through the inverse-generator F . The content loss $\mathcal{L}_{content}$ based on vgg-19 is defined between the original and reconstructed images x , and $\tilde{x} = F(y) = F \circ G(x)$, respectively. In addition, adversarial color loss \mathcal{L}_{color} is defined to compare enhanced

image and high-quality image. D_c and D_t are two adversarial discriminators, which involve the comparison of color and texture between the enhanced image and the high-quality image respectively. According to [125], $\mathcal{L}_{content}$, \mathcal{L}_{color} and $\mathcal{L}_{texture}$ are defined as follows:

$$\mathcal{L}_{content} = \frac{1}{C_j H_j W_j} \left\| \Psi_j(x) - \Psi_j(\tilde{x}) \right\| \tag{60}$$

$$\mathcal{L}_{color} = - \sum_i \log D_c(G(x)_b) \tag{61}$$

$$\mathcal{L}_{texture} = - \sum_i \log D_t(G(x)_g) \tag{62}$$

where Ψ_j denotes the feature map from j -th VGG-19 convolutional layer. C_j , H_j and W_j are the number, height and width of the feature maps, respectively. A total variation (TV) loss is also defined to impose spatial smoothness of the generated images [126] and its expression is described as follows,

$$\mathcal{L}_{tv} = \frac{1}{CHW} \left\| \nabla_x G(x) + \nabla_y G(x) \right\| \tag{63}$$

where C , H and W are dimensions of the generated image $G(x)$. Finally, WESPE loss is defined as the linear weighting of the four component losses.

Chen et al. [127] proposed a unpaired learning method based on GAN. In their method, two-way generative adversarial networks (GANs) with improvement are employed as the baseline. In addition, U-Net with global feature is use as the generator of GAN model and an adaptive weighting scheme is proposed to improve Wasserstein GAN (WGAN), training converges faster and better. Finally, to make generators better adapt to their own input distributions, they proposed to use the individual batch normalization layers in the two-way GANs model. Although Chen et al. [127]’s method has the advantages of stable performance, fast speed, and outstanding effects for image enhancement, it also has many potential shortcomings. If the input image is very dark and contains a lot of noise, the noise may be amplified in the enhanced image. For unpaired network training, the choice of domain of the training set has great limitations. For the exterior scenes in the paper, lots of images of blue sky and sea were selected. If we do not choose to use, the enhanced images will appear blue for unknown reasons.

Jiang et al. [128] proposed an EnlightenGAN method for deep light enhancement. EnlightenGAN uses attention-guided U-Net as a generator, and uses dual discriminators (global and local) to balance global and local low-light enhancement. In addition, due to the lack of ground truth, the paper proposes an adaptive perceptual loss including global and local to constrain the feature difference between

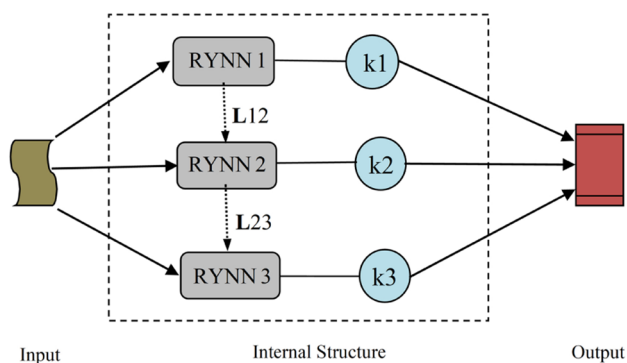


Fig. 10 The architecture of HRYNN

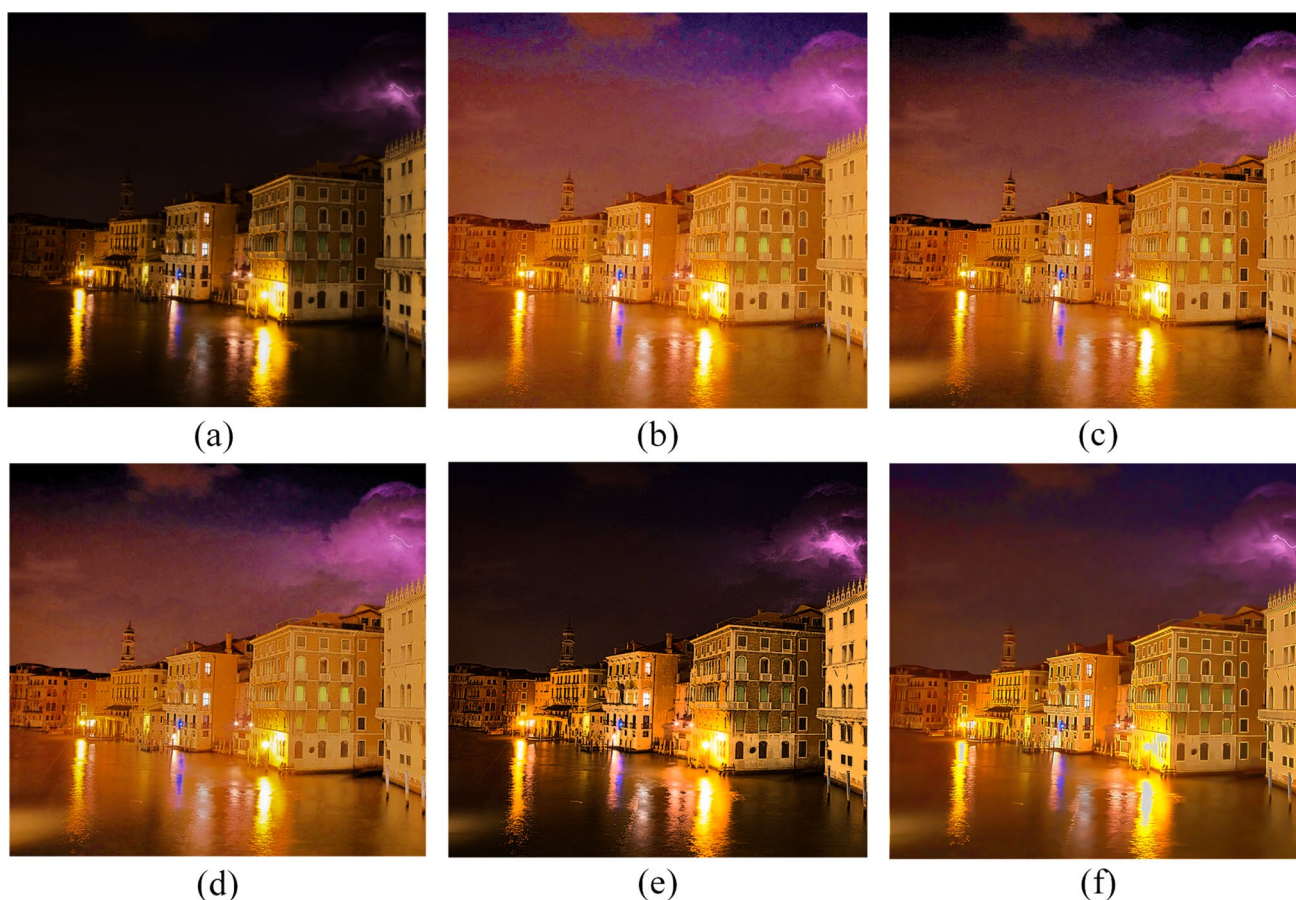


Fig. 11 Some examples of image enhancement. **a** is the original image. **b–f** are the enhancement results by PCNN, SCM, FLM, LSCN and HRYNN, respectively

the low-light input image and its enhanced image, and together with the adversarial loss used to train EnlightenGAN to form the final loss. Furthermore, Xiong et al. [129] proposed a full unsupervised method for real-world low-light image enhancement based on the two-stage GAN frame.

2.5.2 Other Weakly Supervised Methods

In addition to the unsupervised image enhancement methods mentioned above, In this section, we will mainly introduce several weakly-supervised enhancement methods for low-light enhancement.

Yang et al. [130] proposed a semi-supervised method for low-light image enhancement. In their work, firstly, supervised learning is employed to obtain the characteristics of reconstructed high-quality images. Then these characteristics are used as the intermediate variables to assist unsupervised training. In 2019, Shin et al. [131] proposed a novel deep learning method for power-constrained contrast enhancement (PCCE). They constrain the power

consumption by reducing the brightness and the perceived visual quality is preserved by CNN enhancement.

In summary, most of unsupervised methods usually use some indirect training methods, such as supervised methods to obtain high-quality image features, and then assist unsupervised training based on these features. In the future, generative adversarial network (GAN) may be widely used in unsupervised image enhancement. This can be better optimized by combining some essential attributes of the image such as retinex and low rank, etc.

3 Supervised Methods

For image enhancement, supervised methods of image enhancement are usually trained based on paired dataset and achieve a good effect. In recent years, a large number of supervised enhancement methods have emerged, such as fully convolutional networks (FCN) [132, 133], reinforcement learning [134–136] and U-Net [137], etc.

In this section, we mainly focus on a detailed overview of supervised deep learning methods for low-light image enhancement.

3.1 Multi-Level Features Fusion Methods

As we know, low-quality images often exhibit low contrast, artifacts, and noise due to some extreme conditions. Based on the excellent feature extraction capabilities of deep convolutional neural networks, many methods for low-brightness image enhancement based on multi-level features fusion have emerged. Here we will give a brief overview of low-brightness image enhancement methods based on multi-level features fusion.

Gharbi et al. [138] proposed a deep bilateral learning based enhancement method with the idea of multi-level features fusion. In their work, Three main strategies are proposed to fuse features of different levels to effectively enhance low-brightness images. Firstly, most of the predictions are performed under a low-resolution bilateral grid; Secondly, the entire neural network structure learns an affine transformation; Thirdly, most of the operations are performed at low resolution, but the loss function is finally established on the original resolution, in other words, the original resolution image can be optimized based on low resolution operations. In 2018, Lv et al. [139] proposed a multi-branch low-light enhancement network (MBLLEN). They employed the multiple subnets to extract different level features and finally conducted a multi-branch fusion, achieving an outstanding enhancement effect. In addition, Cai et al. [140] proposed a single image enhancement method based on multi-exposure image fusion. However, if the input image is severely overexposed and the area is large, CNN can use little neighborhood information. Thus, it is difficult to synthesize these missing details. Lv et al. [141] designed a multi-branch fusion network guided by the attention map, which can perform image denoising and enhancement simultaneously. Zamir et al. [142] proposed a MIRNet model, which executes parallel resolution convolution streams to extract multi-scale features and exchanges information across resolution streams. Finally, attention-based multi-scale feature aggregation is proposed for image enhancement. In addition, similar work was also conducted by Zhu et al. [143].

In conclusion, the image enhancement methods based on multi-level features fusion fully take into account the importance of different scale features of the image for image restoration. For example, high-frequency features determine the preservation of the details of the image, while low-frequency features play a key role in preserving the naturalness of the image. Therefore, multi-level features fusion is an effective method in the enhancement of low-light images.

3.2 Retinex-Based Deep Learning Methods

In Sect. 2.2, we have mentioned that Retinex theory plays an important role in image enhancement applications. The Retinex model is the basic theory of human visual imaging, and increasing researchers are beginning to work on its combination with deep learning for image enhancement [144]. Shen et al. [145] thought that the traditional multi-scale Retinex (MSR) [63] algorithm can be regarded as a feedforward convolutional neural network with different Gaussian convolution kernels. Therefore, they proposed MSR-net to directly learn end-to-end enhanced mapping. MSR-net consists of three modules: multi-scale logarithmic transformation, convolution difference and color restoration. In addition, the training data uses high-quality images adjusted by PS and corresponding synthetic low-light images (randomly reduced brightness, contrast, and gamma correction). The loss function is the F-norm square of the error matrix with regular terms.

In 2018, Li et al. [14] proposed a LightenNet for low light image enhancement, which only learns to predict the mapping relations between weakly illuminated image and the corresponding illumination map of Retinex model. Although their method has the advantages of easy training and short time-consuming, it shows great limitations in enhancing low-quality images with noise, because the training set only uses high-quality images. Wei et al. [146] proposed a deep Retinex-Net, including a Decom-Net for decomposition and an Enhance-Net for illumination adjustment. In their work, a low-light dataset (LOL) containing low/normal-light image pairs was collected. In addition, Shi et al. [147] combined the Retinex model and the Generative Adversarial Network (GAN) to enhance low-light images. Specifically, the decomposition part divides the image into a illumination image and reflected image, and the enhancement part is used to generate a high-quality image.

Noise is an interference factor that must be considered in low-brightness image enhancement. Inspired by this fact, many scholars have proposed noise suppression low-brightness image enhancement methods based on the Retinex model. Wang et al. [148] considered that contrast enhancement and noise removal are coupled problems, proposing a novel progressive Retinex framework, which makes illumination and noise of low-light image be perceived in a mutually reinforced manner, achieving noise reduction low-light enhancement results. However, Wang et al.'s method has a limitation. Because it only considers the statistical distribution of the pixel space and ignores the structural properties. In addition, the receptive field of the 1×1 convolution kernel is relatively small, resulting in no neighborhood information during convolution. Liang et al. [149] proposed a deep learning method for low-light images enhancement. Their methods particularly

focused on measurement noise. Specially, a neural network is trained to generate a set of pixel-wise operators, which can simultaneously predict the noise and the illumination layer in the bilateral space.

3.3 HDR Image Reconstruction

In general, camera sensors can only capture images with a limited range of lightness and a high dynamic range (HDR) image, in practice, is necessary. Therefore, many researches focus on predicting its saturated pixel value for the reconstruction of HDR images. In 2017, Gaberiel et al. [150] proposed a deep convolutional neural network (CNN) to predict the HDR values of images. In Gaberiel et al.'s work, a large dataset of HDR images is collected to train CNN. In particular, these data are augmented by simulating the sensor saturation of a series of cameras. Yang et al. [151] proposed a Deep Reciprocating HDR Transformation (DRHT). In their work, the missing details in the HDR domain firstly are reconstructed. Then, the LDR image with the recovered details is generated by performing tone mapping on the predicted HDR data. Therefore, their method framework mainly consists of two parts: HDR reconstruction and tone mapping.

4 Quality Evaluation

The quality evaluation of image processing algorithms is essential, which involve people's evaluation of the visual perception of an image. In image enhancement, an appropriate quality evaluation strategy plays a key role in the evaluation of algorithm performance. In this section, we will give a detailed overview of the qualitative and quantitative evaluation for image enhancement.

4.1 Qualitative Assessment

It is difficult to quantify an improved perception of an image. Therefore, it is not easy to assess image enhancement [152]. But in general, it is desirable to conduct both quantitative and qualitative assessments. Qualitative evaluation refers to intuitively expressing people's visual perception of enhanced images, and is the most accurate evaluation method. However, subjective evaluation methods lack stability. It is often affected by various factors such as experimental conditions, the observer's knowledge background, mood, motivation, and fatigue.

In this section, we will conduct a comprehensive and systematic experimental evaluation of image enhancement methods. The methods involved mainly include the

following representative methods: histogram equalization, Retinex-based enhancement and visual cortical neural network. The experimental evaluation of deep learning-based methods will be presented in Sect. 4.2 in the form of quantitative comparison. In addition, due to the limitation of the paper length, here we only perform experimental evaluations of representative methods for each image enhancement type. Detailed experimental settings, which can be found in the original papers, are missed. We compare various image enhancement methods on randomly selected low-brightness test images. The evaluated approaches include Histogram Equalization (HE), Adaptive Histogram Equalization (AHE), partially overlapped sub-block histogram equalization (POSHE) [27], WTHE [29] and AGCWD [33], SRIE [85], LIME [86], SCM [113] and HRYNN [119]. An example of visual comparison based on different image enhancement is shown in Fig. 12.

We can see from Fig. 12 that AHE and SRIE have better performance in detail enhancement, but HE, WTHE and AGCWD cause serious loss of details, such as the edge of a teacup, etc. Obviously, HRYNN has the best detail preservation performance, as marked by red boxes. In addition, SCM seems to have an artifact effect and LIME cause a illumination over-enhancement, which adds to the noise amplification. AHE and AGCWD cannot achieve satisfactory results in overall contrast, such as a low contrast of flowers etc.

In summary, the histogram equalization enhancement is based on the fact that it represents the statistical relationship between each gray level in the image and the number of pixels that appear in the gray level, and reflects the frequency of each gray level in the image. In practice, histogram equalization still has certain limitations. It is usually necessary to consider combining a variety of simple and effective algorithms, or fusing other enhancement techniques to achieve the final enhancement effect, so comprehensive image enhancement technology will also become a key of follow-up study. Retinex model assumes that image lightness depends on illumination and reflectance. Illumination is the radiant flux received by the scene, and reflectivity is the effectiveness of reflected radiant energy. In addition, the perception of color has a strong correlation with reflectivity [47]. Finally, the visual cortex neural network is based on the principle of human visual characteristics and has great potential in the field of image enhancement.

4.2 Quantitative Assessment

The quantitative assessment method adopts the error between the processed image and the original image to measure the quality of the processed image. Specifically, the greater the grayscale difference from the standard image, the more severe the image quality degradation. Representative

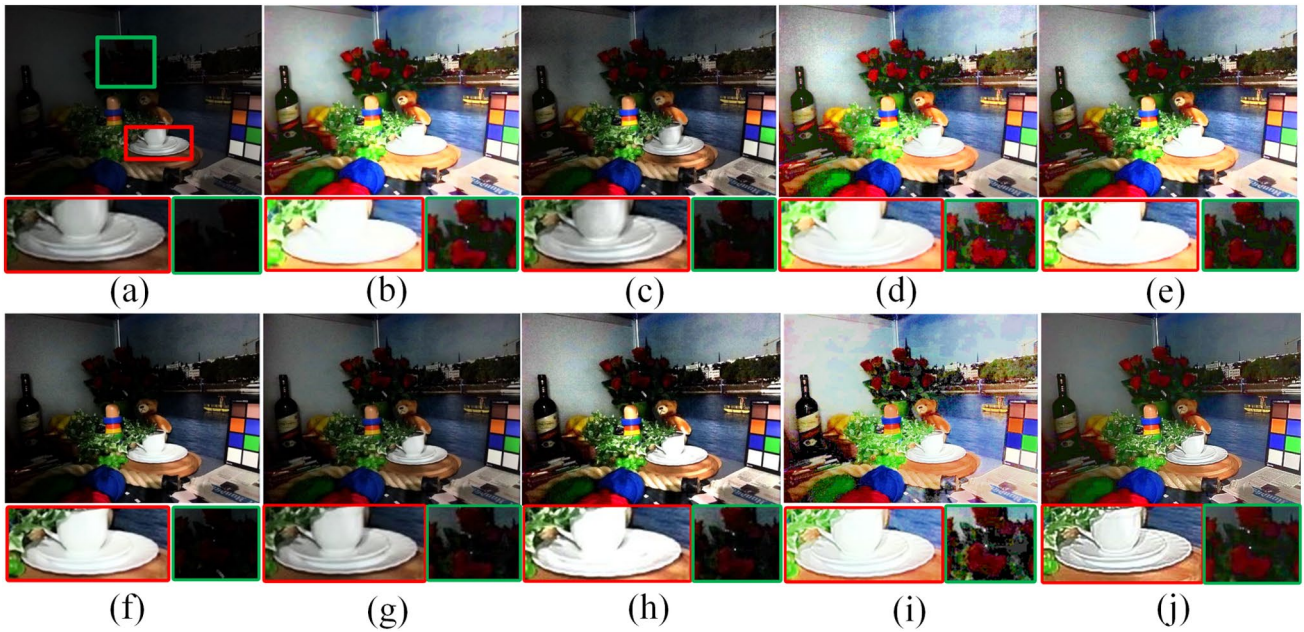


Fig. 12 Visual comparison of different image enhancement based on HE-like methods. **a** is the original image. Enhanced images generated by: **b** HE; **c** AHE; **d** POSHE; **e** WTHe; **f** AGCWD; **g** SRIE; **h** LIME; **i** SCM; **j** HRYNN

methods include mean square error (MSE) and peak signal-to-noise ratio (PSNR). In this section, we first elaborates the quantitative evaluation index for the enhanced image. Then, the experimental evaluation of deep learning based image enhancement methods is conducted.

4.2.1 Quantitative Index

The quantitative measures we introduced here mainly involve absolute mean brightness error (AMBE) [153], structural similarity of image (SSIM) [154] and peak signal to noise ratio (PSNR), discrete entropy (DE) [155], enhancement (EME) [156] and lightness order error (LOE) [157].

First of all, according to literature [153], the absolute mean brightness error(AMBE) is defined as:

$$AMBE(X, Y) = |MB(X) - MB(Y)| \tag{64}$$

where $MB(X)$ and $MB(Y)$ represent the mean brightness of original image X and enhanced image Y , respectively. In addition, the lower the value AMBE the better is preservation of the original image luminance.

Then, structural similarity of image (SSIM) is used to evaluate the similarity of two images and is proposed by Wang et al. [154]. It is usually described as follows:

$$SSIM(x, y) = l(x, y)^\alpha \cdot c(x, y)^\beta \cdot s(x, y)^\gamma \tag{65}$$

where $l(x, y)$, $c(x, y)$ and $s(x, y)$ denote brightness comparison part, contrast comparison part and structure comparison

part of image, respectively. In addition, α , β and γ are parameters employed to adjust the relative importance of the three components. Generally, they are set to 1 and related parameters are defined as follows:

$$l(x, y) = \frac{2\mu_x\mu_y + c_1}{\mu_x^2 + \mu_y^2 + c_1}, \quad c(x, y) = \frac{2\sigma_x\sigma_y + c_2}{\sigma_x^2 + \sigma_y^2 + c_2}, \tag{66}$$

$$s(x, y) = \frac{\sigma_{xy} + c_3}{\sigma_x\sigma_y + c_3}$$

where x and y represent the original image and enhanced image, respectively. μ_x , μ_y , σ_x^2 , σ_y^2 and σ_{xy} denote the mean, variance and covariance of image x and y respectively. In addition, c_1 , c_2 and c_3 are small constants, just to avoid the fact that the denominator is zero. The larger the value of SSIM, it means that the structure of the enhanced image is similar to the original image and the quality of the enhanced image is better. Note $SSIM(x,y) \in (0, 1)$.

Peak signal-to-noise ratio (PSNR) is usually employed to roughly estimate the human perception of reconstruction quality [158]. PSNR is calculated as follows:

$$PSNR = 10\log_{10}\left(\frac{peakval^2}{MSE}\right) \tag{67}$$

where $peakval$ is the maximal variation of the input image data. If it has an 8-bit unsigned integer data type, $peakval$ is 255. In addition, MSE represent the mean square error of input image I and enhanced image I_0 with resolution $m \times n$:

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|I(i, j) - I_0(i, j)\|^2 \tag{68}$$

The discrete entropy (DE) of an image X is

$$DE(X) = - \sum_{i=0}^{255} p(x_i) \log p(x_i) \tag{69}$$

where $p(x_i)$ is the probability of pixel intensity x_i , which is estimated from the normalized histogram. A higher value of DE indicates the image has richer details.

If we divide the image X into $k_1 k_2$ non-overlapping sub-blocks X_{ij} with size $\omega_1 \times \omega_2$. Then EME can be computed as

$$EME(X) = \frac{1}{k_1 k_2} \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} 20 \ln \frac{\max(X_{ij})}{\min(X_{ij})} \tag{70}$$

where $\max(X_{ij})$ and $\min(X_{ij})$ represent the maximum and minimum grey levels in block X_{ij} , respectively. A different sub-block size (ie., $\omega_1 \times \omega_2$) will result in different EME value. High contrast sub-blocks give a high EME value and the homogeneous sub-blocks can result in an EME value close to 0. Note that EME is highly sensitive to noise. Specifically, if the algorithm generates an output image, which introduces noise over homogeneous regions of the image, then although the output image looks unnatural, its corresponding EME value is very high [152].

Finally, the lightness order error (LOE) is of great significance for describing the naturalness of the image, which is proposed to evaluate the quality of the image. If I represents the original image and I_e represents the enhanced image, the lightness of the image is represented by the highest brightness among the three channels:

$$L(x, y) = \max_{c \in \{r, g, b\}} I^c(x, y) \tag{71}$$

For each pixel, before and after image enhancement, the lightness order error related to that pixel is defined as follows:

$$RD(x, y) = \sum_{i=1}^m \sum_{j=1}^n (U(L(x, y), L(i, j)) \oplus U(L_e(x, y), L_e(i, j))) \tag{72}$$

where m and n represent the height and width of the image respectively. $U(x, y)$ is step functions, and \oplus represent the exclusive OR operation. Thus, LOE is defined as follows:

$$LOE = \frac{1}{m * n} \sum_{i=1}^m \sum_{j=1}^m RD(i, j). \tag{73}$$

Due to the large amount of calculation of the LOE algorithm, in practical applications, the image is generally down-sampled with a sampling rate of 50/min(m, n) to achieve the purpose of reducing the amount of calculation. It can be seen from equation (73) that the smaller the LOE value, the higher the naturalness of the image and the higher the quality.

Due to the limitation of the paper length, we only elaborated the detailed algorithm of the above image quality evaluation indicators. There are some other evaluation indicators, such as average brightness (AB) [159], visual information fidelity (VIF) [160], tone mapped image quality index (TMQI) [161] and learned perceptual image patch similarity metric (LPIPS) [162]. Detailed algorithm content, which can be found in the original papers, are missed.

4.2.2 Experimental Evaluation

In this section, we compare various image enhancement methods on synthetic dataset (SYD) [141]. SYD is a large scale low-light simulation dataset with 22,656 scenes, which has diverse scenes and lighting conditions. In quantitative comparison, some indicators such as PSNR, SSIM [154], average brightness (AB) [159], visual information fidelity (VIF) [160], lightness order error (LOE) [157], tone mapped image quality index (TMQI) [161] and learned perceptual image patch similarity metric (LPIPS) [162] are employed to evaluate the representative low-lightness image enhancement methods we investigated. All the results are from the original paper, and they are recorded in Table 1. Note that implementation is done with Keras and Tensorflow.

The evaluated image enhancement methods cover BIMEF [164], LIME [86], MSRCR [10], MF [163], SRIE [85], Dong [165], NPE [157], DHECI [166], BPDHE [40], HE, Ying [168], WAHE [31], JED [167], Robust [87], LLNet [171], DeepUPE [169], GLADNet [170], MBLLEN [139] and Lv [141]. Note that in the Table 1 below, bold and underlined indicate the best and second place results, respectively. We can see from Table 1 that Lv et al. [141] achieved the best overall performance. In their work, a multi-branch network with a ue-attention map and noise map was proposed to enhancement low light images in a region adaptive manner. In addition, supervised methods such as LLNet [171], DeepUPE [169], GLADNet [170], MBLLEN [139] and Lv [141] basically achieved a better performance. For example, their metric SSIM reach above 0.6. It is worth noting that JED [167] and Robust [87] require a lot of computing resources, which can cause insufficient memory when processing large images.

Table 1 Quantitative comparison of synthetic low-light image data sets (without additional noise) enhanced. Bold and underlined indicate the best and second place results, respectively. "↑" means higher is better, "↓" means lower is better, "↓" means lower absolute value is better

	PSNR ↑	SSIM ↑	LPIPS ↓	VIF ↑	LOE ↓	TMQI ↑	AB ↓
Input	11.99	0.45	0.26	0.33	677.85	0.80	− 59.22
MF [163]	15.89	0.68	0.18	0.44	766.00	0.83	− 36.88
MSRCR [10]	14.87	0.72	0.15	0.52	1249.24	0.82	35.07
SRIE [85]	13.83	0.56	0.21	0.37	787.42	0.82	− 47.86
BIMEF [164]	18.28	0.76	0.11	0.49	550.20	0.85	− 28.06
NPE [157]	14.93	0.66	0.18	0.42	875.15	0.83	− 41.35
LIME [86]	15.80	0.68	0.20	0.48	1121.17	0.80	<u>− 2.46</u>
Dong [165]	15.37	0.65	0.22	0.35	1228.49	0.81	− 33.80
DHECI [166]	18.13	0.76	0.17	0.39	547.12	0.87	− 17.37
BPDHE [40]	13.62	0.60	0.24	0.34	609.89	0.82	−47.82
MBLLEN [139]	24.21	<u>0.90</u>	0.08	<u>0.63</u>	536.75	<u>0.91</u>	− 3.66
Lv [141]	25.24	0.94	0.08	0.67	495.48	0.93	2.04
WAHE [31]	15.46	0.65	0.18	0.44	564.83	0.84	− 39.38
JED [167]	16.11	0.65	0.21	0.41	1212.66	0.82	− 25.95
Robust [87]	16.83	0.69	0.20	0.47	1052.22	0.82	− 22.09
HE	17.88	0.76	0.18	0.47	596.67	0.88	19.24
Ying [168]	19.21	0.80	0.11	0.56	778.67	0.83	− 9.28
DeepUPE [169]	16.55	0.64	0.17	0.55	516.47	0.84	− 30.48
GLADNet [170]	<u>24.57</u>	<u>0.90</u>	<u>0.09</u>	0.62	<u>513.18</u>	<u>0.91</u>	5.52
LLNet [171]	20.11	0.80	0.39	0.40	1088.43	0.87	4.30

From these results, we can draw the conclusion that deep learning-based methods are in the leading position. The performance can be improved by the following strategies: designing novel optimization loss, modifying channel attention mechanism, Retinex combination, adapting scale-aware enhancement, and learning multi-level fused CNN features.

5 Conclusion

Image enhancement plays important roles in image processing. In this paper, we give a comprehensive review to analyze image enhancement methods from a supervised and unsupervised perspective. There are three main aspects. We first survey the unsupervised image enhancement methods, including histogram specification, Retinex model, deep learning and visual cortex neural network. Then we introduce supervised image enhancement methods involving deep convolutional neural network. In addition, we also provide main quality evaluation methods for image enhancement. In the future, weakly supervised or unsupervised strategies will probably generate new image enhancement frameworks and bring a rapid progress than state-of-the-art algorithms.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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