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Stabilization of Polynomial Fuzzy Large-Scale System : Sum-of-Square approach

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Abstract: This paper presents a polynomial fuzzy method approach to stabilize the nonlinear large-scale system. Unlike conventional T-S fuzzy system, polynomial fuzzy system contains polynomial matrix. Since polynomial fuzzy system has polynomial matrix, this system has less IF-THEN rules than T-S fuzzy system. Based on the proposed method, stabilization condtiion is derived in the form of polynomial and solved by sum-of-square method. Finally, some examples demonstrate that the proposed condition is adoptable in modeling and stabilization for the nonlinear large-scale system.

Keywords: Large-scale system, Polynomial fuzzy system, Sum-of-squares, polynomial Lyapunov function.

1. INTRODUCTION

Takagi-Sugeno (T-S) fuzzy system represents nonlinear system by fuzzy blending of fuzzy set and linear system with respect to consequence part of IF-THEN rule. For a quite general class of nonlinear systems, a systematic modeling methodology is available to exactly transform them into the so-called T-S fuzzy system [1]. This methodology is known as sector-nonlinearity [4]. Thus, we can apply linear control theory to T-S fuzzy system, because it represents the nonlinear system to several linear consequence parts, that is the reason why various researches are progressed [1] - [7]. In the previous researches, stability and stabilization condition for the T-S fuzzy system is presented by linear matrix inequalities (LMIs) [2]. However, LMI condition cannot contain state variables and have conservative results.

Large-scale system means the interconnected system of several sub-systems. Each sub-system influences each other. Complicated system can be decentralized into interconnected systems to easily design the controller. That is the merits, so various researches have been progressed on developing stabilization method for T-S fuzzy large-scale systems [5]- [6]. However, there still exist a large number of design problems that the stability and stabilization condition obtained through LMIs are too conservative.

Recently, to relax the conservativeness of stability and stabilization condition for T-S fuzzy systems in terms of LMIs, some authors proposed sum-of-square (SOS) approach [8] - [10]. Polynomial fuzzy system which is similar to T-S fuzzy system has polynomial system matrices and input matrices in the consequence part of IF-THEN rules. Moreover, stability and stabilization condition of polynomial fuzzy system are derived from polynomial Lyapunov function which can be solved by SOSTOOLS, and this result is more relaxed condition than traditional T-S fuzzy system appraoch. The new concept is a generalization of T-S fuzzy system and has less IF-THEN rule than T-S fuzzy case in some nonlinear systems. Since it has aforementioned merits, polynomial fuzzy method is adopted in this paper. To the best of our knowledge, this paper represents the first attempt to apply SOS techniques to stabilization and modeling of polynomial fuzzy large-scale system.

2. POLYNOMIAL FUZZY LARGE-SCALE SYSTEM

From sector nonlinearity concept, the nonlinear large-scale system can be exactly converted to polynomial fuzzy large-scale system. Consider a polynomial fuzzy large-scale system (S) which consist of N subsystems S_i ($i = 1, 2, \cdots, N$). The i^{th} polynomial fuzzy subsystem S_i is described by the following polynomial fuzzy IF-THEN rules:

i^{th} sub-plant rule k:

IF $z_{i_1}(t)$ is $M_{i_{k1}}$ and $\cdots$ and $z_{i_p}(t)$ is $M_{i_{kp}}$

$$\text{THEN } \dot{x}_i(t) = A_{ik}(x_i(t))\hat{x}_i(x_i(t)) + B_{ik}(x_i(t))u_i(t) + \sum_{j=1,\, j\neq i}^{N} A_{ijk}(x_j(t))\hat{x}_j(x_i(t)) \quad (1)$$

where $x_i(t)$ is state vector, $z_{i_c}(t)$ $(c = 1, 2, \cdots, p)$ is premise variable of i^{th} subsystem, $M_{i_{kc}}$ $(k = 1, 2, \cdots, r_i)$ is fuzzy set, $A_{ik}(x_i(t))$ and $B_{ik}(x_i(t))$ are system matrix and input matrix, respectively, and $A_{ijk}(x_j(t))$ is k^{th} rule of interconnection rate with respect to i^{th} subsystem to j^{th} subsystem, $\hat{x}_i(x_i(t))$ is column vector whose entries are monomials in $x_i(t)$, and $u_i(t)$ is input vector.

Stability and stabilization conditions of polynomial fuzzy system is derived from polynomial Lyapunov function [8]. Polynomial Lyapunov function is represented as follows.

$$V_i(x_i(t)) = \hat{x}_i^T(x_i(t))P_i(\tilde{x}_i)\hat{x}_i(x_i(t)) \quad (2)$$

where $P_i(\tilde{x}_i(t))$ is polynomial matrix in $\tilde{x}_i(t)$. The information of $\tilde{x}_i$ is obtainable in the next section.

Polynomial fuzzy large-scale system can be stabilized by following controller using parallel distributed compensation method. Controller of i^{th} subsystem is represented following polynomial fuzzy IF-THEN rules:

i^{th} controller rule k:

IF $z_{i_1}(t)$ is $M_{i_{k1}}$ and $\cdots$ and $z_{i_p}(t)$ is $M_{i_{kp}}$
THEN $u_i(t) = -K_{ik}(x_i(t))\hat{x}_i(t)$ (3)

where $K_{ik}(x_i(t))$ is the controller gain of i^{th} controller with respect to k^{th} rule.

From now on, for notation simplicity, the notation with respect to time will be omitted. t. Using above controller (3) and applying singleton fuzzifier, product inference engine, and center-average defuzzifier to subsystem (1), the defuzzified output of i^{th} fuzzy subsystem is described as follows:

$$\dot{x}_i = \sum_{k=1}^{r_i}\sum_{m=1}^{r_i} h_k(z_i)h_m(z_i)\Bigg\{\Big(A_{ik}(x_j) - B_{ik}(x_i) \times K_{im}(x_i)\Big)\hat{x}_i(x_i) + \sum_{j=1,j\neq i}^{N} A_{ijk}(x_i)\hat{x}_j(x_j)\Bigg\} \tag{4}$$

where $h_k(z_i)$ and $h_m(z_i)$ are fuzzy weighting function such that following conditions are satisfied.

$$h_k(z_i) \geq 0, \ \sum_{k=1}^{r} h_k(z_i) = 1. \tag{5}$$

To prove our main result, the following lemmas are needed.

Lemma 1 ([11]) For some scalar $\epsilon > 0$, and arbitrary matrix X and Y with proper dimension, following inequality is always satisfied.

$$X^TY + Y^TX \preceq \epsilon X^TX + \frac{1}{\epsilon}Y^TY \tag{6}$$

Lemma 2 ([9]) (Schur's complements) Given symmetric matrices N, L, and positive definite matrix O, following two inequalities are equivalent.

$$N + L^TOL \prec 0 \tag{7}$$

$$\begin{bmatrix} N & L^T \\ L & -O^{-1} \end{bmatrix} \prec 0 \tag{8}$$

3. MAIN RESULT

The main result on the global asymptotic stabilization of the closed-loop polynomial fuzzy large-scale system is summarized in the following theorem.

Theorem 1: The overall system consisted of each subsystem (4) is globally asymptotically stabilizable if there exist a symmetric polynomial matrix $X_i(\tilde{x}_i) = P_i^{-1}(\tilde{x}_i)$ and a polynomial matrix $N_{im}(x_i)$ satisfying following SOS conditions. Where $\epsilon_{1i}(x_i) > 0$ and $\epsilon_{2ikm}(x_i) > 0$ are positive polynomials in all x.

$$v_1^T\left(X_i(\tilde{x}_i) - \epsilon_{1i}(x_i)I\right)v_1 \quad \text{is SOS} \tag{9}$$

$$-v_2^T\left(\gamma_{ikm}(x_i) + \gamma_{imk}(x_i) + \epsilon_{2ikm}(x_i)I\right)v_2 \ \text{is SOS} \quad \text{for } k \leq m \tag{10}$$

where,

$$\gamma_{ikm}(x_i) = \begin{bmatrix} \alpha_{ikm}(x_i) + (N-1)I & X_i(\tilde{x}_i)\beta_{ik}^T(x_i) \\ \beta_{ik}(x_i)X_i(\tilde{x}_i) & -I \end{bmatrix},$$

$$\gamma_{imk}(x_i) = \begin{bmatrix} \alpha_{imk}(x_i) + (N-1)I & X_i(\tilde{x}_i)\beta_{im}^T(x_i) \\ \beta_{im}(x_i)X_i(\tilde{x}_i) & -I \end{bmatrix},$$

$\alpha_{ikm}(x_i) = X_i(\tilde{x}_i)G_{ikm}^T(x_i)T_i^T(x_i) + T_i(x_i)G_{imk}(x_i) \times X_i(\tilde{x}_i) - \sum_{l_i\in L_i}\Big(\partial X_i(\tilde{x}_i)/\partial x_{il_i}\Big)A_{ik}^{l_i}(x)\hat{x}(x)$,

$G_{ikm}(x_i) = A_{ik}(x_i) - B_{ik}(x_i)K_{im}(x_i)$,

$\beta_{ik}(x) = \Big[T_i(x_i)A_{i1k}(x_i) \quad \cdots \quad T_i(x_i)A_{iJk}(x_i)\Big]$,

$A_{ik}^l(x_i)$ is the l^{th} row of $A_{ik}(x_i)$,

$L_i = \{l_{i1}, l_{i2}, \cdots, l_{iy}\}$ denote the row indices of $B_{ik}(x_i)$ whose corresponding row is equal to zero, and define $\tilde{x}_i = [x_{l_1}, x_{l_2}, \cdots, x_{l_y}]$, v_1 and v_2 are column vectors whose entries are monomials regardless of x_i,

$$T_i(x_i) = \begin{bmatrix} \frac{\partial\hat{x}_{i1}(x)}{\partial x_{i1}} & \cdots & \frac{\partial\hat{x}_{i1}(x)}{\partial x_{in_i}} \\ \vdots & \ddots & \vdots \\ \frac{\partial\hat{x}_{in_i}(x)}{\partial x_{i1}} & \cdots & \frac{\partial\hat{x}_{in_i}(x)}{\partial x_{in_i}} \end{bmatrix},$$

where n_i is number of state variables.

If $X_i(\tilde{x}_i)$ and $N_{im}(x_i)$ hold (9) and (10), we can obtain a stabilizing feedback gain $K_{im}(x)$ from below relationship.

$$K_{im}(x) = N_{im}(x_i)X_i^{-1}(\tilde{x}_i) \tag{11}$$

Proof: The proof is omitted due to lack of the paper. ■

4. NUMERICAL EXAPMLE

4.1 Example 1

To prove suitability of our proposed method, we design the controller for the following numerial system with $\hat{x}_i = \begin{bmatrix} x_{i1} & x_{i2} \end{bmatrix}^T$, and initial conditions are $x_1(0) = \begin{bmatrix} 1.5 & -1 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^T$, and $x_3(0) = \begin{bmatrix} 0.7 & -0.3 \end{bmatrix}^T$.

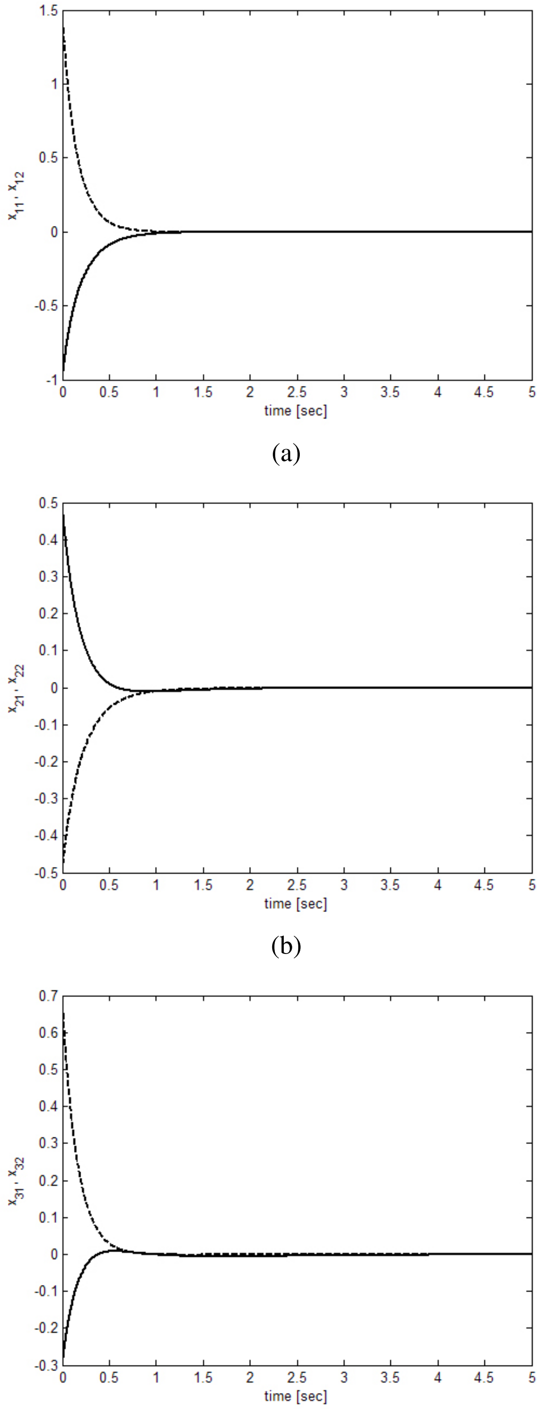
Subsystem 1:

$$A_{11}(x) = \begin{bmatrix} -2 + x_{11} + x_{12} & 3 \\ 1.5 & -2.2 \end{bmatrix},$$

$$A_{12}(x) = \begin{bmatrix} -2 + x_{11} + x_{12} & 3 \\ 1.5 & -2.2 \end{bmatrix},$$

$$B_{11}(x) = \begin{bmatrix} 0.15 \\ 0.1 \end{bmatrix}, B_{12}(x) = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix},$$

$$A_{121}(x) = \begin{bmatrix} 0.5 & 0.9 \\ 0.4 & 0.4 \end{bmatrix}, A_{122}(x) = \begin{bmatrix} 0.1 & 0.2 \\ 0.15 & 0.2 \end{bmatrix},$$



(a)

(b)

(c)

Fig. 1 State response of polynomial fuzzy large-scale system (example 1). (a) x_{11} (dotted line), x_{12} (solid line). (b) x_{21} (dotted line), x_{22} (solid line). (c) x_{31} (dotted line), x_{32} (solid line).

$$A_{131}(x) = \begin{bmatrix} 0.3 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, A_{132}(x) = \begin{bmatrix} 0.2 & 0.5 \\ 0.1 & 0.3 \end{bmatrix}$$

Subsystem 2:

$$A_{21}(x) = \begin{bmatrix} -4 + x_{21} & 1 \\ 5 & -3 \end{bmatrix},$$
$$A_{22}(x) = \begin{bmatrix} -3 + x_{22} & 1 \\ 3 & -0.3 \end{bmatrix},$$
$$B_{21}(x) = \begin{bmatrix} 0.1 \\ 0.6 \end{bmatrix}, B_{22}(x) = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},$$
$$A_{211}(x) = \begin{bmatrix} 0.2 & 0.5 \\ 0.1 & 0.2 \end{bmatrix}, A_{212}(x) = \begin{bmatrix} 0.1 & 0.5 \\ 0.2 & 0.3 \end{bmatrix},$$
$$A_{231}(x) = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.2 \end{bmatrix}, A_{232}(x) = \begin{bmatrix} 0.3 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$$

Subsystem 3:

$$A_{31}(x) = \begin{bmatrix} -5 + x_{31} + x_{32} & 1 \\ 4 & -2 \end{bmatrix},$$
$$A_{32}(x) = \begin{bmatrix} -7 + x_{31} + x_{32} & 1 \\ 3 & -1 \end{bmatrix},$$
$$B_{31}(x) = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}, B_{32}(x) = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},$$
$$A_{311}(x) = \begin{bmatrix} 0.8 & 0.6 \\ 0.3 & 0.7 \end{bmatrix}, A_{312}(x) = \begin{bmatrix} 0.6 & 0.4 \\ 0.1 & 0.3 \end{bmatrix},$$
$$A_{321}(x) = \begin{bmatrix} 0.4 & 0.2 \\ 0.5 & 0.6 \end{bmatrix}, A_{322}(x) = \begin{bmatrix} 0.3 & 0.3 \\ 0.6 & 0.5 \end{bmatrix}$$

Fuzzy weighting functions of each subsystems are given by

$$h_{11} = \frac{-x_{11} + 1}{2}, h_{12} = \frac{x_{11} + 1}{2},$$
$$h_{21} = \frac{-x_{21} + 1}{2}, h_{22} = \frac{x_{21} + 1}{2},$$
$$h_{31} = \frac{-x_{31} + 1}{2}, h_{32} = \frac{x_{31} + 1}{2}.$$

By applying theorem 1 to our system, controller gain is obtained as follows:

$$K_{11}(x_1) = \begin{bmatrix} 10.2 + 4.75x_{11}^2 + 0.52x_{11}x_{12} + 4.55x_{12}^2 \\ 10.8 + 5.02x_{11}^2 + 0.55x_{11}x_{12} + 4.81x_{12}^2 \end{bmatrix}^T,$$
$$K_{12}(x_1) = \begin{bmatrix} 2.58 + 2.32x_{11}^2 + 0.01x_{11}x_{12} + 1.74x_{12}^2 \\ 2.72 + 2.44x_{11}^2 + 0.01x_{11}x_{12} + 1.84x_{12}^2 \end{bmatrix}^T,$$
$$K_{21}(x_2) = \begin{bmatrix} 2.15 + 1.11x_{21}^2 + 1.05x_{22}^2 \\ 1.68 + 0.86x_{21}^2 + 0.82x_{22}^2 \end{bmatrix}^T,$$
$$K_{22}(x_2) = \begin{bmatrix} 5.42 + 1.43x_{21}^2 + 1.49x_{22}^2 \\ 4.24 + 1.12x_{21}^2 + 1.17x_{22}^2 \end{bmatrix}^T,$$
$$K_{31}(x_3) = \begin{bmatrix} 0.49 + 0.34x_{31}^2 + 0.24x_{31}x_{32} + 0.34x_{32}^2 \\ 0.35 + 0.24x_{31}^2 + 0.17x_{31}x_{32} + 0.24x_{32}^2 \end{bmatrix}^T,$$
$$K_{32}(x_3) = \begin{bmatrix} 0.80 + 0.68x_{31}^2 + 0.15x_{31}x_{32} + 0.68x_{32}^2 \\ 0.57 + 0.48x_{31}^2 + 0.11x_{31}x_{32} + 0.48x_{32}^2 \end{bmatrix}^T.$$

Fig. 1 shows state responses of polynomial fuzzy large-scale system (4). As shown in Fig. 1, we can know that proposed method can stabilize polynomial fuzzy large-scale system.

4.2 Example 2

This example shows the modeling method and its stabilization progress. Nonlinear system is presented as

$$
\begin{aligned}
\dot{x}_1 =& -7x_1 - x_2^2 + x_3 + x_4 + \frac{3-\sin x_1}{2}u_1 \\
\dot{x}_2 =& -2x_1 - x_2 + (\sin x_1)x_1 + x_4 \sin x_1 \\
& - \frac{1+3\sin x_1}{2}u_1 \\
\dot{x}_3 =& -3x_1 - x_4^2 + x_1 \cos x_3 + \frac{3-\cos x_3}{2}u_2 \\
\dot{x}_4 =& x_1 + x_2 - 4x_3 - 3x_4 + (\cos x_3)x_3 \\
& - \frac{1+3\cos x_3}{2}u_2. \qquad (12)
\end{aligned}
$$

Above nonlinear system (12) can be represented as polynomial fuzzy large-scale system, given in,

Subsystem 1:

$$A_{11}(x) = \begin{bmatrix} -7 & -x_2 \\ -3 & -1 \end{bmatrix},$$

$$A_{12}(x) = \begin{bmatrix} -7 & -x_2 \\ -1 & -1 \end{bmatrix},$$

$$B_{11}(x) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, B_{12}(x) = \begin{bmatrix} 1 \\ -2 \end{bmatrix},$$

$$A_{121}(x) = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, A_{122}(x) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

Subsystem 2:

$$A_{21}(x) = \begin{bmatrix} -3 & -x_4 \\ -5 & -3 \end{bmatrix},$$

$$A_{22}(x) = \begin{bmatrix} -3 & -x_4 \\ -3 & -3 \end{bmatrix},$$

$$B_{21}(x) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, B_{22}(x) = \begin{bmatrix} 1 \\ -2 \end{bmatrix},$$

$$A_{211}(x) = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}, A_{212}(x) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$

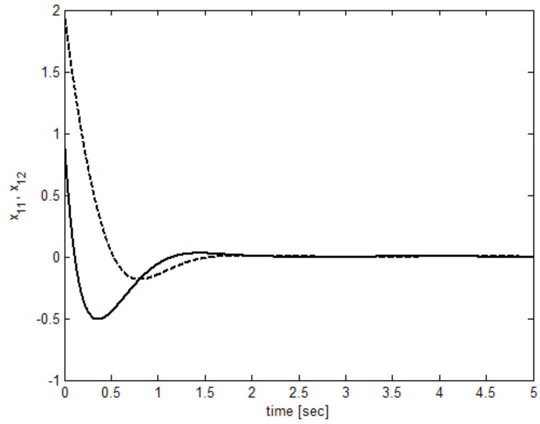
$$\hat{x}_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \hat{x}_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}.$$

Fuzzy weighting functions are derived as

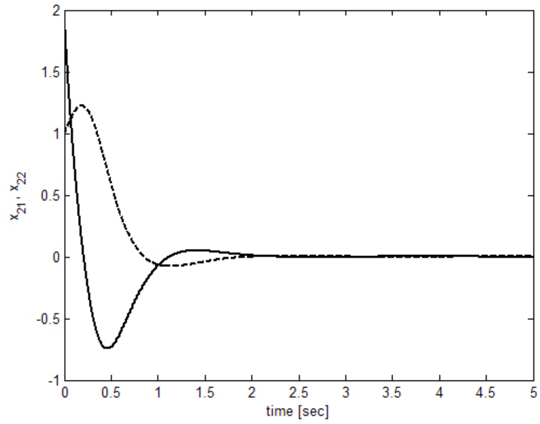
$$
\begin{aligned}
h_{11} =& \frac{1-\sin x_1}{2}, h_{12} = \frac{1+\sin x_1}{2}, \\
h_{21} =& \frac{1-\cos x_3}{2}, h_{22} = \frac{1+\cos x_1}{2}.
\end{aligned}
$$

By applying proposed Theorem 1 to (12) which is represented as polynomial fuzzy large-scale system, each controller gains are obtained as

$$
\begin{aligned}
K_{11} &= \begin{bmatrix} -1.552 & -1.683 \end{bmatrix}, \\
K_{12} &= \begin{bmatrix} -1.849 & -2.005 \end{bmatrix}, \\
K_{21} &= \begin{bmatrix} -1.603 & -1.786 \end{bmatrix}, \\
K_{22} &= \begin{bmatrix} -1.922 & -2.142 \end{bmatrix}.
\end{aligned}
$$



(a)



(b)

Fig. 2 State response of polynomial fuzzy large-scale system (12). (a) x_1 (dotted line), x_2 (solid line). (b) x_3 (dotted line), x_4 (solid line).

Note that controller gains of the polynomial fuzzy large-scale system can be constant term. Fig. 2 shows that polynomial fuzzy large-scale system (12) is asymptotically stable. It is evident that our proposed method properly represents nonlinear system as polynomial fuzzy large-scale system, effectively.

5. CONCLUSION

In this paper, we have proposed the modeling method and stabilization condition for polynomial fuzzy large-scale system. Stabilization condition of polynomial fuzzy large-scale system is derieved as SOS condition by using polynomial Lyapunov function. Finally, numerical examples have presented to prove reasonability of proposed method.

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