A kernel-based ICI self-cancellation scheme using constrained subcarrier combiners

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Conventional ICI self-cancellation methods are spectral consuming because they modulate a single data on a group of subcarriers. To improve the spectral efficiency, the proposed approach uses a kernel-based precoder that maps at most \( L/C_0 \) data symbols to a group of \( L \) consecutive subcarriers. On the receive side, the carrier-frequency-offset-directed (CFO-directed) structure of the precoded signal enables the proposed approach to estimate the CFO in the frequency domain. Then, based on this CFO estimate, the proposed approach develops a set of constrained-subcarrier-combiners (CSC) to eliminate intra-group interference. Computer simulations show that in addition to achieving a high spectral efficiency proportional to the precoder order, the proposed approach can effectively eliminate the ICI caused by a large-frequency-error because of the CSC.

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1. Introduction

Orthogonal frequency division multiplexing (OFDM) has become a key technology for wireless communication systems because of its high spectral efficiency, robustness against multipath fading, and reliable high speed transmission under serious channel conditions [1–3]. However, OFDM systems are very sensitive to frequency discrepancies [4].

Frequency synchronization error is caused by the Doppler frequency shifts and carrier frequency offset (CFO) between the local oscillators in the transmitter and the receiver. To investigate the performance degradation caused by the frequency error, Sathananthan and Tellambura [5] developed a two-dimensional characteristic function (2D-CHF) of ICI to express the probability of OFDM decoding error. Other researchers have developed useful methods to combat ICI [6–14]. Armstrong [6] analyzed some ICI reduction approaches that improved decoding reliability. Barbancho et al. [7] used the autocorrelation properties of the code-division-multiplex-access (CDMA) codes to mitigate the ICI in the OFDM-CDMA system. Sun et al. [8] proposed a spectral-periodogram-based algorithm to estimate and then compensate the CFO to eliminate the ICI. Liu and Li [9] used the mirror spectrum induced by I/Q imbalance to estimate the CFO in a blind manner. Zhao and Häggman [10] proposed a simple yet effective ICI self-cancellation scheme that uses a weight sequence to map each data symbol to a band of subcarriers. Seyedi and Saulnier [11] proposed a general ICI self-cancellation scheme that used a window-based method to suppress ICI in the time domain to achieve high decoding accuracy. Chang [12] proposed a diversity-based ICI self-cancellation scheme approach that periodically extends the time domain signal to generate diversity to the frequency domain symbols, which are combined to mitigate the ICI. Wang and Huang [13] proposed a phase-rotation-based scheme that needs the CFO information before determining the optimal rotation phase of the transmitter. Orlik et al. [14] designed a transmit signal structure to achieve the optimal
signal-to-interference power ratio in the presence of ICI. The approaches in [11,14] can express Zhao's method as a special case of their general ICI self-cancellation schemes. However, both methods in [11,14] have no closed-form solution. Dwivedi and Singh [15] combined Zhao's method and the repeated correlated coding to combat the ICI; achieving a better carrier-to-interference ratio (CIR) over Zhao's method. Huang et al. [16] proposed a factor-graph-based progressive parallel inter-carrier interference canceller (PPIC) for OFDM systems. By exchanging messages in both the space domain and the frequency domain, Huang's algorithm iteratively suppresses inter-antenna interferences and the ICI. However, the PPIC suffers from high computational complexity caused by the associated message passing. Besides, the PPIC approach can only handle the ICI caused by small frequency errors, such as the Doppler shifts. Lin et al. [17] proposed a time domain ICI self-cancellation method from the view of equivalent channel time variation mitigation. Assuming a linear time-varying channel model, Lin's approach uses a set of windowing coefficients to equalize the channel time variation and thus suppress the ICI. However, the linearity of the time-varying channel model disappears when the CFO increases; limiting the application of the method in [17] to small scale frequency discrepancy cases.

A critical drawback of existing ICI self-cancellation scheme algorithms [6–17] is that they all suffer from a spectral efficiency degradation, because of using multiple subcarriers/time samples to modulate a single data symbol. Besides, they can hardly handle the ICI caused by a large frequency error. For example, under the 8-phase-shift-keying (8-PSK) modulation, the performance of Zhao's ICI self-cancellation scheme seriously deteriorates when the CFO exceeding 0.45 subcarrier spacing. However, practical wireless applications often suffer from a large CFO. For example, with a 2.6 GHz carrier frequency and a 15 kHz subcarrier spacing (or a called a normalized frequency), the long-term-evolution (LTE) system may experience a CFO of 0.87 normalized frequency when the receiver oscillator imposes a 5 ppm frequency bias [18]. To expand the CFO tolerance of general ICI self-cancellation approaches, Hao [19] proposed a CFO tracking algorithm that uses the intrinsic periodic structure of the precoded signal, generated by Zhao's precoder, to estimate the CFO through a decision feedback manner. However, Hao's CFO estimate is sensitive to noise perturbation. Wang [20] proposed a subspace-based algorithm to estimate the CFO of Zhao's precoder generated signal, achieving better estimation accuracy than Hao's method.

Kernel methods have been found applications in various disciplines, such as statistical signal processing, and computer science [21–24]. In statistical signal processing, kernel methods provide a diverse framework for developing non-parametric detection procedures [22,23]. In computer science, kernel methods are a class of algorithms for pattern analysis, whose most known member is the support vector machine [24]. This study presents a kernel-based precoder to suppress the ICI in OFDM. The proposed approach employs a set of kernel sequences to construct an ICI self-cancellation scheme with high spectral efficiency. Unlike conventional ICI self-cancellation methods [10–12] modulating only a single data symbol to \( L \) subcarriers, the proposed approach can modulate at most \( L-1 \) data symbols to a group of \( L \) consecutive subcarriers; achieving high spectral efficiency in the precoded OFDM. The key challenge of the proposed approach is the interference caused by the multiple kernel-sequence-modulated data symbols. This study classifies the interferences among subcarriers into two categories: (1) the inter-group interference, which is caused by the signals in other subcarrier groups and (2) the intra-group interference, which results from the loss of orthogonality between the mapping sequences because of the CFO. The proposed kernel-based precoder can easily suppress the inter-group interference using the smooth response feature of the interference. The suppressed inter-group interference enables the proposed approach to represent the received signal in each subcarrier group as a CFO-directed signature vector. On the receive side, the proposed approach first uses the multiple-signal-classification (MUSIC) algorithm [25] to estimate the CFO. Based on the CFO estimate, this study suppresses the intra-group interference through a set of constrained subcarrier combiners (CSC). Unlike conventional ICI self-cancellation scheme methods suffering a limited spectral efficiency less than 0.5, the proposed ICI self-cancellation scheme of order \( L \) can effectively eliminate the large-CFO-caused ICI and achieves a higher spectral efficiency of \( (L-1)/L \).

The rest of this paper is organized as follows. Section 2 introduces the system model of the OFDM system with frequency errors. Section 3 presents the proposed ICI self-cancellation scheme. Section 4 presents computer simulation results. Section 5 draws a conclusion of the paper.

### 1.1. Mathematical notation

Throughout this paper, vectors and matrices are denoted by lower case and upper case boldface letters. In addition, the mathematical notations are denoted as follows:

- \( (\cdot)^T \) transpose operation;
- \( (\cdot)^* \) complex conjugation operation;
- \( (\cdot)^H \) Hermitian operation;

- \( N \) OFDM symbol size;
- \( N_{CP} \) length of cyclic prefix insertion;
- \( L \) order of the ICI self-cancellation scheme;
- \( \epsilon \) CFO normalized to the subcarrier spacing;

\[
\begin{align*}
E[\cdot] & \quad \text{statistical expectation operation;}
\operatorname{diag}(\mathbf{v}) & \quad \text{diagonal matrix with vector } \mathbf{v} \text{ as the diagonal elements;}
\mathbf{a} = [(a_i)_{i=1,...,N}] & \quad \text{vector formed by stacking the elements } a_i^\ast, \quad i = 1, \ldots, N
\end{align*}
\]

### 2. System model

For a \( \nu \)-tap baseband finite-impulse-response (FIR) wireless channel, the channel output at time instance \( n \)
can be expressed as
\[ y_n = \sum_{l=0}^{v-1} h_l x_{n-l} + z_n, \]  
where \( x_n \) is the transmit signal, \( h_l \) denote the channel response, and \( z_n \) is an additive white Gaussian noise (AWGN) with zero mean and a power spectral density of \( N_0/2 \). At the transmitter, the OFDM modulator inserts a cyclic prefix (CP) at the beginning of each OFDM symbol to avoid inter-symbol-interference (ISI). At the receiver, after discarding the CP, an OFDM symbol can be expressed in a vector form as
\[
\mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{z}.
\]  
where \( \mathbf{x} = [x_0, \ldots, x_{N-1}]^T \) denotes the transmit signal vector, \( \mathbf{z} \) is the noise vector, and \( \mathbf{H} \) denotes the response matrix of the FIR channel, which can be expressed as an \( N \times N \) circulant matrix with the vector \( \mathbf{u}^T = [h_0, h_1, \ldots, h_{N-1}]^T \) as its first row. The CFO distorts each receive time sample with a phase rotation. As a result, the CFO-distorted signal vector can be expressed as
\[
\mathbf{y} = \Phi \mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{z}.
\]  
where \( \Phi = \text{diag}[1, e^{-j2\pi/2}, \ldots, e^{-j2\pi(N-1)/N}] \) is the CFO-caused phase-rotation matrix with \( \epsilon \) being the CFO normalized to the subcarrier spacing, and \( \mathbf{z} \) denotes the noise vector. Fig. 1 shows the block diagram of the OFDM system using the proposed ICI self-cancellation scheme.

Because the column vectors of the fast Fourier transform (FFT) matrix are eigenvectors of a circulant matrix [29], the channel matrix can be decomposed as
\[
\mathbf{H}_k = \mathbf{F} \mathbf{h}_k \mathbf{F}^H,
\]  
where \( \mathbf{H}_k = \text{diag}[H_0, \ldots, H_{N-1}] \) is a diagonal matrix with \( H_k = (1/\sqrt{N}) \sum_{n=0}^{N-1} h_k e^{-j2\pi k n/N} \) denoting the \( k \)th eigenvalue of \( \mathbf{H}_k \), and \( \mathbf{F} = (1/\sqrt{N}) \mathbf{e}^{j2\pi/n} \) denotes the inverse-FFT (IFFT) matrix. In (3), the transmit signal vector
\[ \mathbf{x} = \mathbf{F} \mathbf{x}_f, \]  
where \( \mathbf{x}_f = [x_0, \ldots, x_{N-1}]^T \) denotes the frequency domain signal vector, in which \( \mathbf{x}_f \) is the transmit signal of the \( k \)th subcarrier. Using (4), in the CFO-free case, the OFDM system can transform the ISI channel as a set of mutually orthogonal subcarriers. However, the presence of the CFO destroys the orthogonality among subcarriers, and thus incurs the ICI in the frequency domain. This seriously degrades the system performance in data decoding. Substituting (4) into (3), after the FFT, the frequency domain OFDM symbol can be expressed as
\[
\mathbf{y} = \mathbf{F} \mathbf{y} = \mathbf{H}_k \mathbf{x}_f + \mathbf{z},
\]  
where \( \mathbf{H}_k = \mathbf{F} \mathbf{h}_k \mathbf{F}^H \) is the ICI matrix with \( \mathbf{H}_k \) being its \( (l, m) \)th element. Here, the term \( \mathbf{H}_k \) refers to the ICI factor between subcarriers \( l \) and \( m \). Specifically, this factor can be expressed as
\[
\mathbf{H}_{l,m} = \delta(\epsilon) \frac{\sin (\pi (l-m) / N)}{N \sin (\pi (l+m) / N)} e^{-j\pi (l-m) / N}.
\]  

When subcarrier \( l \) is the desired one to decode and \( m \neq l \), \( \mathbf{H}_{l,m} \) represents the ICI magnitude that the desired subcarrier \( l \) experiences. When \( m=l \)
\[
\mathbf{H}_{l,l} = \delta(\epsilon) \frac{\sin (\pi \epsilon)}{N \sin (\pi \epsilon)} e^{-j\pi (l-m) / N}.
\]  

denotes the CFO-caused attenuation suffered by the desired subcarrier \( l \). The magnitude of \( \mathbf{H}_{l,m} \) depends on the indices of subcarriers \( l \) and \( m \), between which this study defines subcarrier \( l \) as the desired one and subcarrier \( m \) as the one that contributes ICI to subcarrier \( l \).

Conventional ICI self-cancellation methods [10,11,14] use a processing sequence in the frequency domain to alleviate the ICI. For the OFDM using these approaches, the \( n \)th time sample of (3) can be expressed through the IFFT as
\[
\tilde{y}_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} d_k c_k \mathbf{H}_k e^{j2\pi k n/N} + z_n,
\]  
where \( d_k \) denotes the data symbol, \( c_k \) is the processing weight of subcarrier \( k \) to suppress the ICI, and the transmit symbol of subcarrier \( k \) is \( X_k = d_k c_k \). For example, assuming that the channel coherence bandwidth is greater than the frequency coverage of \( L \) consecutive subcarriers, Zhao’s method [10] modulates a common data symbol \( d_k = d_k \delta_0 \), \( \forall l = 0, \ldots, L-1 \), on the \( k \)th group of \( L \) consecutive subcarriers. The associated processing weight of the \( k \)th subcarrier in each subcarrier group is \( c_k = \delta (L-1)! / l! (L-l-1)! \), \( \forall l = 0, \ldots, L-1 \) [10]. At the receiver, Zhao’s method uses the same processing weights, \( \{P_l\}_{l=0}^{L-1} \), to combine the subcarrier signals in each group to further suppress the ICI before data decoding. Zhao’s method is based on the ICI factors remaining almost stationary over \( L \) subcarriers. Therefore, this method can use a set of processing weights of zero-sum to suppress the ICI factors. Similar assumptions appear in Chang’s method [12], which entails

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**Fig. 1.** The block diagram of the OFDM using the proposed ICI self-cancellation scheme.
using the frequency diversity, generated by repeating the OFDM symbol in the time domain, to suppress the ICI. However, most ICI self-cancellation schemes (including Zhao’s method and Chang’s method) are only applicable to cases involving a small frequency error. These methods cannot handle the large-CFO-caused ICI, because the stationary assumption of the ICI factors no longer holds when the CFO increases. In addition, conventional methods suffer a serious decrease in spectral efficiency because of their high consumption of subcarrier resources. For example, some methods in [10,11] modulate only a single data symbol on a group of multiple subcarriers in the frequency domain, and Chang’s method [12] repetitively transmits the same signal in the time domain.

This study proposes an ICI self-cancellation scheme capable of a high spectral efficiency. The proposed approach entails using a kernel-based precoder to modulate at most \( L \) in-group subcarriers, with the ICI factors \( \hat{R}_{lm} \) for \( m \leq l + L - 1 \). Also, define the inter-group interference as the ICIs from the subcarriers in other groups, excluding the belonging group of subcarrier \( l \). Fig. 2 shows the distribution of the ICI factors of the OFDM corresponding to various CFOs. The size of the OFDM symbol is \( N = 128 \), the subcarrier group size is \( L = 4 \), and the desired subcarrier is \( l = 64 \). Fig. 2(a) and (b) shows that, under the influence of a small CFO \( \epsilon = 0.1 \), the ICI factors for both the intra-group and the inter-group subcarriers remain almost stationary. This stationary property supports the effectiveness of the ICI self-cancellation methods in [6–14]. However, Fig. 2(c) and (d) shows that, for a larger CFO \( \epsilon = 0.73 \), the magnitudes of the ICI factors in the intra-group subcarriers vary dramatically, whereas those of the inter-group subcarriers remain stationary. This dramatic variance in the intra-group ICI factors prevents conventional ICI self-cancellation methods from working in the OFDM with a large frequency discrepancy.

### 3. The proposed approach

Fig. 3 shows the proposed ICI self-cancellation scheme. The proposed approach groups \( L \) consecutive subcarriers...
to deal with the ICIs, and calls the group size as the order of the proposed ICI self-cancellation scheme. The proposed approach employs a set of kernel-based mapping sequences to mitigate the inter-group interference at the receive side. For the ICI self-cancellation approach employs a set of kernel-based mapping sequences to suppress the intra-group interference at the receive side.

### 3.1. The kernel-based precoder

Using the feature that the inter-group interferences correspond to those ICI factors with smooth responses over subcarriers as Fig. 2 shows, the basic idea of the proposed precoder is to find a set of zero-sum mapping sequences to modulate data symbols and suppress the inter-group interference. For the ICI self-cancellation scheme of order $L$, the mapping sequences $(c_m, m = 0, \ldots, L − 2)$ can thus be selected from the kernel of an all-one vector of size $L$,

$$c_m = \begin{bmatrix} c_{m,0} \\ \vdots \\ c_{m,L−1} \end{bmatrix} \in \ker \{1_L\}, \tag{9}$$

where $1_L$ denotes the all-one vector of size $L$, $\ker \{1_L\} = \{c_m| c_{0,1_L} = 0, m = 0, \ldots, L − 2\}$ is the kernel of $1_L$. Note that each sequence $c_m$ is of zero-sum, $\sum_{k=0}^{L−1} c_{m,k} = 0$, and the dimension of $\ker \{1_L\}$ is $L−1$, which ensures the proposed precoder capacity to accommodate at most $L−1$ data symbols in each subcarrier group. Unlike the traditional ISI self-cancellation [6–17] mapping only a single data onto a group of $L$ subcarriers, the proposed kernel-based precoder can thus improve the spectral efficiency.

The upper part of Fig. 3 shows the data modulation scheme of the proposed precoder. To investigate how the kernel-based precoder handle the inter-group interference, the $l$th subcarrier signals in the $k$th subcarrier group can be expressed as

$$X_{kL+l} = \sum_{m=0}^{L_p−1} d_{m,k} c_{m,l} l = 0, \ldots, L−1 \quad \text{and} \quad k = 0, \ldots, \frac{N}{L}−1 \tag{10}$$

where $d_{m,k}$ denotes the $m$th data symbol modulated by the sequence $c_m$, and $L_p \leq L − 1$ is the loading factor representing the number of data symbols in each subcarrier group.

Based on (10), the receive time signal in (8) can be rewritten as

$$y_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N/L−1} \sum_{l=0}^{L−1} X_{kL+l} + H_{kL+l} e^{j2\pi/N(kL+l+\epsilon)n} + z_n,$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N/L−1} H_{kL} \sum_{l=0}^{L−1} \sum_{m=0}^{L_p−1} d_{m,k} c_{m,l} e^{j2\pi/N(kL+l+\epsilon)n} + z_n. \tag{11}$$

Eq. (11) is based on the assumption that $H_{kL+l} = H_{kL}$, for $l = 0, \ldots, L−1$, which indicates that the channel coherence bandwidth is greater than the frequency coverage of a subcarrier group. This assumption of constant channel response is reasonable. For example, the channel model for the LTE system [18] has a coherence bandwidth ranging from 100 kHz (for an Extended Typical Urban (ETU) model) to 1220 kHz (for an Extended Pedestrian A (EPA) model). For a 15 kHz subcarrier space, the LTE system has a coherence bandwidth covering $L = 6$ subcarriers for the ETU model and $L = 81$ for the EPA model.

After the FFT, the $l$th subcarrier signal in the $k$th subcarrier group can be expressed as

$$r_{kL+l+i} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N−1} y_n e^{−j2\pi/nN(kL+i)n}, \quad k = 0, \ldots, \frac{N}{L}−1 \quad \text{and} \quad l = 0, \ldots, L−1,$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{(N/L)−1} H_{kL} \sum_{m=0}^{L_p−1} d_{m,k} c_{m,l} e^{j2\pi/N(kL+i+\epsilon)n} + z_{kL+l+i},$$

$$\quad \quad \quad \quad \quad + \sum_{k=0}^{N/L−1} H_{kL} \sum_{m=0}^{L_p−1} d_{m,k} e^{j2\pi/N(kL+i+\epsilon)n} + z_{kL+l+i}, \tag{12}$$

where $S_{m,l}(e) = e^{j2\pi/N(kL+i+\epsilon)n} c_m$ denotes the intra-group-interference coefficient, which represents the magnitude of the interference from the $m$th data symbol to the $l$th data symbol in the desired subcarrier group $k$. The term $S_{m,0}(kL, kL+\tilde{l}) = d_{m,k} e^{j2\pi/N(kL+\tilde{l})e} c_m$, $\forall k \neq \tilde{k}$, denotes the inter-group-interference coefficient, representing the magnitude of the interference from the $n$th data symbol in other subcarrier groups $k \neq \tilde{k}$. The associated vector $\alpha_{l, k, kL+\tilde{l}}$ for both coefficients is defined as

$$\alpha_{l, k, kL+\tilde{l}} = \begin{bmatrix} \frac{l(kL−(kL+1)+\epsilon)}{l(kL+1−(kL+\tilde{l})+\epsilon)} \\ \frac{l(kL+L−1−(kL+\tilde{l})+\epsilon)}{l(kL+L−1−(kL+\tilde{l})+\epsilon)} \end{bmatrix}_{l \times 1}. \tag{13}$$
The first term in (12) contains the $L_p$ signals of interest, whereas the second term represents the inter-group interference. Because the response of the ICI-factors associated with the inter-group interference is almost stationary in both the real and the imaginary part (as Fig. 2 shows), the vector $\alpha_{kl,k+1}$ can be rewritten as

$$\alpha_{kl,k+1} = I(kL-(kL+1)+\epsilon)\mathbf{1}_L + \delta_{kl,k+1}, \forall k \neq \tilde{k},$$

(14)

where $\delta_{kl,k+1} = [\delta_{kl,k+1,0} \cdots \delta_{kl,k+1,L-1}]^T$ is the biased vector with the element $\delta_{kl,k+1,l}$ defined as

$$\delta_{kl,k+1,l} = \frac{I(kL-(kL+1)+\epsilon)-I(kL-(kL+1)+\epsilon)}{l(kL-(kL+1)+\epsilon)}$$

(15)

These biases $\delta_{kl,k+1,l}$’s usually have small magnitudes, because $I(kL+1-(kL+1)+\epsilon) \approx I(kL-(kL+1)+\epsilon)$. The stationary property in (14) enables the proposed approach to mitigate the inter-group interference using the zero-sum feature of the mapping sequence, $\sum_{l=1}^{L} c_{ml} = 0$. Thus, the inter-group interference coefficients in (12) are small and can be expressed as

$$S_m(kL, kL+1) = \delta_{kl,k+1}^T c_m \neq 1, \forall k \neq \tilde{k} \text{ and } m = 0, \ldots, L_p - 1.$$  

(16)

Fig. 4 shows the magnitudes of $S_m(kL, 1)$ when the CFO $\epsilon = 0.73$. The parameters of the proposed kernel-based precoded OFDM are $L_c = 4$, $L_p = 3$, $N = 256$, and the desired subcarrier is assumed to be the first subcarrier (i.e., $\tilde{k} = 0$, and $l = 1$). Fig. 4 shows that the magnitude of $S_m(kL, 1)$ decreases as $k$ increases. This decrease shows that the inter-group interference is inversely proportional to the separation between the desired subcarrier and the subcarrier causing the interference.

3.2. The constrained subcarrier-combiner (CSC)

The CFO destroys the orthogonality among the kernel-based mapping sequences at the receiver, which in turns leads to the intra-group interference. The loss of orthogonality among the mapping sequences makes it infeasible to decode data using the same sequences as the combining sequences. This section presents the CSC to suppress the intra-group-interference at the receiver. The proposed approach first employs the MUSIC algorithm [25–27] to estimate the CFO, and then, based on the CFO estimate, describes the intra-group-interference suppression as a constrained optimization problem.

3.2.1. The CFO estimation

This study assumes that the receiver exactly knows the channel response. After channel equalization, the signals in subcarrier group $\tilde{k}$ can be expressed in a vector form as

$$\mathbf{r}_k = H_{kL}^{-1} \frac{L_p-1}{ \sum_{m=0}^{L_p-1} \sum_{k \neq \tilde{k}} d_{m,k} S_m(kL, kL) + \mathbf{Z}_k,}$$

(17)

where $\mathbf{s}_m(kL, kL+1) = \mathbf{S}_m\mathbf{d}_k + \mathbf{Z}_k$

(18)

In (18), $\mathbf{s}_m = ([s_{m,0}(\epsilon), \ldots, s_{m,L_p-1}(\epsilon)])^T$ is the spectral signature of the $m$th data symbol in subcarrier group $k$, $\mathbf{S}_m = [s_{0}(\epsilon), \ldots, s_{L_p-1}(\epsilon)]$ denotes the signature matrix, $\mathbf{Z}_k$ is the associated noise vector. Because the kernel-based mapping sequence suppresses the inter-group-interference as (16) shows, thus $\mathbf{Z}_k \approx \mathbf{1}$, and we have neglected the associated term in (18).

The MUSIC algorithm is a subspace-based algorithm that estimates the parameter of interest using the eigenvector to receive signals. To estimate the CFO, observe that the covariance matrix of $\mathbf{r}_k$ can be expressed as

$$\mathbf{R} = \mathbf{E}[\mathbf{r}_k \mathbf{r}_k^H] = \mathbf{S}(\epsilon) \Sigma \mathbf{S}(\epsilon)^H,$$

(19)

where $\Sigma = \text{diag}(\sigma_1^2, \ldots, \sigma_{L_p}^2)$ with $\sigma_m = \mathbf{E}[d_{m,k}^2]$, and we have ignored the noise term in (19) for simplicity. Eq. (19) shows that the signature matrix $\mathbf{S}(\epsilon)$ shares the same column-space with $\mathbf{R}$. Based on this, the MUSIC algorithm uses the eigenvector of $\mathbf{R}$ to estimate the CFO. The eigendecomposition of $\mathbf{R}$ can be expressed as

$$\mathbf{R} = \begin{bmatrix} \mathbf{E}_s & \mathbf{e}_n \end{bmatrix} \begin{bmatrix} \Lambda_s & 0 \\ 0 & \Lambda_d \end{bmatrix} \begin{bmatrix} \mathbf{E}_s^H \\ \mathbf{e}_n^H \end{bmatrix},$$

(20)

where $\mathbf{E}_s = [\mathbf{e}_{s,1}, \ldots, \mathbf{e}_{s,L_p}]$ denotes the signal subspace matrix of $\mathbf{R}$ in which $\mathbf{e}_{s,m}$ is the $m$th eigenvector corresponding to the non-zero eigenvalue $\lambda_m$, $\Lambda_s = \text{diag}(\lambda_1, \ldots, \lambda_{L_p})$, and $\mathbf{e}_n$ is the eigenvector corresponding to zero eigenvalue. The columns of $\mathbf{E}_s$ are also the bases of the column-space of $\mathbf{R}$. Because the columns of $\mathbf{E}_s$ and $\mathbf{e}_n$ are mutually orthogonal, the MUSIC algorithm defines the cost function for the CFO estimation as

$$P(\omega) = \frac{1}{\|\mathbf{S}(\omega)^H \mathbf{e}_n\|}.$$  

(21)
where $\| \cdot \|$ denotes the 2-norm of the embraced vector. The cost function $P(\omega)$ is generally called the pseudo-spectrum of the MUSIC algorithm. The CFO estimate can be obtained by finding the argument that maximizes $P(\omega)$ as

$$\hat{\omega} = \arg \max_{\omega} P(\omega). \quad (22)$$

In practical realization, the MUSIC algorithm approximates the covariance matrix $\mathbf{R}$ through its sample average counterpart as

$$\mathbf{R} = \frac{1}{N} \sum_{k=0}^{N-1} \mathbf{r}_k \mathbf{r}_k^H. \quad (23)$$

This sample averaged covariance matrix approaches $\mathbf{R}$ in (19) with an approximation error reciprocal to the number of sample vectors [28]. Therefore, a large number of $N/L$ in (23) are required to obtain a precise estimate of $\mathbf{R}$. The proposed approach can easily achieve this requirement even using a single OFDM symbol because the ratio between the OFDM symbol size and the ICI self-cancellation order, $N/L$, is generally large.

The computational complexity of the MUSIC algorithm includes the eigen-decomposition of $\mathbf{R}$ requiring $O(L^3)$ calculations plus $O(L^2)$ calculations for each grid point in the pseudo-spectrum searching. This computational overload is practically acceptable because the order of the ICI self-cancellation scheme $L$ is generally small for saving spectral efficiency.

### 3.2.2. The CSC

Based on the CFO estimate, the proposed approach can develop a set of subcarrier combiners to suppress the intra-group interference. The $m$th subcarrier combiner can be expressed by a constrained optimization problem as

$$\mathbf{w}_m = \arg \min_{\mathbf{w}} \mathbb{E}[\|\mathbf{w}^H \mathbf{r}_k\|^2],$$

subject to $\mathbf{C}_m \mathbf{w} = \mathbf{f}$ for $m = 0, \ldots, L_p - 1 \quad (24)$

where $\mathbf{C}_m = [s_m(\hat{\omega}), s_m(\hat{\omega})_v]_{\forall v \neq m, \hat{\omega}}$, $\mathbf{r}_k$ is the response vector with $\mathbf{r}_{L_p - 1}$ being a zero vector of size $L_p - 1$. Specifically, the subcarrier combiner in (24) uses the spectral signature $s_m(\hat{\omega})$ to retain the signal of interest, but suppresses the intra-group interference with signatures $[s_m(\hat{\omega})]_{\forall v \neq m}$. Direct manipulations readily yield

$$\mathbf{w}_m = \mathbf{R}^{-1} \mathbf{C}_m^{-1} (\mathbf{C}_m^H \mathbf{R}^{-1} \mathbf{C}_m) \mathbf{f}. \quad (25)$$

After applying (25) to (17), the output of the $m$th subcarrier combiner becomes

$$\tau_{m,k} = \mathbf{w}_m^H \mathbf{r}_k = \mu_m d_{m,k} + \sum_{m \neq m} \bar{\mu}_m d_{m,k} + \sum_{k \neq k} \xi_{k,m} \mu_{m,k} d_{m,k} + \bar{z}_{m,k}, \quad (26)$$

$$\tau_{m,k} = d_{m,k} + \bar{z}_{m,k}. \quad (27)$$

where $\mu_m = \mathbf{w}_m^H s_m(\hat{\omega})$ is the response constant of the desired signal, which is approximately to be 1, when $\hat{\omega} = \omega$. The term $\bar{\mu}_m = \mathbf{w}_m^H s_m(\hat{\omega})$, $m \neq m$, is the residue of the intra-group interference, $\mu_{m,k} = \mathbf{w}_m^H \tilde{s}_{m,k}(\hat{\omega})$ denotes the residue of the inter-group interference, and $\bar{z}_{m,k}$ is the resulting noise. Data decoding for the $m$th data in subcarrier group $k$ is then performed on $\tau_{m,k}$.

Based on (26), the carrier-to-interference power ratio (CIR) of the proposed CSC can be expressed as

$$\text{CIR} = \frac{\mathbb{E}[|\mu_m d_{m,k}|^2]}{\sum_{m \neq m} |\mu_m|^2 + \sum_{k \neq k} \sum_{m} \xi_{k,m} |\mu_{m,k}|^2 |\mu_{m,k}|^2.} \quad (28)$$

4. Computer simulations

This section presents the results of computer simulations to assess the performance of the proposed approach. The OFDM symbol size is $N = 256$, and the length of CP insertion is $N_{CP} = 15$. The differential-phase-shift-keying of size 8 (8-DPSK) is employed as the modulation scheme. This study is based on the assumption of a Rayleigh fading FIR channel with 10 taps. The power profile of the channel taps is $\{\gamma_l\}_{l = 0, \ldots, 9} = \{0.36, 0.42, 0.65, 0.86, 0.62, 0.45, 0.29, \ldots 0.12, 0.07, 0.02\}$, where $\gamma_l$ denotes the variance of tap $l$. The channel response is assumed to be constant over the time duration of one OFDM symbol, and the noise spectral density $N_0$ is adjusted to achieve the specified energy per data symbol over noise spectral density ($E_s/N_0$).

Fig. 5 shows the symbol-error-rate (SER) comparisons of the proposed approach, Zhao’s method [10], and Chang’s method [12]. The order of the proposed precoder is $L = 5$, and the loading factor is $L_o = 4$. The order of both Zhao’s method and Chang’s method is 2 to achieve their maximal spectral efficiency. Thus, under the 8-DPSK modulation scheme, the proposed approach has a transmission throughput of 2.4 bits/subcarrier compared to the 1.5 bits/subcarrier of both Zhao’s method [10] and Chang’s method [12]. This shows that, for $L = 5$, the proposed approach can improve the spectral efficiency by 60% compared to the conventional methods. Fig. 5 shows that, in addition to improving the spectral efficiency, the proposed approach is significantly superior to the conventional methods in

![Fig. 5. Comparisons of the SER of the proposed approach.](image-url)
SER because of the proposed CSC technique. Fig. 5 shows that the proposed approach survives when $\varepsilon = 0.85$, whereas both Zhao's method and Chang's method fail to resist the ICI in the environment of a large CFO.

Fig. 6 presents a comparison of the SERs of the proposed approach corresponding to various loading factors $L_p$ to investigate the influence of the intra-group and the inter-group interference when $\varepsilon = 0.85$. Fig. 6 shows that the SER of the proposed approach increases in the high $E_s/N_0$ region as $L_p$ increases. This is because a heavily loaded system incurs high interference residues, including both the intra-group and the inter-group interference, in the CSC output.

Fig. 7 shows the pseudo-spectra of the proposed MUSIC algorithm. Three CFOs, $\varepsilon_1 = 0.15$, $\varepsilon_2 = -0.65$, and $\varepsilon_3 = 1.15$, were used to demonstrate the CFO estimation under $E_s/N_0 = 10$ dB. Fig. 7 shows that the pseudo-spectra can effectively identify the target CFOs even when the CFO exceeds the subcarrier space.

Fig. 8 shows the mean-square-errors (MSEs) of the CFO estimate corresponding to the precoder orders, $L_p = 2$, 4, and 6. The target CFO in this figure is $\varepsilon = 0.85$, and the loading factor for the three cases is $L_p = L - 1$. Fig. 8 shows that, in the three cases, the proposed CFO estimates have approximately the same performance in MSE.

5. Conclusion

This study proposes an ICI self-cancellation scheme that achieves a high spectral efficiency in an OFDM system. The proposed approach improves the spectral efficiency of the OFDM by mapping $L_p \leq L - 1$ data symbols to a group of $L$ consecutive subcarriers. Using the response of the ICI factors, this study classifies the ICIs into the intra-group interference and the inter-group interference. The proposed approach employs a set of kernel-based weight sequences to suppress the inter-group interference at the transmitter, and uses a set of CSCs to eliminate the intra-group interference at the receiver. Based on the precoded OFDM signal, this study uses the MUSIC algorithm to estimate the CFO in the frequency domain. Based on the CFO estimate, the proposed approach can eliminate the intra-group interference using the CSCs. Computer simulations show that, unlike conventional ICI self-cancellation methods, which have a limited spectral efficiency of less than 0.5, the proposed ICI self-cancellation scheme can effectively eliminate the large-CFO-caused ICI through the CSC and achieve a higher spectral efficiency of $(L - 1)/L$.

References


