Fast communication

Image denoising using modified Perona–Malik model based on directional Laplacian

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Abstract

The Perona–Malik (PM) model, or the anisotropic diffusion, shows good performance for image restoration. However, it suffers from the so-called staircasing effect if the contrast parameter is small and otherwise cannot preserve edges. This paper proposes a modified Perona–Malik (MPM) model based on directional Laplacian, which diffuses image along the edge direction of the original image. A slightly weighted Laplacian is also integrated to suppress noise. The proposed model can alleviate the staircasing effect, preserve sharp discontinuities, and remove noise simultaneously. Experimental results compared with several relevant methods demonstrate the good performance of the proposed algorithm.

1. Introduction

During the past two decades, variational and PDE-based methods have gained popularity for image restoration [1]. The anisotropic diffusion proposed by Perona–Malik (PM) [2] in 1990 is usually taken as the seminal work on partial differential equations (PDEs) for image restoration. Since its introduction, there has been a great deal of research devoted to the theoretical and practical understanding of this model for image enhancement and restoration [3–9]. For example, Sochen et al. proposed the manifold based diffusion [5], Weickert proposed the structure tensor based diffusion [6] and Black et al. proposed the robust anisotropic diffusion based on robust statistics [7]. Tschumperlé and Deriche proposed a unifying anisotropic diffusion equation based on local filtering with spatially adaptive Gaussian kernels for vector-valued image regularization [8]. Very recently, the anisotropic diffusion was reformulated to denoise band pass signals with a constant frequency and discontinuities [9]. The great success of the PM model can be mainly attributed to its excellent performance in edge preservation and noise removal.

Despite its great success, the PM model suffers from the so-called staircasing effect if the contrast parameter is small, which means it transforms ramps into piecewise constant regions. However, if the contrast parameter is large, the PM model cannot preserve edges. This staircasing effect is visually unpleasant and may be falsely recognized as edges by edge detectors. A detailed analysis of such effects was carried out in [10]. An effective way to overcome this shortcoming is to increase the order of derivatives in the diffusion model, and the fourth-order PDEs have been of special interest in recent years [11–17]. Although the fourth-order PDEs are free of staircasing effect, there are still works on alleviating the staircasing effect in the anisotropic diffusion, such as the ramp preserving Perona–Malik (RPPM) model [18], the difference...
eigenvalue based Perona–Malik (DEPM) model [19], the Sobolev anisotropic diffusion [20], the linear regression method [21], the adaptive PM model [22] and the introduction of gradient fidelity term [23].

In this letter, we also aim at alleviating the staircasing effect and preserving edges simultaneously in the PM model. In contrast to the generalization to fourth-order models [11–17] and the improvements of the PM model [18–23], the proposed model is a direct generalization of the PM model using directional Laplacian. This modified PM (MPM) model takes into account the edge vector of the original image and smoothes the image along the edge direction of the original image so that the edges in the original image can be preserved. Since the edge vector may be noisy, a slightly weighted Laplacian is integrated to suppress noise. Comparisons with several models, such as the DEPM [19], RPPM [18], PM [2], TV [3], and the You–Kaveh’s (YK) fourth order models [12], show that the proposed MPM performs effectively on removing noises, preserving edges, and avoiding staircases.

2. Proposed method

2.1. PM: Perona–Malik model

The basic idea of the PM model is to recover an image \( u(x,y) \) from a given noisy image \( I(x,y) \) by solving the following PDE [2]:

\[
\frac{\partial u}{\partial t} = \text{div}(c(\|\nabla u\|)\nabla u) \\
= c(\|\nabla u\|)\text{div}(\nabla u) + \nabla c(\|\nabla u\|) \cdot \nabla u
\]

with initial condition \( u(x,y,0) = I(x,y) \), where \( \nabla \) is the gradient operator, \( \text{div} \) the divergence, \( \| \cdot \| \) Euclidean norm, and \( c(\cdot) \) the diffusion coefficient, which is positive and non-increasing over \( \|\nabla u\| \). One typical diffusion coefficient defined by Perona and Malik is

\[
c(\|\nabla u\|) = \frac{1}{1 + (\|\nabla u\|/K)^2},
\]

where \( K \) is the contrast parameter and determines the contrast of the edges to be preserved.

Fig. 1. (a) Clear corner image, (b) noisy corner image, the noise is white Gaussian noise of variance 20, (c) filtered image with \( c(\cdot) = 1 \) and (d) filtered image with \( c(x) = 1/\sqrt{1+\pi^2} \). The iteration number is 150.
Fig. 2. (a) Noise free test image, (b) noisy images, results by (c) the YK model, (d) the PM model, (e) the RPPM model, (f) the TV model, (g) the DEPM model, (h) the MPMp with $\alpha=0$, (i) the MPMp with $\alpha=0.05$, and (j) the MPMe with $\alpha=0.05$. In each row, from left to right, the noise variance is 20, 60, and 100, respectively.
2.2. MPM: Modified Perona–Malik model using directional Laplacian

It is clear the anisotropic diffusion in Eq. (1) reduces to the following heat flow when \(c(\cdot)\) is a constant

\[
\frac{\partial u}{\partial t} = c \cdot \text{div}(\nabla u) + Vc \cdot \nabla u = c \cdot (u_{xx} + u_{yy}),
\]

where \(V\) is the gradient vector and \(\text{div}(\nabla u)\) the Laplacian operator. The PM model is essentially a generalization of the heat flow using an inhomogeneous weight \(c(||V||)\). Our proposed MPM model generalizes the PM model using directional Laplacian. Before presenting the proposed MPM model, we first introduce the following flow using directional Laplacian:

\[
\frac{\partial u}{\partial t} = \vec{n} \nabla^T \nabla \vec{n}^T, \tag{4}
\]

where \(\vec{n}\) is an arbitrary unit vector, \(H = \nabla^T \nabla u\) is the Hessian matrix and superscript \(T\) denotes the transposition. \(\vec{n}^T H \vec{n}\) is the second order directional derivative of \(u(x,y)\) along \(\vec{n}\), i.e., directional Laplacian. The diffusion (4) smoothes image \(u(x,y)\) along \(\vec{n}\), therefore, it behaves differently with different \(\vec{n}\) [10]. After the inhomogeneous weight \(c(||V||)\) is further incorporated into Eq. (4), we get

\[
\frac{\partial u}{\partial t} = \vec{n} \nabla^T (c(||V||) \nabla u) \vec{n}^T, \tag{5}
\]

This expression can be further expanded as

\[
\frac{\partial u}{\partial t} = c(||V||) \vec{n} \nabla^T \nabla \vec{n} \vec{n}^T + \vec{n} \nabla^T c(||V||) \nabla u \vec{n}^T = c(||V||) \vec{n} \nabla^T \nabla \vec{n} \vec{n}^T + (\nabla c(||V||) \cdot \vec{n}) (\nabla \cdot \vec{n}) \tag{6}
\]

According to the analysis in [10], there are usually two choices for \(\vec{n}\) by associating it with image \(u(x,y)\):

1. \(\vec{n} = (-u_y, u_x) / \|V\|. \) In this case, the second term in the right-hand side of Eq. (6) is zero since \(V u \cdot \vec{n} = (u_x, u_y) \cdot (-u_y, u_x) / \|V\| = 0. \) As a result, diffusion (6) simplifies to

2. \(\vec{n} = \nabla u / \|V\|. \) In this case, diffusion (6) becomes

\[
\frac{\partial u}{\partial t} = c(||V||) \vec{n} \nabla^T \nabla \vec{n} \vec{n}^T + (\nabla c(||V||) \cdot \vec{n}) (\nabla \cdot \vec{n}) \tag{7}
\]

where \(u_{xx} = (u_{xx} u_{yy}^2 + u_{yy} u_{xx}^2 - 2u_{xx} u_{yy} u_{xy}) / (u_{xx}^2 + u_{yy}^2)\) is the second order derivative of image \(u(x,y)\) along the edge direction of \(u(x,y)\), and it tends to preserve edges in image \(u(x,y)\) [10]. When \(c(||V||) = 1 / ||V||\), diffusion (7) becomes the well-known TV model [3]. This fact means that the TV model can be formulated using directional Laplacian. The TV model tends to preserve edges in image \(u(x,y)\), however, it suffers from the so-called staircasing effect.

\[
\frac{\partial u}{\partial t} = c(||V||) \vec{n} \nabla^T \nabla \vec{n} \vec{n}^T + (\nabla c(||V||) \cdot \vec{n}) (\nabla \cdot \vec{n}) \tag{8}
\]

where \(u_{NN} = (u_{xx} u_{yy}^2 + u_{yy} u_{xx}^2 + 2u_{xx} u_{yy} u_{xy}) / (u_{xx}^2 + u_{yy}^2)\) is the second order derivative of image \(u(x,y)\) along the gradient direction of \(u(x,y)\), and it tends to smooth the edges in image \(u(x,y)\). Since diffusion (8) is dominated by \(u_{NN}\), it tends to smooth the edges in image \(u(x,y)\) and
is not appropriate for image denoising [10]. In fact, diffusions (7) and (8) are closely related to the PM model in Eq. (1) since the following equation holds:

\[
\frac{\partial c(Jr_uJ)}{\partial t} + \nabla u (c(\|\nabla u\|) u_	ext{NN}) + c(\|\nabla u\|) u_{TT} = \text{div} \left( c(\|\nabla u\|) \nabla u \right)
\]

(9)

This relation coincides with the decomposition of the PM model in [10].

Since these two candidates for \( \mathbf{n} \) mentioned above are not good enough for image denoising, we propose to choose \( \mathbf{n} = (n_x, n_y) = (-l_x, l_y)/\|\nabla I\| \) in the proposed MPM model. Since the vector \( \mathbf{n} \) may be noisy, a slightly weighted Laplacian is integrated to suppress noise. As a result, the MPM diffusion reads

\[
\frac{\partial u}{\partial t} = \mathbf{n} \nabla^T (c(\|\nabla u\|) \nabla u, \nabla u^T + \alpha \cdot c(\|\nabla u\|) \Delta u
\]

\[
= c(\|\nabla u\|) u_{TT} + (\nabla c(\|\nabla u\|) \cdot \mathbf{n}) \|\nabla u \cdot \mathbf{n}\| + \alpha \cdot c(\|\nabla u\|) \Delta u,
\]

(10)

where \( u_{TT} = (u_{xx} l_x^2 + u_{yy} l_y^2 - 2l_x l_y u_{xy})/(l_x^2 + l_y^2) \) is the second order derivative of image \( I(x,y) \) along the edge direction of image \( I(x,y) \) and \( \alpha \) is a small constant. The weight function \( c(\cdot) \) is slightly modified as \( c(\cdot) = 1/\sqrt{1 + (\|\nabla u\|)^2} \).

Although the Laplacian is isotropic, it does not destroy edges since \( \alpha \) is small and the weight \( c(\cdot) \) reduces the smoothing effect near edges. If there is a noise-free image which possesses the same edge structure as, but different intensity distribution from the noisy image, \( \mathbf{n} \) can be

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Fig. 3. (a) Noise free Pepper image, (b) noisy images, results by (c) the YK model, (d) the PM model, (e) the RPPM model, (f) the TV model, (g) the DEPM model, and (h) the MPM with \( \alpha = 0.5 \). In each row, from left to right, the noise variance is 20, 60 and 100, respectively.
If $c(\cdot)$ is a constant in Eq. (10), say 1, and $\alpha = 0$, the MPM model reduces to

$$\frac{\partial u}{\partial t} = \nu_{ITT} + (\nabla c \cdot \mathbf{n}) (\nabla u \cdot \mathbf{n}) = \nu_{ITT}$$

(11)

The diffusion (11) smooths image $u(x,y)$ along the edge direction of image $I(x,y)$, and this is helpful to preserve the edges in image $I(x,y)$. As a result, if there is no edge in image $I(x,y)$, there would be no edge arising in image $u(x,y)$. The staircases are also (false) edges, thus, if there is no staircase (i.e., false edge) in image $I(x,y)$, the staircase will not arise in image $u(x,y)$. In (10), the diffusion is dominated by $u_{ITT}$, and the inhomogeneous weight $c(\|\nabla u\|)$ reduces the diffusion amount near edges. Just as in the TV model, it is necessary that the weight $c(\|\nabla u\|)$ remains a function of $\|\nabla u\|$ to reduce the diffusion amount near edges. This way, the MPM model performs well on both preserving edges in $I(x,y)$ and alleviating staircases.

2.3. Numerical implementation

The numerical implementation of our model is similar to those of the PM model [2] and the TV model [3]. In order to present the numerical schemes for the MPM model, Eq. (10) is expanded as

$$\frac{\partial u}{\partial t} = \nabla (c(\|\nabla u\|) \nabla u) + \alpha \cdot c(\|\nabla u\|) \Delta u$$
3. Experiments

In this section, we demonstrate the desirable properties of the MPM model, and the performance of the TV [3], PM [2], MPM, RPPM [18], DEPM [19], and the YK [12] models are compared. The peak signal-to-noise ratio (PSNR) and the mean structure similarity (MSSIM) [24] are employed as objective indices to evaluate the image quality of the filtered images. We focus primarily on staircase alleviation and edge preservation. In fact, the staircases are also (false) edges in the filtered image; therefore, to alleviate the staircases is essentially to smooth out these false edges. There are three factors affecting the result: time step, iteration number and the contrast parameter, i.e., parameter \( K \) in the PM and YK models and \( \tau \) in the RPPM model. The parameter \( K \) (similarly \( \tau \) determines the edge contrast to be preserved, that is, if the edge contrast (whatever the edge is false or real) is below \( K \), the edge will be smoothed out when the diffusion amount is large enough. As a result, if \( K \) is large enough, the PM model is free of staircase, but smooth out some useful edges as well. Meanwhile, it remains an open problem in image processing to select the optimal parameters [25]. Just as pointed out in [25], "different criteria may perform better or worse for different classes of denoising algorithms or with different noise

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Table 2
PSNR and MSSIM of the six models on Pepper image.

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Table 3
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<td>20.61</td>
<td>0.503</td>
<td>22.64</td>
<td>0.676</td>
<td>10.15</td>
<td>0.046</td>
<td></td>
</tr>
</tbody>
</table>
distributions or assumptions on the noise model." Therefore, several criteria for stopping time (iteration number) selection, such as the relative variance [26] and the decorrelation criterion [27], are proposed, they are not employed in recent works [12,13,16–20,22,23]. In this paper, the time step is 0.1 in all the experiments and the algorithms terminated when the MSSIM reached maximum. We experimentally single out the optimal parameter $K$ for the PM in each experiment in the following way: the parameter $K$ ranges from 1 to 30 by step 1, the MSSIM increases first as $K$ increases, then reaches the peak and decreases as $K$ increases further. The $K$ value at which the MSSIM reaches the peak is optimal for the PM model. The optimal $K$ for the YK model and

**Fig. 5.** (a) Noise free house image, (b) noisy images, results by (c) the YK model, (d) the PM model, (e) the RPPM model, (f) the TV model, (g) the DEPM model, and (h) the MPMe with $\alpha = 0.5$. The noise variance is 20 in the left column and 30 in the right column.
Table 4
PSNR and MSSIM of the six models on the house and cameraman images.

<table>
<thead>
<tr>
<th>Noise variance</th>
<th>YK</th>
<th>PM</th>
<th>RPPM</th>
<th>TV</th>
<th>DEPM</th>
<th>MPMe</th>
<th>Noise Image</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSNR</td>
<td>MSSIM</td>
<td>PSNR</td>
<td>MSSIM</td>
<td>PSNR</td>
<td>MSSIM</td>
<td>PSNR</td>
</tr>
<tr>
<td>House 20</td>
<td>2</td>
<td>29.53</td>
<td>0.788</td>
<td>15</td>
<td>30.92</td>
<td>0.821</td>
<td>0.07</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>27.81</td>
<td>0.732</td>
<td>18</td>
<td>29.01</td>
<td>0.784</td>
<td>0.07</td>
</tr>
<tr>
<td>Camera 20</td>
<td>2</td>
<td>27.07</td>
<td>0.796</td>
<td>15</td>
<td>28.46</td>
<td>0.817</td>
<td>0.07</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>24.97</td>
<td>0.703</td>
<td>18</td>
<td>26.33</td>
<td>0.760</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Fig. 6. (a) Noise free cameraman image, (b) noisy images, results by (c) the YK model, (d) the PM model, (e) the RPPM model, (f) the TV model, (g) the DEPM model, and (h) the MPMe with $\lambda = 0.5$. The noise variance is 20 in the left column and 30 in the right column.
optimal $\tau$ for the RPPM model are also singled out in a similar way except that the parameter $K$ ranges from 1 to 10 by step 1, and $\tau$ ranges from 0.01 to 0.3 by step 0.01. When calculating $\mathbf{n}$ in the MPM model, the noisy image $l(x,y)$ can be presmoothed using a Gaussian filter of small scale (say 1 in this paper) just as in the structure-based diffusion [6]. In addition, all the noisy images employed in this paper are coined from the IPOL\(^1\) website by uploading noise-free images and specifying the noise variance.

The first example is a synthetic image containing sharp corners. The noise variance is 20, and the MPM model with $z = 0$ is applied to this noisy image. Fig. 1 shows the results. It is clear that when $c(\cdot) = 1$, the sharp discontinuities are smoothed out (see Fig. 1(c)), when $c(\cdot)$ remains a positive and decreasing function over $\|\mathbf{n}\|$, the sharp discontinuities are preserved well (see Fig. 1(d)). The weight $c(\|\mathbf{n}\|)$ reduces the diffusion amount near edges.

The second experiment is a noisy synthetic Test image of ramps, which is shown in Fig. 2(a). There is no complicated structure and texture in the image, however, the ramps and the sharp edges should be preserved. Fig. 2 shows just the results at noise variance 20, 60 and 100 and Table 1 presents the PSNR and MSSIM indices with the noise variance varying from 20 to 100. The results by the YK model are staircase-free, but there are dark spots, and the sharp edges are blurred. The TV model suffers from serious staircases. The PM and RPPM models work well when the noise is low, however, when the noise becomes heavier, the sharp edges are smoothed out. One can also observe impulse-like artifacts in the results by the RPPM model. The DEPM model suffers from impulse-like artifacts and serious staircases. The MPMe and MPMe work very well on preserving the edges, suppressing staircases and removing noises simultaneously. Experiments on synthetic and real images demonstrate the effectiveness of the MPM model for image restoration, and comparisons with other classical models show that it has a good performance overall.

4. Conclusion

We propose a modified Perona–Malik (MPM) model aiming at preserving edges and suppressing staircases. The proposed MPM model directly generalizes the PM model using directional Laplacian with an inhomogeneous weight. The weight function reduces the diffusion amount near edges to preserve edges and the directional Laplacian makes the diffusion along the edge direction of the original image. A slightly weighted Laplacian is also integrated to suppress noise. Consequently, the proposed MPM model can preserve edges, suppress staircases and remove noises simultaneously. Experiments on synthetic and real images demonstrate the effectiveness of the MPM model for image restoration, and comparisons with other classical models show that it has a good performance overall.

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