Multi-messages Broadcasting of Paths by Simultaneously Send/Receive

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ABSTRACT: In a communication network, the multi-originators broadcasting refers to the process of message passing from some processors (called originators) to all other processors. Assuming each originator has all the messages that need to be passed and a processor can send one message it owns to a processor at a time and simultaneously receive one message from another processor. We model this process by a graph where the vertices represent the processors in the network and the edges represent the communication links. In this paper, we establish the minimum broadcasting time with multi-messages in the paths, and give the location for the originators in the paths in order to get the minimum broadcasting time.

KEYWORDS: Multi-originators, multi-messages, broadcasting, path, caterpillar, simultaneously send / receive.

1. INTRODUCTION

In a communication network, message passing is an important operation. There are two kinds of message passing, gossiping [7, 10] and broadcasting [11]. Gossiping refers to the way of message passing when each processor knows a unique item of information and must transfer to all the other processors. Broadcasting refers to the way of message passing such that there is one processor has a set of messages need to pass to all other processors. The original message holder is called originator and the time require for this process is referred as broadcasting time. We use $BT_m(G)$ to represent the minimum broadcasting time of broadcasting $m$ messages. Gossiping is a all-to-all message passing and broadcasting is a one-to-all message passing. If there are more than one originators, then the broadcasting becomes a some-to-all message passing. In this case we use $BT_m^s(G)$ to represent the minimum broadcasting time by using $s$ originators in a given network topology $G$. There is a thoroughly survey on gossiping and broadcasting in [8].

There are three models of broadcasting, namely, simultaneous model [1], non-simultaneous model [11], and bidirectional telephone model [2]. Assuming that each processor sends a message to an adjacent processor also requires one unit of time. In bidirectional telephone model, each processor may communicate with exactly one other processor and exchange messages in one unit of time. In non-simultaneous model, each processor can only receive (or send) a message from (or to) a connected processor at a time. So if processor A has to receive a message from processor B and send a message to processor C, it requires two units of time to complete the process. On the contrary, in simultaneous model, each processor can send a message to one processor and receive a message from another processor at the same time. So in simultaneous model, the whole process for processor A to receive a message from processor B and send a message to processor C requires only one unit of time.

Many works have been done in non-simultaneous model. For single originator and single message passing, several network topologies such as fully connected [3], trees [5], meshes [6] and hypercubes [9] have been investigated. They gave polynomial algorithm to allocate originator and have formula for the broadcasting time. For single originator and multi-messages broadcasting, fully connected [3] and hypercubes [9] network topology are also solved. And given the broadcasting time, the minimum number of originators needed on the rooted tree is provided, see [5].

In simultaneous model, [1] gave the broadcasting time for single originator, multi-message broadcasting on complete graphs and hypercubes, [9] provided the solution for rooted trees.

In this paper, we consider only simultaneous model. We use a graph to represent a communication network such that the vertices indicate the processors and the edges represent the links between processors. We allocate the originator in paths for single originator, multi-messages broadcasting, and present the broadcasting scheme of multi-originators, multi-messages.

2. BROADCASTING ON PATHS

Path is a common network topology, and is also a basis for simple graph. Unlike fully connected network topology with all possible links between
processors and has minimum diameter, path has minimum links with maximum diameter in the communication network. There are quite a few works done with fully connected networks, however, it is interesting to know whether there is a best location to allocate the originator when passing messages in the path.

First, let’s consider single originator with multiple messages broadcasting in the path. Following lemmas give us the broadcasting time when the originator is located in certain position in the path. Lemma 1 deal with the case of number of messages is greater than or equal to the number of processors, lemma 2 deal with the other case.

**Lemma 1** Let \( P_n = v_1, v_2, \ldots, v_n \) be a path of \( n \) vertices. Let \( m \) be the number of messages that are going to be broadcasted on \( P_n \) such that \( m \geq n \). If we assign the originator to one of the processor in \( V^* = \{ v_{1}, v_{n} \} \), then the broadcasting time on \( P_n \) is \( m + n - 2 \).

**Proof:** Since \( v_i \) and \( v_n \) are similar vertices in the path, we know that they have the same broadcasting time. With out loss of generality, let \( v_i \) be the originator. The earliest time that it can send the last message to \( v_n \) is at time \( m \). Then the earliest time for \( v_n \) to get the last message will be \( n - 1 + m - 1 = n + m - 2 \).

**Lemma 2** Let \( P_n = v_1, v_2, \ldots, v_n \) be a path of \( n \) vertices. Let \( m \) be the number of messages that are going to be broadcasted on \( P_n \) such that \( m < n \). If the originator is assigned to one of the processor in \( V^* = \{ v_{[a]}, v_{[a]} \} \), where \( a = \lfloor (n - m + 1) / 2 \rfloor \), \( b = (n + m + 1) / 2 \), then the broadcasting time on \( P_n \) is \( \lceil (n - m - 1) / 2 \rceil + 2m - 1 \).

**Proof:** Since \( v_{[a]} \) and \( v_{[a]} \) are similar vertices, \( v_{[a]} \) and \( v_{[a]} \) are similar vertices in \( P_n \), we only have to show the broadcasting time for \( \{ v_{[a]}, v_{[a]} \} \) as originators are \( \lceil (n - m - 1) / 2 \rceil + 2m - 1 \). First we consider \( v_{[a]} \) as the originator. In order to take the advantage of simultaneous send/receive property, it must send all message to one side than the other. Since \( n - \lceil a \rceil > \lceil a \rceil - 1 \), we know that it will send the messages to \( v_{[a]} \), then \( v_{[a]} \). Then \( v_{[a]} \) will get the last message at time \( m \) and \( v_{[a]} \) will get the last message at time \( 2m \). So the broadcasting time will be either vertex \( v_n \) or \( v_i \) to get the last message from their adjacent vertex, which is \( \max \{ \lceil a \rceil - 1 + 2m - 1, n - \lceil a \rceil + m - 1 \} = n - \lceil a \rceil + m - 1 \). Therefore, we have the broadcasting time as

\[
\begin{align*}
&n - \lceil a \rceil + m - 1 = n - \left( \frac{n + m - 1}{2} \right) + m - 1 \\
&= \left( \frac{n + m - 1}{2} \right) + m - 1 \\
&= \left( \frac{n - m - 1}{2} \right) + 2m - 1.
\end{align*}
\]

Now consider \( v_{[a]} \) to be the originator. Similar to the case above, it will send the messages to \( v_{[a]} \) then \( v_{[a]} \). So the broadcasting time is \( \max \{ \lceil a \rceil - 1 + 2m - 1, n - \lceil a \rceil + m - 1 \} = \lceil a \rceil - 1 + 2m - 1 \). Then we have the broadcasting time as

\[
\begin{align*}
&\lceil a \rceil - 1 + 2m - 1 = \left( \frac{n - m + 1}{2} \right) + 2m - 2 \\
&= \left( \frac{n - m - 1}{2} \right) + 2m - 1.
\end{align*}
\]

So in both cases we have the broadcasting time as \( \lceil (n - m - 1) / 2 \rceil + 2m - 1 \).

Figure 1(a) shows an example of 2-messages broadcasting on \( P_5 \) while locate the originator at \( v_{[a]} = v_3 \). We can see that at time \( t = 5 \), all the messages have been broadcasted to all processors.

- Passing message 1
- Passing message 2

Figure 1(a): 2-messages broadcasting on \( P_5 \)

Figure 1(b) shows another example of 2-messages broadcasting on \( P_5 \) while locate the originator at \( v_{[a]} = v_4 \). In this case, \( v_1 \) receives message 2 at time \( t = 6 \) while \( v_n \) gets all messages by time \( t = 5 \). Note that in this case, locating the originator at \( v_3, v_4, v_5 \) and \( v_6 \) will have the same broadcasting time. If
we locate the originator at \( v_3 \), then \( v_n \) instead of \( v_1 \) will get all the messages at time \( t = 6 \).

Case 1: \( i < \lfloor a \rfloor \) or \( i > \lceil b \rceil \). Without loss of generality, let \( i < \lfloor a \rfloor \). We know that the earliest time for \( v_i \) to send the last message to \( v_{i+1} \) is time \( m \). The message passing through \( v_{i+1} \) to \( v_n \), the earliest time for \( v_n \) to receive the last message from \( v_{i-1} \) will be

\[
m + n - i - 1 > n - \lfloor a \rfloor + m - 1 \\
= n - \left(\frac{n - m + 1}{2}\right) + m - 1 \\
= \left(\frac{n + m - 1}{2}\right) + m - 1
\]

which is a contradiction.

Case 2: \( \lceil a \rceil < i < \lceil b \rceil \). Since \( v_i \) has to send message to both \( v_{i-1} \) and \( v_{i+1} \), the last message it send is at time \( 2m \). Suppose the last message is sent to \( v_{i-1} \), then the earliest time for \( v_i \) to get the last message is

\[
\left(\frac{n - m + 1}{2}\right) + 2m - 2 = i - 1 + 2m - 1 \\
> \lceil a \rceil + 2m - 2 \\
= \left(\frac{n + m - 1}{2}\right) + m - 1
\]

which is a contradiction.

Now suppose the last message is sent to \( v_{i+1} \), then the earliest time for \( v_n \) to get the last message is

\[
n - i + 2m - 1 > n - \lceil b \rceil + 2m - 1 \\
= n - \left(\frac{n + m + 1}{2}\right) + 2m - 1 \\
= \left(\frac{n + m - 1}{2}\right) + m - 1
\]

which is a contradiction.

By case 1 and 2 we know that for \( m < n \), \( BC(P_n) = V^* \). Hence, the theorem is proved.

Figure 2(a) shows the \( BC(P_5) \) with 2-messages broadcasting, which is the case of \( m < n \). In this case, \( BC(P_5) = \{v_3, v_4, v_5, v_6\} \) and \( BT_2(P_5) = 6 \).

Figure 2(a): \( BC(P_5) = \{v_3, v_4, v_5, v_6\} \)

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Figure 2(b) shows the $BC(P_8)$ with 8-message passing which is the case of $m \geq n$. In this case, $BC(P_8) = \{v_1, v_8\}$ and $BT_2(P_8) = 14$.

Figure 2(b): $BC(P_8) = \{v_1, v_8\}$

Applying Lemma 1 and Lemma 2 on Theorem 1, we address the broadcasting time of path in Theorem 2 as follows.

Theorem 2 Let $P_n = v_1, v_2, \ldots, v_n$ be a path of $n$ vertices. Let $m$ be the number of messages that are going to be broadcasted on $P_n$. Then

$$BT_m(P_n) = \left\lceil \frac{n + 3m - 3}{2} \right\rceil.$$

If there are more than one originator in the path, we can simply divide the path into several subpaths then treating each subpath as doing single originator broadcasting. Lemma 3 is trivial.

Lemma 3 Let $H$ be a subgraph of $G$. Then $BT_m^+(H) \leq BT_m^+(G)$ and $BT_m^-(H) \leq BT_m^-(G)$ for positive integers $m$ and $s$.

Theorem 3 Let $P_n = v_1, v_2, \ldots, v_n$ be a path of $n$ vertices, $s$ be the number of originators, and $m$ be the number of messages that are going to be broadcasted on $P_n$. Then the broadcast time of the path is

$$BT_m(P_n) = \left\lceil \frac{\left\lceil \frac{n}{s} \right\rceil + 3m - 3}{2} \right\rceil.$$

By theorem 2 we have

$$BT_m(Q_i) = \left\lceil \frac{\left\lceil \frac{n}{s} \right\rceil + 3m - 3}{2} \right\rceil = BT_m^+(P_n).$$

3. CONCLUSION

In this paper, we establish the allocation of originator in paths when broadcasting multi-messages using simultaneously model by single or multi originator, and provide the minimum broadcasting time in both cases.

Reference