Design of robust sliding mode control with disturbance observer for multi-axis coordinated traveling system

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Abstract

This paper deals with the robust control strategy for multi-axis coordinated motion system. Firstly, the adjacent cross-coupling error control method was introduced to reduce the synchronization error. Then, the sliding mode control (SMC) law based on the mathematical model of the plant was adopted for restraining parameter perturbation. Furthermore, addressing the unknown torque disturbance, a disturbance observer was proposed. The switching gain of the robust control algorithm can be set as a smaller value so that the chattering on the sliding mode plane can be decreased. The simulation results have proved the effectiveness of the proposed control algorithm.

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1. Introduction

Multi-axis systems are widely used in industrial fields such as the multi-motor system in numerical control machines, robotics, and the multi-hydraulic-motor system in the driving system of construction machineries. In mostly application the coordinated motion means generally the synchronization motion. So in a multi-axis coordinated motion system, the elimination of the synchronization errors is an important problem.

The synchronization errors among different axes are caused by many reasons, such as the different initial values, the different parameters, and the different disturbance in each plant. The cross-coupling strategy is introduced for synchronization control by Koren [1] and Perez-Pinal [2], but it is too complicated to use this basic cross-coupling strategy to the systems with more than 2 motors. Sun and Mills proposed the adjacent cross-coupling strategy in robotic synchronization control, which has simplified the structure of the cross-coupling strategy [3,4]. Cao et al. [5], and Zhang et al. [6] respectively studied proportional-synthesized type of and proportional-integral-synthesized type of adjacent coupling error sliding mode control law to be used in the synchronized control of multi-induction-motor systems. Li et al. [7] proposed an adjacent coupling error total sliding mode control used to multi-induction-motors, which can guarantee no sliding mode approaching stage so as to enhance the robustness of the multi-axis coordinated control system. Kempf and Kobayashi studied the design of the disturbance observer in the high-speed-drive-positioning table, and put forward a solution with the feed-forward compensation, disturbance observer, and Q-filter [8]. Kim and Chung [9] proposed a kind of advanced disturbance observer with Q-filter and applied this control law to the mechanical positioning system.

Li investigated the design of the disturbance-observer of the electro-hydraulic simulator and proposed a hybrid control strategy [10]. Pai and Yau [11] focused the parameters uncertainty and disturbance in the chaotic control system with SMC law and proposed a kind of the compensation method by using an augment state variable. Chung and Chen addressed the linear system subject to periodic disturbance and designed an adaptive disturbance/state observer [12]. Liang et al. [13] studied the friction compensation issues in the servo systems. Horng and Lee [14] designed a sliding observer for friction compensation in a linear-motor-driven motion system, and this method can acquire a higher precision than that of the...
disturbance observer (DOB) compensation or without friction compensation. Chen and Li [15] designed a kind of adaptive iteration learning control law based on an observer for a robotic system tracking problem. Liu [16] designed a state observer to estimate the disturbance in a servo system. Shi et al. [17] proposed a stable adaptive fuzzy control law for a nonlinear system, and this control method can increase the ability of the system to reject the disturbance.

For the traveling system with multi-axis driving, obviously, the cross-coupling and the external torque disturbance simultaneously exist. So, in order to improve the coordinated performance of the system, we need to consider the influence of the cross-coupling and the torque disturbance. Addressing this problem, this paper proposed a robust control algorithm which can simultaneously restrain the cross-coupling error and the external torque disturbance.

The rest of the paper is arranged as follows. Firstly, in the next section, a mathematical model to describe the dynamic characteristics of the multi-axis rotational speed system was established and the forward adjacent coupling error strategy was proposed. In Section 3, an SMC law with the disturbance observer compensation was designed for restraining uncertainties of the plant, so that we can select the smaller value of the switching gain of the SMC law and decrease the chattering on the sliding mode plane. Finally, the main conclusions of the paper are summarized.

2. Problem statement

2.1. The objective of multi-axis coordinated motion system

Consider a multi-axis coordinated motion system in [18]. Note that all of the wheels make the steering center while steering, the synchronized relationships among of the rotational speeds of the motors to combine the traveling system are as follows

\[
\frac{\omega_1 x_1}{R_1} = \frac{\omega_2 x_2}{R_2} = \cdots = \frac{\omega_n x_n}{R_n}
\]

where, \(i_r\), \(R_i\) (\(i = 1, 2, \ldots, n\)), and \(\omega_i\) (\(i = 1, 2, \ldots, n\)) are the transmission ratio of the rotational speeds from the driving motor to the vehicle wheel, the rolling radius of the wheel, the steering radius of the \(i\)-th wheel rotating the steering center, and the rotational angle speed of the \(i\)-th wheel, respectively. Supposing that the traveling velocity of the transporter at the geometric center is \(v_{d0}\), and defining \(\omega_d = \frac{v_{d0}}{R_0}\), \(x_d = \frac{\omega d}{R_0}\), \(v_{d0} = \frac{v_{d0}}{R_0}\), and \(x_d = \frac{x_d}{R_0}\), then from the pure-rolling conditions of the vehicle steering kinematics we can derive the following kinematical relationships as

\[
x_{d1} = x_{d2} = \cdots = x_{dn} = x_d
\]

where \(R_0\) and \(\omega_d\) represent the steering radius of the transporter geometric center, and the desired angle speed of the \(i\)-th transporter wheel, respectively.

Using Newton’s second motion law for each motor yields the dynamic model of each axis of the multi-axis system can be described as

\[
J_i \dot{\omega}_i + B_i \omega_i + T_{Li} = T_{ei}.
\]

In (3), \(J_i\) is the rotary inertia of the \(i\)-th axis, \(B_i\) is the damping coefficient of the \(i\)-th axis, \(T_{Li}\) is the unknown torque disturbance, and \(T_{ei}\) is the driving torque.

Let \(x_i = \frac{\omega_i}{R_i}\), \(b_i = \frac{B_i}{J_iR_i}\), \(d_i = \frac{T_{ei}}{J_iR_i}\), \(u_i = \frac{T_{Li}}{J_iR_i}\), and to substitute them into (3) yields

\[
\dot{x}_i + b_i x_i + d_i = u_i.
\]

Defining the tracking error of each axis of the multi-hydraulic motors system as

\[
e_i = x_d - x_i.
\]

Then we can give the goal of the control system described by Eq. (4) is to realize the multi-axis coordinated motion control and the tracking of \(x_i\) to \(x_{di}\), and its expressions are as follows

\[
x_1 = x_2 = \cdots = x_n = x_d.
\]

Note that \(R_1 = R_2 = \cdots = R_n = \infty\) for the synchronization line traveling problem, i.e. all of the wheels have the same wheel velocity, so we may let \(x_i = \omega_i\), \(b_i = \frac{b_i}{R_i}\), \(d_i = \frac{d_i}{R_i}\), \(u_i = \frac{u_i}{R_i}\) in Eq. (4).

2.2. The forward adjacent error strategy

In order to guarantee the coordinated control of the multi-hydraulic-motor traveling system, we must consider the influence of the different characteristics amount on every hydraulic motor driving sub-system. For this goal, we put forward...
the forward adjacent coupling error strategy on the basis of the basic control law of the tracking error, which introduced the tracking errors of the adjacent axis into the controller of the local axis. Defining the forward adjacent coupling error (i.e., forward synchronization error) $\varepsilon_i = e_i - e_{i+1} = x_{i+1} - x_i$, and the adjacent coupling error control law can be expressed as

$$u_i = A(\varepsilon_i, e_i)$$

where, $A$ denotes some of operator for $\varepsilon_i$ and $e_i$. The structure of the multi-axis coordinated motion system based on the adjacent error and the tracking error is shown in Fig. 1. As shown in Fig. 1, the coordinated control law considered the influence of the adjacent error and tracking error comprehensively.

3. An SMC law based on disturbance observer

Seen from (4), the load disturbance existed in the multi-motor traveling system. Therefore, in order to increase the robustness of the control strategy, here we provided a solution of the sliding mode control (SMC) with disturbance observer. This SMC law can restrain the parameter perturbation and the load disturbance, and improve the robust performance of the system. The introduced disturbance observer can allow the switching gain of the SMC law be set as a lower value, so that the chattering problem can be repressed.

3.1. The SMC law for multi-axis coordinated motion

The sliding mode plane is defined with both the tracking error and the coupling error as

$$s_i = e_i + \alpha \int_0^t e_i \, d\tau + \beta \varepsilon_i + \chi \int_0^t \varepsilon_i \, d\tau$$

where $\alpha > 0$, $\beta > 0$, $\chi > 0$. To solve of the derivation of (8) immediately yields

$$\dot{s}_i = \dot{e}_i + \alpha \dot{e}_i + \beta \dot{\varepsilon}_i + \chi \dot{\varepsilon}_i.$$

By substituting (5)–(7) into (9), we can express (9) as

$$\dot{s}_i = (1 + \beta)\dot{x}_d - \beta \dot{x}_{d,i+1} - (1 + \beta)(u_i - b_i x_i - d_i) + \beta(u_{i+1} - b_{i+1} x_{i+1} - d_{i+1}) + \alpha e_i + \chi \varepsilon_i.$$

For the convenience to derive and to design the control law, we define the intermediate variables as

$$e'_i = \alpha e_i + \chi \varepsilon_i$$
$$u'_i = (1 + \beta)u_i - \beta u_{i+1}$$
$$d'_i = (1 + \beta)d_i - \beta d_{i+1}.$$

Substituting (11) into (10) and considering (2), we can simplify (10) as

$$\dot{s}_i = \dot{x}_d + (1 + \beta)b_i x_i - \beta b_{i+1} x_{i+1} + e'_i + d'_i - u'_i.$$
By adopting the switch type of approaching law \( \dot{s}_i = -\rho_i \text{sgn}(s_i) \) and the disturbance compensation for the nominal model of the plant, the intermediate value of the output of the SMC controller can be structured as

\[
u_i' = \rho_i \text{sgn}(s) + x_d + (1 + \beta)b_i x_i - \beta b_{i+1} x_{i+1} + \epsilon_i' + \hat{d}_i'
\] (13)

where, \( \hat{d}_i' \) denotes the intermediate value of the estimation of the unknown disturbance. The final real control vector of all axes \( \mathbf{u} = (u_1, u_2, \ldots, u_n)^T \) and the immediate vector of the control vector \( \mathbf{u}' = (u_1', u_2', \ldots, u_n')^T \) satisfy the following relationship

\[
u' = \mathbf{T} \nu
\] (14)

where, the transaction matrix \( \mathbf{T} \) is defined in

\[
\mathbf{T} = \begin{pmatrix}
1 + \beta & -\beta & & \\
& 1 + \beta & -\beta & \\
& & \ddots & -\beta \\
-\beta & & & 1 + \beta
\end{pmatrix}_{n \times n}
\] (15)

So, if \( \det(\mathbf{T}) > 0 \), we can express the final control vector as

\[
u = \mathbf{T}^{-1} \nu'.
\] (16)

3.2. Design of the disturbance observer

In order to estimate and observe the torque disturbance for each axis, we constructed a disturbance observer as follows [16]:

\[
\begin{pmatrix}
\dot{\hat{d}}_i \\
\dot{\hat{x}}_i
\end{pmatrix} =
\begin{pmatrix}
0 & 0 \\
-1 & -b
\end{pmatrix}
\begin{pmatrix}
\hat{d}_i \\
\hat{x}_i
\end{pmatrix} +
\begin{pmatrix}
0 \\
1
\end{pmatrix} u_i +
\begin{pmatrix}
K_{1i} \\
K_{2i}
\end{pmatrix} (\hat{x}_i - x_i)
\] (17)

where, \( \hat{d}_i \) and \( \hat{x}_i \) respectively represent the estimation values of the torque disturbance and the state variable, \( K_{1i} \) and \( K_{2i} \) are respectively the correction gain of the estimation state error. Note that the state equation of the controlled plant (4) is a one-order system, the output variable can be directly selected as the state variable. To define the estimation errors of the disturbances and the state variables as

\[
\tilde{d}_i = \hat{d}_i - d_i \\
\tilde{x}_i = \hat{x}_i - x_i.
\] (18)

Substituting (18) into (17) yields

\[
\dot{\tilde{d}}_i = K_{1i} \tilde{x}_i \\
\dot{\tilde{x}}_i = -\tilde{d}_i - b \tilde{x}_i + u_i + K_{2i} \tilde{x}_i.
\] (19)

Eq. (19) is the mathematical expression of the error estimation of the observer with the disturbance and state variable.

3.3. Stability analysis

In order to prove the stability of the sliding mode and observer, by substituting (13) into (10) yields

\[
\dot{s}_i = -\rho_i \text{sgn}(s) + d_i' - \hat{d}_i'.
\] (20)

Suppose \( K_{1i} > 0 \), and choose the Lyapunov function as

\[
V = V_{1i} + V_{2i} = \frac{1}{2} s_i^2 + \frac{1}{2 K_{1i}} \tilde{d}_i^2 + \frac{1}{2} \tilde{x}_i^2.
\] (21)

By substituting (13)-(21), the derivative of \( V_{1i} \) is obtained as

\[
\dot{V}_{1i} = s_i \dot{s}_i = s_i[-\rho_i \text{sgn}(s) - \hat{d}_i'].
\] (22)

To choose \( \rho_i > \sup(|\hat{d}_i'|) \), the following of the stability of the sliding mode can be guaranteed, i.e.

\[
\dot{V}_{1i} = s_i \dot{s}_i \leq 0.
\] (23)
The derivative of $V_{2i}$ is as
\[
\dot{V}_{2i} = \frac{1}{K_{ii}} \ddot{d}_i \dot{d}_i + \dot{x}_i \ddot{x}_i
\]
\[
= \frac{1}{K_{ii}} \ddot{d}_i (\dot{d}_i - \dot{d}_i) + \dot{x}_i (\ddot{x}_i - \ddot{x}_i).
\] (24)

By substituting (19) into (24), we can get
\[
\dot{V}_{2i} = \frac{1}{K_{ii}} \ddot{d}_i \dot{d}_i + \dot{x}_i [(-\ddot{d}_i - b\ddot{x}_i + u_i + K_{ii} \ddot{x}_i) - (-b\ddot{x}_i + u_i - \ddot{d}_i)]
\]
\[
= \frac{1}{K_{ii}} \ddot{d}_i \dot{d}_i + \dot{x}_i [-b\ddot{x}_i - \ddot{d}_i + K_{ii} \ddot{x}_i].
\] (25)

Supposing that the torque disturbance changes very slowly, Eq. (25) can be written as
\[
\dot{V}_{2i} = \frac{1}{K_{ii}} \ddot{d}_i \dot{d}_i + \dot{x}_i [-b\ddot{x}_i - \ddot{d}_i + K_{ii} \ddot{x}_i]
\]
\[
= \ddot{d}_i \ddot{x}_i + \dot{x}_i [-b\ddot{x}_i - \ddot{d}_i + K_{ii} \ddot{x}_i]
\]
\[
= -(b - K_{ii}) \dot{x}_i^2.
\] (26)

Choosing $K_{ii} < b_i$ can make $\dot{V}_{2i} \leq 0$ hold. Therefore, the stability of the system is proved.

4. Simulation

Consider a 3-motor coordinated motion system. The numerical simulation of the proposed control law with the disturbance observer and compensation was conducted. The parameters of the three-motor-synchronization system are: $J_1 = 0.016 \text{ kg m}^2$, $J_2 = 0.017 \text{ kg m}^2$, $J_3 = 0.018 \text{ kg m}^2$, $B_1 = 0.1 \text{ N m s}$, $B_2 = 0.11 \text{ N m s}$, $B_3 = 0.09 \text{ N m s}$. The parameters of the controller are set as $\alpha = 10$, $\beta = 0.1$, $\chi = 60$, $\rho_1 = \rho_2 = \rho_3 = 5$. The parameters of the disturbance observer are chosen as $K_{11} = K_{12} = K_{13} = 400$, $K_{21} = K_{22} = K_{23} = -23$.

4.1. Control effect without disturbance

Fig. 2 shows the simulation curves of the control effect of the 3 motors with the initial value vector as $x_0 = [0, 30, 50]$. The reference signal $\omega_d$ is 20 rad/s. The torque applied to the 3 motors is zero. Fig. 2(a) and (b) show the rotational speed responses and the curves of the coupling errors of the 3 motors using the SMC law with the adjacent error, respectively. In Fig. 2(c), let $\beta$ and $\chi$ be 0, it illustrates the rotational speed responses curves errors of the 3 motors using the SMC law without considering the adjacent error.

In Fig. 2(a) and (b), because of the adoption of the cross-coupling strategy, although the motors have different initial values, the 3 response curves coincide with each other very fast firstly, and then converge to the reference speed asymptotically. In Fig. 2(c), the motor controllers do not consider the cross-coupling function, so the 3 response curves converge to the reference speed slowly. Note that the introducing of the cross-coupling error into the motor controller accelerates the transient process, the settling time is about 0.4 s in Fig. 2(c) and it is longer than 0.2 s in Fig. 2(a).

The coupling errors of the system with cross-coupling strategy are shown in Fig. 2(b). The coupling errors converge in 0.1 s, which is faster than the tracking errors. This is because of the different weights of the parameters ($\alpha$, $\beta$, $\chi$) of the SMC law, which reflects different importance between the speed tracking and the synchronization. Fig. 2(b) obviously stresses the synchronization performance of the multi-axis rotational speed system.

4.2. Control effect with disturbance

Fig. 3(a) shows the speed responses of the 3 motors with the initial values $x_0 = [20, 20, 20]$. The reference signal $\omega_d$ is 40 rad/s. When motors suffer from the torque disturbances from 0.5 s, the response curves deviate from the reference signal at first, but after a while regulate back to the reference signal. During the whole process the 3 motors respond synchronously. Fig. 3(b) illustrates the convergence process of the estimation of the disturbance to motor 1.

In Fig. 3(b) and (b), because of the compensation function of the disturbance observer, the torque disturbances do not affect steady-state response of the motors. However, when we remove the disturbance compensation function, the torque disturbances will result in the static errors, shown in Fig. 4. Note that when the torque disturbances are applied to motors, the static errors are remarkable and cannot converge, but because of the forward adjacent coupling error strategy the three motors also respond synchronously.
5. Conclusions

Addressing the uncertainties and different characteristic of each axis in a multi-axis coordinated traveling system, we designed a disturbance observer based sliding mode control algorithm. For the elimination of the synchronization errors, the forward adjacent coupling error is introduced into the sliding mode plane. To improve the robustness and static precision of the system under disturbance torque, a disturbance observer is designed and its compensation signal is introduced into the controller. The simulation results have proved the effectiveness of the proposed algorithm. The main conclusions in the paper are as follows:
(1) The introduction of the forward adjacent coupling error strategy can reduce the synchronization errors remarkably, and can accelerate the whole transient process.

(2) The proposed SMC law with the disturbance observer and compensation acquired the satisfactory coordinated control performance under the different parameters uncertainties and the torque disturbance among a multi axes rotational speed system.

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References