MIQP-BASED MPC IN THE PRESENCE OF CONTROL VALVE STICTION

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ABSTRACT
Stiction in control valve is the most common and long standing problem in process industry, resulting in oscillations in process variables which subsequently lower product quality and productivity. In this paper, we investigate the effect of avoiding the dead-band region of stiction using Mixed-Integer Quadratic Programming (MIQP) based Model Predictive Controller (MPC). Different scenarios of control valve stiction are considered. Simulation studies using an industrial case study of fluid catalytic cracking unit (FCCU) show that, if the dead-band value is known a-priori, the MIQP-based MPC can effectively improve the closed-loop performance in the presence of stiction through the reduction of oscillations in the process variables.

INTRODUCTION
Model Predictive Control (MPC) is widely acknowledged as one of the dominant advanced control algorithms in industry (Qin & Badgwell, 2003). Its applications can be found in a great variety of industrial areas, including petrochemical, pulp and paper, gas pipeline, food processing, automotive, aerospace and metallurgy (Qin & Badgwell, 2003, Ghazzawi et al, 2001). The popularity of MPC may be contributed largely to its ability in handling inputs, states and/or output constraints explicitly. In addition, MPC calculates optimal control input movements online using an explicit internal model while taking into account the process behavior over a future, finite time horizon (Maciejowski, 2002). However, a potential limitation of MPC design methods is its sensitivity towards model uncertainties. The uncertainties are due to the modeling errors of the process itself, sensor dynamics, and/or its actuator, e.g. control valves.

Control valves constitute an important element in chemical process control systems. Through a control valve, control actions are implemented on the process. They manipulate energy flows, mass flows or forces as a response to low energy input signals, for example, electrical voltages or currents, pneumatic and hydraulic pressures or flows. Due to their continuous operations, control valves tend to undergo wear and aging. In general, they contain static and dynamic nonlinearities including saturation, backlash, stiction, dead-band and hysteresis (Zabiri & Samyudia, 2006). A survey reported in (Miller & Desborough, 2001) found that 30% of the loops were oscillatory due to control valve problems.

Mixed-Integer Quadratic Programming (MIQP)-based design of MPC (Zabiri & Samyudia, 2006) is one of the recent approaches that have been developed to compensate for, in particular, backlash nonlinearity in the control valve. This method involves the integration of the actuator backlash approximate nonlinear inverse and its
associated dead-band values within the framework of MPC controller algorithm itself. This method has been shown to be able to handle effectively actuator saturation and backlash nonlinearities simultaneously. However, among many types of nonlinearities in control valves, stiction is the most commonly encountered in the process industry (Choudhury et al, 2005, Kano et al, 2004, Kariwala et al, 2004). Stiction in general is a phenomena that describes the valve’s stem (or shaft) sticking when small changes in valve movements are attempted. It may be considered as the generalized version of backlash, i.e., a pure dead-band type of stiction is similar in behavior to backlash.

Stiction causes fluctuation of process variables in a more serious manner than backlash due to the presence of the stick-band and slip-jump regions. The variability of process variables makes it difficult to keep operating conditions close to their constraints, and hence causes excessive or unnecessary energy consumption. Since stiction and backlash nonlinearities both contain dead-band region, it is interesting to investigate whether the MIQP-based MPC can minimize or reduced stiction deleterious effect from being propagated. The outline of this paper is as follows: Section II describes stiction and its model used in this study. In Section III, the MIQP-based MPC formulation is described. The results and discussions are presented in Section IV. Finally, conclusions are made in Section V.

STITION AND STITION MODEL

For the purpose of simulation study, the stiction model developed by Choudhury et al (2005) is used. Fig. 1 shows the flow chart for the data-driven stiction model as developed by Choudhury et al (2005). Fig. 2 illustrates the input-output behavior for control valve with stiction. The dashed line represents the ideal control valve without any friction. Stiction consists of primarily of dead-band, stick-band, slip jump and the moving phase. For control valve under stiction resting at point (a), the valve position remains unchanged even when the controller output increases due to the dead-band caused by the static friction. Only when the controller output exceeds the maximum static frictional force, $f_S$, the valve starts to respond (point(b)). A slip jump of magnitude $J$ is incurred when the valve starts to move at point (b) when the frictional force $f_S$ converts to kinetic force $f_D$. From (c) to (d), the valve position varies linearly. The same scenario happens when the valve stops at point (d), and when the controller output changes direction. Parameter $S$ represents the dead-band plus stick-band regions.

MIQP-BASED MPC DESIGN

Fig. 3 describes an input-output map of the actuator backlash. The backlash input at any time $t$, $u(t)$, may be either positive or negative value, with respect to the backlash axis. Two quantities to represent these situations are defined; (i) if $u(t)$ is inside the backlash and positive, the distance from the positive boundary is defined as $d_p$, and (ii) if $u(t)$ is inside the backlash and negative, the distance from its negative boundary is defined as $d_n$. Both $d_p$ and $d_n$ can be calculated as follows:
\begin{equation}
\begin{aligned}
  d_p(t) &= \left[ \frac{u_p(t)}{m_b} + d \right] - u_f(t) \\
  d_n(t) &= \left[ \frac{u_p(t)}{m_b} + (-d) \right] - u_f(t)
\end{aligned}
\end{equation}

Fig. 1: Signal and flow chart for the data driven stiction model developed by Choudhury et al (2005).
The mathematical model of the actuator backlash can then be presented as follows:

\[
\begin{align*}
\Delta u_p(t) = \begin{cases} 
  m_b [u_I(t) - d] : & \Delta u_I(t) > d_p(t-1) \\
  m_b [u_I(t) - (-d)] : & \Delta u_I(t) < d_p(t-1) \\
  u_p(t-1) : & \text{otherwise}
\end{cases}
\end{align*}
\] (2)

The backlash model involves a set of logical rules that represent three regions of the backlash dynamics. The input change \( \Delta u_I(t) \) with respect to the dead-band size \( d \) would determine what the backlash output would be. It should be noted that the backlash is active if \( u_p(t) = u_p(t-1) \), i.e. the input signal is travelling within the dead-band.

The principle strategy of the MIQP-based design of MPC developed by Zabiri & Samyudia (2006) is to include the actuator backlash within the MPC design framework as illustrated in Fig. 4. This is achieved via the reformulation of the input constraints.

With the assumption of unity backlash gradient, the backlash inverse model can be written as:
Using the simplified backlash inverse model above, Zabiri & Samyudia (2006) introduced a set of logical variables \( \delta_{ij} \), for \( j=1,2,3 \), for representing the input change conditions (e.g. positive, negative, or zero). The following propositional logics is imposed on the inputs \( \Delta u(k) \):

\[
\sum_{j=1}^{3} \delta_{ij} = 1
\]  

where for \( i=1,2, \ldots, m \), with \( m \) is the number of manipulated inputs in the system.

The MPC formulation is now imposed with the following set of constraints for the \( i^{th} \) input (assuming unconstrained outputs):

\[
\begin{align*}
    u_{i,\text{min}} & \leq u_i(k) \leq u_{i,\text{max}} \\
    \Delta u_{i,\text{min}} & \leq \Delta u_i(k) \leq \Delta u_{i,\text{max}} \\
    \delta_{i,1} & \leftrightarrow \Delta u_i(k) \geq d_i \\
    \delta_{i,2} & \leftrightarrow \Delta u_i(k) \leq -d_i \\
    \delta_{i,3} & \leftrightarrow \Delta u_i(k) = 0
\end{align*}
\]  

The propositional logics specified in (3) are then transformed into a set of mixed-integer linear inequalities, i.e. linear inequalities involving both continuous variables \( u \in \mathbb{R}^m \) and logical variables \( \delta \in \{0,1\} \), by adopting the framework described in Bemporad & Morari (1999). Let us define:

\[
\begin{align*}
    u_{i,\text{max}} &= u_{i,\text{max}} - u(k-1) \\
    u_{i,\text{min}} &= u_{i,\text{min}} - u(k-1)
\end{align*}
\]  

Combining (6) with (4-5), the final MPC optimization problem is subjected to the full constraints of:

\[
\begin{align*}
    \Delta u_{i,\text{min}} & \leq \Delta u_i(k) \leq \Delta u_{i,\text{max}} \\
    \delta u_i(k) - d_i & \geq (u_{i,\text{min}} - d_i)(1 - \delta_{i,1}) \\
    \delta u_i(k) - d_i & \leq (u_{i,\text{max}} - d_i)\delta_{i,1} \\
    \delta u_i(k) + d_i & \geq (u_{i,\text{min}} + d_i)(1 - \delta_{i,2}) \\
    \delta u_i(k) + d_i & \leq (u_{i,\text{max}} + d_i)\delta_{i,2} \\
    \delta u_i(k) & \geq u_{i,\text{min}}(1 - \delta_{i,3}) \\
    \delta u_i(k) & \leq u_{i,\text{max}}(1 - \delta_{i,3})
\end{align*}
\]  

The final MPC optimization problem is expressed as:
\[
\min_{z} z^T Qz + b^T z \\
\text{s.t.} \\
Cz + d \leq 0 \\
z = \begin{bmatrix} z_c \\ z_d \end{bmatrix}; \quad z_c \in \mathbb{R}^{n_c} \\
z_d \in \{0,1\}^{n_d}
\]

where \(Cz + d \leq 0\) represents the mixed linear inequalities of (7), and \(z_c\) are the continuous variables \(u\) and \(z_d\) are logical variables \(\delta_i\). The problem defined by (8) is a Mixed Integer Quadratic Programming (MIQP) optimization problem. As pointed earlier, this optimization problem has shown successful implementation for backlash cases. In the next section, the capability of the developed method to generalize to the stiction nonlinearity is investigated. The software used for the simulation purposes are MATLAB/SIMULINK, and the optimization problem is solved by interfacing between MATLAB/SIMULINK with GAMS/SBBQP.

**AN INDUSTRIAL FCCU CASE STUDIES**

There are two main characteristics of stiction that normally give rise to a poor control loop performance, namely: (1) its dead-band - the larger the deadband width, the stronger the stiction effect would be, and (2) its slip-jump - the larger the jump, the worse the impact would be (Kariwala et al., 2004). In the absence of slip-jump region, stiction essentially behaves as a pure dead-band or backlash (please refer to Fig. 2). As shown in the earlier section, MIQP-based MPC formulation incorporates the dead-band value within its framework. It would be interesting to study whether by considering only the dead-band region in the MIQP-based MPC can help in reducing the stiction effect to the control loop performance or not. Based on this idea and using the industrial fluidized catalytic cracking unit, the effectiveness of the method will be demonstrated. The four cases of stiction are investigated, namely, dead-band \((J=0)\), stiction undershoot \((S>J)\), stiction no offset \((S=J)\) and stiction overshoot \((S<J)\).

The industrial FCC unit is taken from Grosdidier et al. (1993), where the unit operates under a full combustion mode and MPC is cascaded with some PI controllers. Three PI controllers were applied in the flows of the combustion air, the hot gas oils and the combined cold gas and recycle oils \((u_1, u_2, u_3)\). Two PI controllers were to control the feed preheat \((u_4)\) and the riser outlet temperature \((u_5)\). Recycle flow \((u_6)\) is regulated by adjusting the output of a hand controller. The MPC is used to control seven variables by manipulating six input variables, please refer to Table 1. In all the cases below, stiction is assumed to exist in the hot gas oil flow \((u_2)\) control loop. It is assumed that the dead-band value of the stiction is known \textit{a-priori}.
Fig. 5: An industrial FCCU adapted from Grosdidier et al (1993).

Tab. 1: FCC unit control variables and limit values

<table>
<thead>
<tr>
<th>Variable description and abbreviation</th>
<th>Tag</th>
<th>Limit values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustion air flow (air)</td>
<td>$u_1$</td>
<td>$140 - 155 \text{T/h}$</td>
</tr>
<tr>
<td>Hot gas oil flow (hot feed)</td>
<td>$u_2$</td>
<td>$190 - 110 \text{m}^3/\text{h}$</td>
</tr>
<tr>
<td>Combined cold gas and recycle oils (cold feet)</td>
<td>$u_3$</td>
<td>$290 - 110 \text{m}^3/\text{h}$</td>
</tr>
<tr>
<td>Feed preheat temperature (feed T)</td>
<td>$u_4$</td>
<td>$230 - 250 ^\circ\text{C}$</td>
</tr>
<tr>
<td>Riser outlet T (riser T)</td>
<td>$u_5$</td>
<td>$4515 - 535 ^\circ\text{C}$</td>
</tr>
<tr>
<td>Recycle oil flow controller output (recycle)</td>
<td>$u_6$</td>
<td>$0.9%$</td>
</tr>
<tr>
<td>Flue gas $\text{O}_2$ concentration ($\text{O}_2$)</td>
<td>$y_1$</td>
<td>$520 - 80 %$</td>
</tr>
<tr>
<td>Regenerator bed T (bed T)</td>
<td>$y_2$</td>
<td>$^*6705 - 735 ^\circ\text{C}$</td>
</tr>
<tr>
<td>Fuel gas flow (fuel gas)</td>
<td>$y_3$</td>
<td>$^*7\text{Max 15 T/h}$</td>
</tr>
<tr>
<td>Wet gas compressor suction pressure controller output (WGC)</td>
<td>$y_4$</td>
<td>$^*8\text{Max 70%}$</td>
</tr>
</tbody>
</table>
### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riser outlet temperature controller output (fresh cat VP)</td>
<td>$y_5$</td>
<td>$y_5^{*9}$ Max 80%</td>
</tr>
<tr>
<td>Regenerated catalyst slide valve pressure differential (fresh cat dP)</td>
<td>$y_6$</td>
<td>$y_6^{*10}$ Min 22 kPa</td>
</tr>
<tr>
<td>Spent catalyst slide valve pressure differential (spent cat dP)</td>
<td>$y_7$</td>
<td>$y_7^{*11}$ Max 24 kPa</td>
</tr>
</tbody>
</table>

In all figures of simulation results, $u_{MPC}$ refers to the control output signal from the MPC and $u_{toplant}$ refers to the signal from the control valve.

### Case Study 1: Deadband ($J=0$)

In this case, $S=0.5$ and $J=0$. These values represent the % of valve travel or the span of the control signal. The value of $d$ supplied to the MIQP-based MPC algorithm is taken to be approximately half of the stiction stick-band value, i.e. $d = 0.25$. Fig. 6 shows the phase-plane plots of the inputs, i.e. $u_{toplant}$ vs. $u_{MPC}$, for (a) ideal case of standard MPC with no stiction, (b) standard MPC in the presence of stiction, and (c) MIQP-based MPC in the presence of stiction under the same step change in the disturbance.

In the case of standard MPC with perfect control valve (i.e. no stiction nonlinearity), a small oscillation is incurred during the transient response under the step disturbance change, however the system eventually reaches its steady-state as shown in Fig. 6(a). When stiction is introduced in the hot feed flow valve ($u_2$) under the control of standard MPC, Fig. 6(b) clearly shows deterioration in the closed-loop performance, where permanent oscillations is suffered and no steady-state is achieved. This continuous oscillatory behaviour suffered by the control valve proves detrimental to the output variable as shown in Fig. 7. (In Fig. 7, “ideal” refers to standard MPC with no stiction, “with stiction” refers to standard MPC with stiction, and “MIQP-MPC” refers to MIQP-MPC with stiction.)
When MIQP-MPC is applied to the system with stiction, significant reduction in the oscillation can be seen as shown in Fig. 6(c). It is seen that the input intelligently reaches its terminal value so that sustained oscillation can be reduced significantly. The corresponding output performance is as shown in Fig. 7 where much of the oscillation in the controlled variable $y_3$ is eliminated.

**Case Study 2: Stiction undershoot ( $S>J$)**

In this case, $S=0.5$, $J=0.3$ and $d=0.25$ are used. Figs. 8 and 9 show the phase plane plot of the inputs and the corresponding output responses, respectively. Again, a similar improved performance is achieved using MIQP-MPC. The MIQP-MPC effectively eliminates the stiction effect and substantially improved the overall closed-loop responses.
Case Study 3: Stiction no offset ($S=J$)
For stiction with no offset, the values of $S$ and $J$ both are set to 0.5. The value of $d$ is again set to 0.25. Similar improved performances are obtained with MIQP-MPC, as shown in Figs. 10 and 11.

Fig. 8: Phase plane plot of $u_2$ for stiction undershoot.

Fig. 9: Output response for $y_5$ under stiction undershoot nonlinearity.

Case Study 4: Stiction overshoot ($S<J$)
For stiction overshoot, $S$ is taken to be 0.3, and $J$ is set to be equal to 0.5. For this case, $d = 0.15$. Figs. 12 and 13 show that the oscillations caused by stiction have been reduced significantly. An acceptable closed-loop performance is regained where the steady-state is subsequently achieved.
CONCLUSIONS
In this paper, it has been shown that the deleterious effect of stiction on the closed loop performance can be significantly reduced if the stick-band (or dead-band) region is avoided. MIQP-based MPC, which accounts for control valve dead-band as part of its formulation, has been shown to successfully improve the closed loop behavior under stiction provided that the stick-band value is known a-priori.

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Fig. 12: Phase plane plot of $u_2$ for stiction overshoot

Fig. 13: Output response for $y_5$ under overshoot nonlinearity

REFERENCES


BRIEF BIOGRAPHY OF PRESENTER

Haslinda Zabiri graduated from Loughborough University, UK with B.Sc. (1998) in Chemical Engineering (Honors). She received M.A.Sc. in Advanced Process Control from McMaster University, Canada in 2004 with a thesis on “MPC design for linear multivariable systems under actuator saturation and backlash”. She is currently a lecturer at the Universiti Teknologi PETRONAS (UTP) since 2004 in the Department of Chemical Engineering, UTP. Her research interests include process modelling, simulation, and advanced process control.