Deformation and profile measurement using the
digital projection grating method

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Abstract

This paper presents a digital projection grating method for full field measurement of out-of-plane deformation and shape of an object. Two grating patterns on an object before and after deformation are captured by a CCD camera and stored in a computer. With the aid of Fast Fourier Transform (FFT) and signal demodulating techniques, a wrapped phase map is generated. The phases are expanded in the range of \(0 - 2\pi\) and compared with the resulting moiré pattern. An unwrapping procedure is used to obtain a continuous phase. In addition, a digital method for fractional fringe multiplication is also developed. Results on deformation and object profile measurements are presented. © 1998 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The white light grating projection method has been widely used to measure object shape [1–4] and out-of-plane deformation [5–7]. This method has the advantages of being full-field, non-contacting, and is able to measure objects of large sizes. However, the sensitivity of this method is often limited by the pitch of the projected grating when the moiré fringe is directly used to analyse contour or deformation data. In previous works [8–12], there have been some attempts to improve the sensitivity and accuracy of the method by incorporating computer vision and the Fourier transform algorithm.

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This paper discusses a study to further improve the sensitivity of the digital projection grating method. Two grating patterns are projected on an object and the images are captured by a CCD camera. The analysis which was carried out using a PC involves the use of a FFT algorithm to generate a wrapped phase map. A phase expansion method and an unwrapping algorithm are utilised to obtain the actual phases. The sensitivity of the method is also discussed.

2. Method

2.1. Relationship between grating phase and out-of-plane deformation

As shown in Fig. 1, a parallel grating with cosine function modulated intensity is projected on a reference plane. This grating pattern can be captured by a digital camera and stored into a computer. \( A_1 \) and \( A_2 \) are two points on two neighbouring fringes projected on an undeformed reference plane. The pitch of the projected fringes (between points \( A_1 \) and \( A_2 \)) is \( p \) and the incident angle is \( \alpha \). The observation direction is normal to the reference plane. When the object is deformed (as shown in Fig. 1) a point \( Q \) on the object surface would have two corresponding shadow points on the reference plane, in the observation direction (point \( R \)) and the projection direction (point \( S \)). Using point \( A_2 \) as a reference, the phase angle \( \phi_0 \) of point \( R \) on the reference plane is

\[
\phi_0 = 2\pi A_2 R / p
\]

and the phase angle \( \phi \) of point \( S \) is

\[
\phi = 2\pi A_2 S / p
\]

As the projected light has a cosine function, and \( QS \) is parallel to the incident direction, point \( Q \) on the object and point \( S \) on the reference surface would have the same phase angle. From simple geometry, the out-of-plane displacement \( RQ \) is given by

\[
RQ = RS / \tan \alpha
\]

Substituting Eqs. (1) and (2) into Eq. (3a) we have

\[
RQ = p(\phi - \phi_0) / 2\pi \tan \alpha
\]

Thus from Eq. (3b), the out-of-plane deformation or the profile can be obtained if the phase difference between the reference and the deformed grating can be measured.

2.2. Phase analysis by the FFT method

As shown in Fig. 1, a reference grating is projected on the reference plane and the light intensity of the fringes can be expressed as:

\[
I_1(x, y) = a(x, y) + b(x, y) \cos[2\pi f_0 x + \phi_0(x, y)]
\]
where \( f_0 = 1/p \) and \( b(x, y) \) are the spatial frequency and amplitude of the gratings, respectively, \( \varphi_0(x, y) \) is an initial undeformed phase and \( a(x, y) \) represents the background intensity. When an object is placed against the reference plane, the projection grating is deformed resulting in a phase change \( \varphi(x, y) \). The intensity of the deformed grating becomes

\[
I_2(x, y) = a(x, y) + b(x, y)\cos(2\pi f_0 x + \varphi_0(x, y) + \varphi(x, y))
\] (5a)

This expression shows that the “signal” \( \varphi(x, y) \) is modulated by a constant high-frequency signal \( f_0 \). To obtain the deformation phase, a demodulating technique is used.

Eq. (5a) can also be written as

\[
I_2(x, y) = a(x, y) + c(x, y)\exp(j2\pi f_0 x) + c^*(x, y)\exp(-j2\pi f_0 x)
\] (5b)

where \( c(x, y) = [b(x, y)/2]\exp\{j[\varphi_0(x, y) + \varphi(x, y)]\} \) and \( c^*(x, y) \) is the complex conjugate of \( c(x, y) \). The Fourier transform of \( I_2(x, y) \) with respect to \( x \) becomes

\[
F[I_2(x, y)] = A(f, y) + C(f - f_0, y) + C^*(f + f_0, y)
\] (6)

where the \( F, A \) and \( C \) represent the Fourier spectra, \( C^* \) represents the complex conjugate of \( C \) and \( f \) represents the spatial frequency in the \( x \) direction. As the frequencies of \( a(x, y) \), \( b(x, y) \) and \( \varphi(x, y) \) are much lower than \( f_0 \), the function \( C(f - f_0, y) \) can be isolated by a filtering process centred on \( f_0 \). Taking the inverse
Fourier transform of \( C(f - f_0, y) \) with respect to \( x \), we obtain \( c(x, y)\exp(j2\pi f_0 x) \). The deformed grating phase \( \Phi(x, y) \) becomes

\[
\Phi(x, y) = 2\pi f_0 x + \varphi_0(x, y) + \varphi(x, y)
\]

\[
= \arctan \left[ \frac{\text{Im}\{c(x, y)\exp(j2\pi f_0 x)\}}{\text{Re}\{c(x, y)\exp(j2\pi f_0 x)\}} \right]
\]

\[
= \arctan \frac{G}{F}
\]  

(7)

where \( F = \text{Re}\{c(x, y)\exp(j2\pi f_0 x)\} \) and \( G = \text{Im}\{c(x, y)\exp(j2\pi f_0 x)\} \) represent the real and imaginary parts of \( c(x, y)\exp(j2\pi f_0 x) \), respectively.

The distribution of the phase angles \( \Phi(x, y) \) is in the range of \(-\pi/2 - \pi/2\). However in reality, the phase change corresponding to one fringe is in the range of \( 0 - 2\pi \). Hence, the phase angles calculated using Eq. (7) should be expanded. Considering the signs of \( G \) and \( F \), the phase angles obtained using Eq. (7) can be expanded such that they are in the range of \( 0 - 2\pi \). The values of \( \Phi'(x, y) \) for various values of \( G \) and \( F \) are

\[
\Phi'(x, y) = \begin{cases} 
\Phi(x, y), & \text{for } G \geq 0 \text{ and } F > 0 \\
\Phi(x, y) + \pi, & \text{for } F < 0 \\
\Phi(x, y) + 2\pi, & \text{for } G < 0 \text{ and } F > 0 \\
3\pi/2, & \text{for } G > 0 \text{ and } F = 0 \\
\pi/2, & \text{for } G < 0 \text{ and } F = 0
\end{cases}
\]

(8)

Similarly, we can obtain the phases of the reference gratings using the above algorithm. Thus, \( \Phi_0(x, y) = 2\pi f_0 x + \varphi_0(x, y) \) and their expanded phases \( \Phi'_0(x, y) \) can be calculated by an equation similar to Eq. (8). Hence, the out-of-plane deformation phases, which are distributed in the range of \( 0 - 2\pi \) can be obtained:

\[
\phi(x, y) = \begin{cases} 
\Phi'(x, y) - \Phi_0(x, y), & \text{for } \Phi'(x, y) \geq \Phi'_0(x, y) \\
\Phi'(x, y) - \Phi_0(x, y) + 2\pi, & \text{for } \Phi'(x, y) < \Phi'_0(x, y)
\end{cases}
\]

(9)

When the above procedure is applied to the whole image, the phase map of the object is obtained. The above analysis utilises the subtraction method which results in better accuracy than the previous method [8] which utilises shifting of the Fourier spectra and hence its accuracy is dependent on the digital frequency resolution.

The grey values of the frame grabber varies between 0 and 255 and the light intensity \( (K) \) of the phase map can be generated using the expression

\[
K(x, y) = 255\phi(x, y)/2\pi
\]

(10)

The phase map obtained using Eq. (10) is still in the wrapped configuration. To obtain a continuous phase configuration, an unwrapping procedure is used.
2.3. Phase unwrapping analysis

The phases obtained using Eq. (10) is in the wrapped configuration. To unwrap the phase map, the following algorithm is developed. Considering an arbitrary point \( a(m, n) \) on a fringe which is assumed to have zero order, i.e. \( N(m, n) = 0 \). A scanning process is then carried out to determine the fringe order for the rest of the data points. Using a threshold value of \( T \) as an indication of phase discontinuity, the image data is scanned along each column and row. The fringe order \( N \) are determined using the following algorithm:

2.3.1. Vertical scanning

(a) Vertical downward scanning

From the reference point \( a(m, n) \) at the \( n \)th column, all the data points below the reference point will assume the following fringe order \( (N) \) values:

\[
N_{i,n} = \begin{cases} 
N_{i-1,n} & \text{for } |a_{i,n} - a_{i-1,n}| \leq T \\
N_{i-1,n} - 1 & \text{for } a_{i,n} - a_{i-1,n} > T \\
N_{i-1,n} + 1 & \text{for } a_{i,n} - a_{i-1,n} < -T 
\end{cases} 
\]

\((i = m + 1, m + 2, \ldots, 512)\)

(b) Vertical upward scanning

Similarly, for data points above the reference point, all data points will assume the following fringe order values

\[
N_{i,n} = \begin{cases} 
N_{i+1,n} & \text{for } |a_{i,n} - a_{i+1,n}| \leq T \\
N_{i+1,n} - 1 & \text{for } a_{i,n} - a_{i+1,n} > T \\
N_{i+1,n} + 1 & \text{for } a_{i,n} - a_{i+1,n} < -T 
\end{cases} 
\]

\((i = m - 1, m - 2, \ldots, 1)\)

2.3.2. Horizontal scanning

Data points are scanned both to the left and right of the \( n \)th column and the following fringe order values are assumed:

(a) Right scanning

\[
N_{i,j} = \begin{cases} 
N_{i,j-1} & \text{for } |a_{i,j} - a_{i,j-1}| \leq T \\
N_{i,j-1} - 1 & \text{for } a_{i,j} - a_{i,j-1} > T \\
N_{i,j-1} + 1 & \text{for } a_{i,j} - a_{i,j-1} < -T 
\end{cases} 
\]

\((i = 1, 2, 3, \ldots, 512; j = n + 1, n + 2, \ldots, 512)\)
When the unwrapping procedure is completed, the deformation phases are obtained from

\[
\Phi_{i,j} = 2\pi N_{i,j} + \varphi_{i,j}
\]

\[(i = 1, 2, 3, \ldots, 512; \ j = n - 1, n - 2, \ldots, 1)\] (12)

Hence a whole field out-of-plane displacement or profile distribution can be obtained.

2.4. Fringe multiplication

When the order of a moiré fringe number is less than unity, it would be difficult to obtain deformation information using the conventional fringe analysis. To obtain and display the fractional fringe order, particularly in areas of rapidly changing profiles or stress gradients, a fringe multiplication method is necessary.

The general method for fringe multiplication [13, 14] involves the use of a Fourier filtering process which alters the diffraction orders on the transform plane. In the present study, the fringe multiplication for obtaining the fractional fringe order is incorporated in the phase analysis. For an original fringe pattern given by

\[
I(x, y) = \sin^2(\varphi(x, y)/2)
\]

the fringe order may be readily increased by incorporating a magnification factor \(k\). Thus Eq. (13) becomes

\[
I'(x, y) = \sin^2(k\varphi(x, y)/2)
\]

(14)

Hence, from Eq. (14), a fringe pattern with higher accuracy and sensitivity can be generated.

3. Experiments and discussion

3.1. Deformation measurement

As shown in Fig. 2, a vertical grating is projected on a reference plane \(AA\) at an incident \(\alpha\) of 26.6° using a white light source and the reference grating is recorded using a CCD camera with a resolution of 0.504 mm/pixel. The test specimen used in this case is a circular flat plate fixed at the boundary and loaded at the centre. The
applied load results in an out-of-plane displacement of 0.51 mm at the centre of the plate. The projected grating pitch ($p$) on the specimen is 5.45 mm. In the conventional projection moiré method, the sensitivity in measuring out-of-plane displacement is about one pitch of a grating. However, using the proposed technique, an out-of-plane displacement of less than one pitch of the grating can be accurately measured. Since the displacement in this case is much smaller than the pitch of the grating, the use of the conventional method would result in a moiré pattern (using digital subtraction method) with an indistinct fractional fringe as shown in Fig. 3. However, using the proposed phase analysing method, a distinct fringe pattern (Fig. 4a with magnification factor $k = 40$) is obtained. To increase the sensitivity, the fringe order can further be multiplied using a higher value of $k$. As shown in Fig. 4b there is a twofold increase in the number of fringes when the magnification factor is increased to 80. Thus, displacement smaller than one pitch is observable.

In the proposed technique, according to the sampling theorem the sensitivity is only dependent on the density of the array of pixels in the CCD camera. For an object of length $L$ with a CCD sensor consisting of $M$ pixels, the measurable out-of-plane displacement ($d$) is given by

$$d = \frac{L}{M \tan \alpha}$$

As an example, for $L = 50$ mm, $M = 512$ pixels and an incident angle $\alpha = 30^\circ$, the measurable out-of-plane displacement $d$ would be 0.17 mm. While using the...
conventional projection moiré method, the minimum measurable out-of-plane displacement would be greater than the pitch of the projected grating which is about 9.44 mm.

3.2. Profile measurement

In addition to deformation measurement, profile can also be measured using a similar procedure as described above. A conical object of 80 mm base diameter and 80 mm height is placed in front of a reference plane and the deformed grating on the object is subsequently recorded using a CCD camera with a resolution of 0.5 mm/pixel in the out-of-plane direction. The incident angle is 25.1°. Processing of the images is carried out using a PC and the phase corresponding to each point on the object is computed using Eq. (9). The computed phase map is shown in Fig. 5a. It is seen that the fringes generated in the phase map are generally smooth. However, on the right-hand side of the object the fringes appear uneven. This is because the deformed grating frequencies on this section of the object are similar to the moiré fringe frequencies and these deformed grating frequencies are not filtered out during the FFT process. The noises generated is thus an inherent limitation of the FFT method. The phase map (Fig. 5a) generated using Eq. (9) is further compared with the moiré fringes shown in Fig. 5b (magnification \( k = 1 \)). As can be seen, the moiré fringes compare well with that of the phase map.
Fig. 4. (a) Moiré fringe pattern with magnification factor ($k = 40$). (b) Moiré fringe pattern with magnification factor ($k = 80$).
Fig. 5. (a) Phase map of a conical surface. (b) Contour fringes without multiplication. (c) Contour fringes with multiplication. (d) Unwrapped phase map of a conical surface.
The sensitivity of the technique can be further enhanced by increasing the value of the magnification $k$. As shown in Fig. 5c, when the magnification $k$ increases from $k = 1$ (Fig. 5b) to $k = 2$ (Fig. 5c), the fringe density is doubled. As mentioned earlier an unwrapped phase map is required to obtain a continuous phase configuration. The unwrapped phase map generated using Eqs. (11a)–(11d) is shown in Fig. 5d.

4. Conclusions

In this study, the digital projection grating method has been applied to the measurement of out-of-plane deformation and object profile. Using the proposed method a highly accurate phase map from two grating patterns can be obtained. The fractional phases are computed by a signal demodulating technique and an expanding algorithm. The proposed technique enables the determination of small deformation with high accuracy and sensitivity.

References