A Variable Neighbourhood Descent Algorithm for the Redundancy Allocation Problem

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Abstract. This paper presents the first known application of a meta-heuristic algorithm, variable neighbourhood descent (VND), to the redundancy allocation problem (RAP). The RAP, a well-known NP-hard problem, has been the subject of much prior work, generally in a restricted form where each subsystem must consist of identical components. The newer meta-heuristic methods overcome this limitation and offer a practical way to solve large instances of the relaxed RAP where different components can be used in parallel. The variable neighbourhood descent method has not yet been used in reliability design, yet it is a method that fits perfectly in those combinatorial problems with potential neighbourhood structures, as in the case of the RAP. A variable neighbourhood descent algorithm for the RAP is developed and tested on a set of well-known benchmark problems from the literature. Results on 33 test problems ranging from less to severely constrained conditions show that the variable neighbourhood descent method provides comparable solution quality at a very moderate computational cost in comparison with the best-known heuristics. Results also indicate that the VND method performs with little variability over random number seeds.

Keywords: variable neighbourhood descent, redundancy allocation problem, series-parallel system, combinatorial optimisation

1. INTRODUCTION

The most studied configuration of the redundancy allocation problem (RAP) is a series system of $s$ independent $k$-out-of-$n$: $G$ subsystems. Because of the need for reliability and increased security requirements, a series-parallel system has been widely used. The RAP is NP-hard (Chern 1992) and has been studied in many forms as summarized in Tillman et al., (1977a), and by Kuo and Prasad (2000).

As shown in Figure 1, a $k$-out-of-$n$: $G$ subsystem $i$ is functioning properly if at least $k_i$ of its $n_i$ components are operational. In the formulation of a series-parallel system problem, for each subsystem, multiple component choices are used in parallel, and each subsystem may have different component selections. Thus, the RAP can be formulated to select the optimal combination of components and redundancy levels to meet system level constraints, cost of $C$ and weight of $W$ while maximizing system reliability. It is assumed that system weight and system cost are represented by linear combinations of component weight and cost.

$$\text{Max} \quad R = \prod_{i=1}^{s} R_i(y_i, k_i)$$

Subject to

$$\sum_{i=1}^{s} C_i(y_i) \leq C,$$  \hspace{1cm} (2)

$$\sum_{i=1}^{s} W_i(y_i) \leq W,$$  \hspace{1cm} (3)

In addition, if the maximum number of components allowed in parallel is pre-determined, the following constraint is added:

$$k_i \leq \sum_{j=1}^{y_i} n_{ij} \leq n_{i \text{max}} \quad \forall i = 1, 2, \ldots, s.$$  \hspace{1cm} (4)

Figure 1. Series-parallel system configuration
1.1 Notations and Assumptions

$k$ minimum number of components required to function as a pure parallel system
$n$ total number of components used in a pure parallel system
$k$-out-of-$n$: $G$ a system that functions when at least $k$ of its $n$ components function
$R$ overall reliability of the series-parallel system
$C$ system cost constraint
$W$ system weight constraint
$S$ number of subsystems
$i$ index for subsystem, $i = 1, \ldots, s$
$j$ index for components in a subsystem
$a_i$ number of available component choices for subsystem $i$
$r_{ij}$ reliability of component $j$ available for subsystem $i$
$c_{ij}$ cost of component $j$ available for subsystem $i$
$w_{ij}$ weight of component $j$ available for subsystem $i$
$y_{ij}$ quantity of component $j$ used in subsystem $i$
$N_i$ total number of components used in subsystem $i$
$n_i = \sum_{j=1}^{a_i} y_{ij}$, total number of components used in subsystem $i$
$n_{max}$ maximum number of components that can be in parallel (user assigned)
$k_i$ minimum number of components in parallel required for subsystem $i$ to function
$R_i(y_i | k_i)$ reliability of subsystem $i$, given $k_i$
$C_i(y_i)$ total cost of subsystem $i$
$W_i(y_i)$ total weight of subsystem $i$
$R_u$ un-penalized system reliability of solution $u$
$R_u^p$ penalized system reliability of solution $u$
$C_u$ total system cost of solution $u$
$W_u$ total system weight of solution $u$
$\gamma_C$ amplification parameter of cost constraint in the penalty function
$\gamma_W$ amplification parameter of weight constraint in the penalty function
$l_{max}$ number of neighbourhood structures
$l$ index for neighbourhood structures
$N_l$ the $l^{th}$ neighbourhood structure in the search sequence
$y$ current solution
$y'$ the best neighbouring solution of $y$ in a neighbourhood structure
$e$ constant that controls the lower bound of number of components used in parallel
$f$ constant that controls the upper bound of number of components used in parallel

Typical assumptions are considered as follows:
- The state of the components and the system either function or fail.
- Failed components do not damage the system, and are not repaired.
- The failure of a component does not lead to other components failing.
- Components are active redundant, i.e., the failure rates of components when not in use are the same as when in use.
- Component reliability, weight and cost are known and deterministic.
- The supply of components is unlimited.

1.2 Literature Review

Exact methods of the RAP include dynamic programming (Bellman and Dreyfus 1958, Fyffe et al. 1968, Nakagawa and Miyazaki 1981, Li 1996), integer programming (Ghare and Taylor 1969, Bullfin and Liu 1985, Misra and Sharma 1991, Coit and Liu 2000), and mixed-integer and nonlinear programming (Tillman et al. 1977b). Since exact methods in practice are limited to the increase in problem size, meta-heuristics have become a popular alternative to exact methods. Meta-heuristic approaches to the RAP vary among Genetic Algorithm (GA) (Coit and Smith 1996a,b, Levitin et al. 1998), Tabu Search (TS) (Huang et al. 2002, Kulturel-Konak et al. 2003), Ant Colony Optimization (ACO) (Liang and Smith 1999, Huang et al. 2002, Liang 2001, Liang and Smith 2004), hybrid Neural Network (NN) and GA (Coit and Smith 1996c), and hybrid ACO with TS (Huang 2003). In particular, Levitin et al. (1998) generalize a redundancy allocation problem to multi-state systems, where the system and its components have a range of performance levels—from perfectly functioning to complete failure. Levitin’s paper implements a universal moment generating function to estimate system performance (capacity or operation time), and a GA is employed as the optimization technique. Furthermore, considering the problem type for maximizing system reliability, past studies such as TS proposed by Kulturel-Konak et al. (2003), GA developed by Coit and Smith (1996a), and ACO by Liang and Smith (2001, 2004) have provided comparable results over a set of 14-subsystem benchmark problems. Huang et al. (2002, 2003) proposed ACO, TS, and hybrid ACO/TS algorithms respectively to solve the system cost minimization RAP.

Our study considers one of the latest meta-heuristics, Variable Neighbourhood Descent (VND), to solve the system reliability maximization RAP. A variation of the variable neighbourhood search (VNS) method was introduced first by Mladenović (1995). The VND method explores search space based on the systematic change of