A stochastic optimization model for transit network timetable design to mitigate the randomness of traveling time by adding slack time

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ABSTRACT

Transit network timetabling aims at determining the departure time of each trip of all lines in order to facilitate passengers transferring either to or from a bus. In this paper, we consider a bus timetabling problem with stochastic travel times (BTP-STT). Slack time is added into timetable to mitigate the randomness in bus travel times. We then develop a stochastic integer programming model for the BTP-STT to minimize the total waiting time cost for three types of passengers (i.e., transferring passengers, boarding passengers and through passengers). The mathematical properties of the model are characterized. Due to its computational complexity, a genetic algorithm with local search (GALS) is designed to solve our proposed model (OPM). The numerical results based on a small bus network show that the timetable obtained from OPM reduces the total waiting time cost by an average of 9.5%, when it is tested in different scenarios. OPM is relatively effective if the ratio of the number of through passengers to the number of transferring passengers is not larger than a threshold (e.g., 10 in our case). In addition, we test different scale instances randomly generated in a practical setting to further verify the effectiveness of OPM and GALS. We also find that adding slack time into timetable greatly benefits transferring passengers by reducing the rate of transferring failure.

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1. Introduction

1.1. Research background

Transit network planning is an extremely complex problem. This whole problem is usually divided into several subproblems, such as line planning, timetable generation, vehicle scheduling, and crew scheduling (Ceder, 2007; Desaulniers and Hickman, 2007; Guihaire and Hao, 2008). These subproblems are solved sequentially. We are interested in the bus timetabling problem (BTP), in which the departure time of each trip of all lines is determined. Bus timetabling is a principal stage, since its solution determines transit service quality and subsequent subproblems (i.e., the vehicle and crew scheduling). The BTP mainly focuses on timetable synchronization which aims at maximizing the number of synchronized bus arrivals at transfer nodes or minimizing the total transfer waiting time experienced by passengers. Other factors (e.g., even spaced...
an attractive bus transit service. In reality, passengers are often forced to transfer either to or from a bus to complete a trip. This phenomenon commonly exists in large bus transit systems. During the transfer, passengers spend a lot of time waiting for connecting bus that serves on the line without well-designed timetable. Petersen et al. (2013) reported that the waiting time for one transfer is an average of about 9.75 min on each weekday in the Greater Copenhagen area. Therefore, a well-designed timetable, with good coordination between buses so that passengers can enjoy smooth transfers with minimal delay, is quite important for an attractive bus transit service.

A well-designed timetable can significantly improve the service quality by providing passengers effective transfers. However, variations in the passenger demand, in the performance of buses and in weather, etc., all contribute to the uncertainty of bus travel times (Chowdhury and Chien, 2001, 2002). It is difficult for buses to strictly adhere to the predesigned bus arrivals at transfer nodes. The randomness in bus travel times may result in passengers missing their planned transfers entirely. In the case, passengers are forced to wait longer for the following bus, which discourages passengers from choosing the bus transit service. Therefore, transit service reliability is an indispensable factor in bus timetabling. It is necessary to take into account stochastic travel times in the BTP. A reliable bus timetable can largely ensure the planned transfers.

1.2. Literature review

There are a number of studies on the BTP. Ceder et al. (2001) developed a mixed integer programming (MIP) model for the BTP to maximize the number of simultaneous bus arrivals at transfer nodes. An extension has been made by Eranki (2004), where synchronization is redefined as the arrivals of two trips at a transfer node with a separation time within a small time window instead of simultaneous arrivals. Based on the work of Ceder et al. (2001) and Eranki (2004), Ibarra-Rojas and Rios-Solis (2012) studied a flexible BTP that is characterized with oriented synchronization, almost evenly spaced departures and preventing bus bunching. Several studies seek to minimize the total transfer waiting time, and consider evenly spaced departures in the BTP (e.g., Shafahi and Khani, 2010; Cevallos and Zhao, 2006; Jansen and Nielsen, 2002; Chakrobority et al., 1997, 1995; Daduna and Voß, 1995). Wong et al. (2008) investigated the timetable synchronization problem in a railway system. The authors proposed a MIP model to minimize the total transfer waiting time by adjusting train's run times and station dwell times. Guihaire and Hao (2010) addressed the problem of modifying a bus network timetable to improve the number and quality of transfers while maintaining the vehicle trip assignment. Mollanejad et al. (2011) also developed a MIP model for the BTP. Unlike the abovementioned models, this model also considers uneven headways (i.e., separation time between consecutive trips) and the trips before and after a planning horizon. Khani and Shafahi (2011) studied the BTP in order to minimize the transfer waiting time by changing headways and departure times of intersecting lines. The proposed model in this work includes two parts: the headway setting model and the departure time setting model. Parbo et al. (2014) proposed a bus timetabling approach to minimize the weighted transfer waiting time. With this approach, passengers' route choice is considered to obtain accurate passenger weights in the timetable optimization. There are also studies that address the timetable synchronization between different transport systems (e.g., Bruno et al., 2009; Shrivastava and Dhingra, 2002; Clever, 1997).

Most of the existing literature on the BTP, however, is based on deterministic setting, with constant travel times. Although the timetable designed in this situation ensures that passengers enjoy smooth transfers at planning stage, the uncertainty of bus travel times may result in passengers missing their planned transfers entirely at operational stage. This leads to deterioration in transit service reliability. Comparatively, the BTP with stochastic travel times (BTP-STT) has received little attention at the planning stage. Lee and Schonfeld (1991) derived optimal slack time for a transport system with stochastic travel times. They analyzed the impact of stochastic travel times on the transfer optimization. Ting and Schonfeld (2005) proposed a heuristic algorithm to optimize the headways and the amount of slack time for connecting lines. Younan (2010) studied the problem of determining departure time and slack time for a typical bus corridor. Several studies (e.g., Yu et al., 2012; Hadas and Ceder, 2010; Ting and Schonfeld, 2007; Dessouky et al., 2003) introduce operational control tactics to realize the scheduled synchronization of bus arrivals at transfer nodes. Ceder et al. (2013) examined the impact of two operational tactics (i.e., holding at or skipping certain stops) on the total passenger travel time and the number of simultaneous transfers.

In this paper we investigate the BTP-STT for a given bus transit network at the planning stage. Slack time is added into timetable to mitigate the randomness of bus travel times. Bus timetabling with the involvement of uncertainties at the planning stage may yield a much better timetable behavior. Furthermore, a well-designed timetable considering stochastic travel times at the planning stage can reduce the need of numerous corrective actions using control tactics in the coming operation stage.

1.3. Research contributions and outline

The contributions made in this paper are summarized as follows. Firstly, this paper establishes a BTP for the bus transit network, in which the bus travel times are assumed stochastic. By adding slack time into each transfer node, the randomness of bus travel times can be mitigated. Secondly, we develop a stochastic integer programming model for the BTP-STT to
minimize the total waiting time cost for three types of passengers (i.e., transferring passengers, boarding passengers and through passengers). In the model, the departure time of each trip of all lines and the amount of slack time allocated to each transfer node are determined. Thirdly, we analyze the mathematical properties of our proposed model (OPM). Finally, a genetic algorithm with local search (GALS) to solve OPM is designed. The solutions for OPM and the existing deterministic timetabling model (i.e., in which the bus travel times are assumed constant) are compared. For ease of presentation, the deterministic timetabling model is referred to as DTM. In addition, different scale instances randomly generated in a practical setting are tested to verify the effectiveness of OPM and GALS, from which some insights are obtained.

The rest of this paper is organized as follows. In the next section, the BTP-STT is described in detail. Section 3 formulates OPM, and analyzes the mathematical properties of OPM. Section 4 develops the GALS. Section 5 provides instances to illustrate the effectiveness of OPM and GALS. Finally, Section 6 provides conclusions and suggestions for future studies.

2. Problem description

In this section, we make a detailed description of the BTP-STT. The bus network is represented by a set of lines and a set of nodes. The nodes along the lines in our case refer to the bus stations where passenger transfers occur. This type of key nodes is called transfer nodes. Scheduled-based control strategy is adopted at these transfer nodes.

Fig. 1 shows the passenger transfer event at a transfer node by a time–space diagram. Consider one passenger transferring from the $m$-th trip of line $i$ to line $j$ at transfer node $k$. Under the deterministic setting, the optimal arrival time of the $m$-th trip of line $i$ at transfer node $k$ is scheduled at $s_0$, and the optimal departure time of the $n$-th trip of line $j$ from transfer node $k$ is scheduled at $s_1$. Therefore, the passenger's transfer waiting time is $s_1 - s_0$ at transfer node $k$. However, the bus travel times are uncertain at the operational stage. The $m$-th trip of line $i$ may reach transfer node $k$ at $s_2$, and the $n$-th trip of line $j$ may leave at $s_3$. In the case, the passenger entirely misses the planned connecting bus. The passenger has nothing to do but waits much longer for following bus. In the case of stochastic travel times, pure timetable synchronization may not reduce the transfer waiting time effectively. It is intuitive to mitigate the randomness of travel times in bus timetable design. Slack time is literally thought as a safety factor to absorb the travel time randomness (Daganzo, 2009; Mazloumi et al., 2012).

To solve this problem, slack time is added into the bus network timetable to ensure that scheduled transfers can be largely realized at the operational stage. For each transfer node, the scheduled departure time is the sum of the mean bus arrival time and the slack time allocated to the node. Although adding slack time into timetable improves the transit service reliability that benefits transferring passengers and boarding passengers, it increases through passengers' waiting time (Mazloumi et al., 2012). Therefore, timetable design involves a tradeoff between various waiting time components. This paper investigates the BTP-STT in which stochastic travel times and adding slack time are considered. We aim at developing a stochastic programming model for the BTP-STT. In the model, the departure time of each trip of all lines and the amount of slack time allocated to each transfer node are determined to minimize the total passengers' waiting time cost.

3. Model formulation

In this section, the basic components of OPM for the BTP-STT are described, and then OPM is formulated. To facilitate the presentation of the essential ideas of this paper, some assumptions are made as follows:

1. The bus network is given as an input, and lines in the network usually have large headways (i.e., averagely greater than 10 min) in the planning period (e.g., off-peak hours).

2. The scheduled headway for each line is given. Constant headway is not only attractive to passengers, but is also practical for implementation (Shafahi and Khani, 2010). The buses depart from the terminal of each line on time. Therefore, for each line the departure time of each trip from the terminal of the line is multiple of the scheduled headway plus the first trip departure time.
In the case of large headway services, passengers who get on bus at a transfer node know the scheduled departure time before, and they synchronize their arrivals with the posted timetable (Dessouky et al., 1999; Mazloumi et al., 2012). If the bus arrives at the transfer node before the scheduled departure time, the passengers’ waiting time is zero. Otherwise, the waiting time is the difference between the actual departure time and the scheduled departure time.

The total slack time allocated to each line does not exceed a required value (i.e., recovery time at the terminal of the line), which does not significantly change the half-cycle time (Liu and Wirasinghe, 2001; Yan et al., 2012). Hence, extra operational cost caused by adding slack time into timetable is ignored.

The average number of passengers (i.e. transferring passengers, boarding passengers and through passengers) for each trip is given.

The walking time between buses in two lines at a transfer node is known. This value is the time that a passenger spends getting off a bus, walking to another bus and boarding it.

Before formulating the BTP-STT, we first define the following notations.

Indices

\( i, j \) Indexes of lines in the bus network
\( k, s \) Indexes of transfer nodes
\( m, n \) Indexes of trips

Sets

\( L \) Set of lines in the bus network
\( K_i \) Set of transfer nodes along line \( i \)
\( J_{ik} \) Set of lines connecting with line \( i \) at transfer node \( k \)
\( B_k \) Set of transfer nodes placed in line \( i \) before transfer node \( k \)

Parameters

\( T \) The planning period in minutes
\( H_i \) The scheduled headway for line \( i \)
\( F_i \) The number of trips of line \( i \) during \( T \) (i.e., \( \lfloor T/H_i \rfloor \))
\( t_i \) The threshold value of the total slack time allocated to each trip of line \( i \)
\( a_{ik} \) The earliest travel time for each trip from starting node of line \( i \) to transfer node \( k \)
\( b_{ik} \) The latest travel time for each trip from starting node of line \( i \) to transfer node \( k \)
\( TT_{imk} \) Bus travel time of the \( m \)-th trip from starting node of line \( i \) to transfer node \( k \)
\( E(\{TT_{imk}\}) \) The mean value of \( TT_{imk} \)
\( \delta_{ik} \) Time required to serve passengers at transfer node \( k \) of line \( i \)
\( w_{ijk} \) The walking time per transferring passenger between buses on line \( i \) and line \( j \) at transfer node \( k \)
\( PA_{ijmk} \) The average number of passengers transferring from the \( m \)-th trip of line \( i \) to line \( j \) at transfer node \( k \)
\( PT_{imk} \) The total number of passengers transferring from the \( m \)-th trip of line \( i \) to transfer node \( k \) to lines which synchronize with line \( i \)
\( PB_{imk} \) The average number of boarding passengers waiting for the \( m \)-th trip of line \( i \) at transfer node \( k \)
\( PR_{imk} \) The average number of through passengers for the \( m \)-th trip of line \( i \) at transfer node \( k \)
\( \alpha \) The value of a unit waiting time for passengers
\( \beta \) The value of a unit waiting time for boarding passengers
\( \gamma \) The value of a unit waiting time for through passengers

Decision variables

\( x_i \) Scheduled departure time of the first trip from starting station of line \( i \)
\( \tau_{ik} \) The amount of slack time allocated to transfer node \( k \) along line \( i \)

Auxiliary variables

\( AT_{imk} \) Actual arrival time of the \( m \)-th trip of line \( i \) at transfer node \( k \)
\( AT_{imk}^e \) The earliest possible arrival time of the \( m \)-th trip of line \( i \) at transfer node \( k \)
\( D_{imk} \) Departure time of the \( m \)-th trip of line \( i \) from transfer node \( k \)
\( S_{imk} \) Scheduled departure time of the \( m \)-th trip of line \( i \) from transfer node \( k \)
\( W_{ijmk} \) The waiting time per passenger transferring from the \( m \)-th trip of line \( i \) to line \( j \) at transfer node \( k \)
\( E(W_{ijmk}) \) The expected value of \( W_{ijmk} \)
\( E(W_{Bimk}) \) The expected waiting time per boarding passenger for the \( m \)-th trip of line \( i \) at transfer node \( k \)
\( E(W_{Timk}) \) The expected waiting time per through passenger for the \( m \)-th trip of line \( i \) at transfer node \( k \)

\( TT_{imk} \) is a continuous random variable with a given probability density function. The probability density function can be actually calibrated or estimated from historical data gathered from the automatic vehicle location (AVL) system (Yan et al., 2012). The dwell times that buses use to load and unload at the nodes before transfer node \( k \) are incorporated into \( TT_{imk} \).
According to Bookbinder and Desillets (1992) and Adamski (1995), the bus arrival time at each transfer node is assumed to have a shifted truncated exponential distribution. The bus arrival time at a transfer node is the sum of the travel time from the starting node to the transfer node and the departure time from the starting node. In this paper, we assume that $\text{TT}_{imk}$ follows a shifted truncated exponential distribution. The probability density function of $\text{TT}_{imk}$ is shown as Eq. (1):

$$f(t) = \begin{cases} (\rho \lambda) \exp[-\lambda(t-a_k)], & \text{if } t \in (a_k, b_k) \\ 0, & \text{otherwise} \end{cases}$$

(1)

where $\rho = 1/0.95$ and $\lambda = 3/(b_k - a_k)$. The bus travel times in the bus network are independent of each other.

### 3.1. Departure times at transfer nodes

The scheduled departure time from one transfer node is defined by adding a slack time to the expected arrival time at the node (Mazloumi et al., 2012). According to assumption (2), the slack time is the same for all trips of a certain line $i$ at transfer node $k$. Therefore, the scheduled departure time of the $m$-th trip of line $i$ from transfer node $k$ can be formulated as follows:

$$S_{imk} = E(\text{AT}_{imk}) + \tau_{ik}$$

(2)

where $E(\text{AT}_{imk})$ is the expected value of $\text{AT}_{imk}$. $\text{AT}_{imk}$ can be calculated by

$$\text{AT}_{imk} = x_i + (m-1) \cdot H_i + \sum_{x \in K^i_k} \tau_{ix} + \text{TT}_{imk}$$

(3)

Eq. (3) means that $\text{AT}_{imk}$ is equal to the sum of the $m$-th scheduled departure time from the starting node of line $i$, the total slack time allocated to the transfer nodes before transfer node $k$ and $\text{TT}_{imk}$. From Eq. (3), we can obtain

$$E(\text{AT}_{imk}) = x_i + (m-1) \cdot H_i + \sum_{x \in K^i_k} \tau_{ix} + E(\text{TT}_{imk})$$

(4)

Under the scheduled-based control strategy, early arriving buses are held until the scheduled departure time, and late arriving buses depart immediately after they serve passengers. According to Mazloumi et al. (2012), the actual departure time $D_{imk}$ can be expressed as Eq. (5).

$$D_{imk} = \begin{cases} S_{imk}, & \text{AT}_{imk} \leq S_{imk} - \delta_k \\ \text{AT}_{imk} + \delta_k, & \text{AT}_{imk} > S_{imk} - \delta_k \end{cases}$$

(5)

It can be seen that $S_{imk}$ plays an essential role in the on-time performance of buses, which is related to $x_i$ and $\tau_{ik}$.

### 3.2. Waiting times for impacted passengers

Passenger waiting time is associated with service reliability and depends on passengers’ arrival pattern at node (Ceder, 2007). To reliably design the timetable for a given bus transit network, three types of waiting time components are considered including the waiting time of transferring passengers, the waiting time of through passengers, and the waiting time of boarding passengers.

According to assumption (3), the expected waiting time for each boarding passenger $E(\text{WB}_{imk})$ is the integral of the difference between $D_{imk}$ and $S_{imk}$ multiplying by the possibility distribution function of $\text{AT}_{imk}$, which can be represented as Eq. (6).

$$E(\text{WB}_{imk}) = \int_{S_{imk} - \delta_k}^{\infty} (\text{AT}_{imk} + \delta_k - S_{imk}) f(\text{AT}_{imk}) d\text{AT}_{imk}$$

(6)

Using integral transformations and bounds of integration, Eq. (6) is rewritten as below.

$$E(\text{WB}_{imk}) = \int_{\text{TT}_{imk} + \tau_{ik} - \delta_k}^{b_k - \delta_k} (t + \delta_k - E(\text{TT}_{imk}) - \tau_{ik}) f(t) dt$$

(7)

The waiting time of through passengers is related to the waiting time of passengers on board of the vehicle that is held. Similar to Eq. (6), $E(\text{WT}_{imk})$ can be calculated by

$$E(\text{WT}_{imk}) = \int_{S_{imk} - \delta_k}^{\text{AT}_{imk}} (S_{imk} - \text{AT}_{imk} - \delta_k) f(\text{AT}_{imk}) d\text{AT}_{imk}$$

(8)

$\text{AT}_{imk}$ is equal to the $m$-th trip departure time from the starting node of line $i$ plus the earliest travel time from the starting node of line $i$ to transfer node $k$. From Eqs. (1)–(5) and (8), we can obtain Eq. (9) using integral transformations to calculate $E(\text{WT}_{imk})$. 

\[ E(\mathbb{W}_{imk}) = \int_{\mathcal{A}_k} E(T_{imk}) + \tau_{ik} - \delta_{ik} - t \mathbb{I}(t)dt \]  

(9)

Two cases are considered in defining the waiting time experienced by passengers transferring from the \(m\)-th trip of line \(i\) to line \(j\) at transfer node \(k\) (Younan, 2010). First, passengers from the \(m\)-th trip of line \(i\) successfully transfer to the \(n\)-th trip of line \(j\) at transfer node \(k\). Second, the \(n\)-th trip of line \(j\) departs before the passengers transferring from line \(i\) complete their connection. These transferring passengers have to wait for the \((n+1)\)-th trip of line \(j\). For each case, the departure time of each trip of line \(j\) has two possibilities on the basis of Eq. (5). Following Bookbinder and Desilets (1992), the waiting time for each transferring passenger \(W_{imk}\) can be specified as Eq. (10).

\[
W_{imk} = \begin{cases} 
S_{imk} - (\mathcal{A}_{imk} + W_{ijk}), & \text{if } \mathcal{A}_{imk} \leq S_{imk} - \delta_{ik}, \quad \mathcal{A}_{imk} + W_{ijk} \leq S_{imk} \\
\mathcal{A}_{imk} + \delta_{ik} - (\mathcal{A}_{imk} + W_{ijk}), & \text{if } \mathcal{A}_{imk} > S_{imk} - \delta_{ik}, \quad \mathcal{A}_{imk} + \delta_{ik} \geq \mathcal{A}_{imk} + W_{ijk} \\
S_{j(n+1)k} - (\mathcal{A}_{imk} + W_{ijk}), & \text{if } \mathcal{A}_{imk} + W_{ijk} > \mathcal{A}_{j(n+1)k} + \delta_{ik}, \quad \mathcal{A}_{imk} + W_{ijk} > S_{jk}, \quad \mathcal{A}_{j(n+1)k} \leq S_{j(n+1)k} - \delta_{ik} \\
(\mathcal{A}_{imk} + W_{ijk}), & \text{otherwise} 
\end{cases}
\]  

(10)

Therefore, the expected waiting time for each transferring passenger \(E(W_{imk})\) depends on \(\mathcal{A}_{imk}\), \(\mathcal{A}_{jnk}\) and \(\mathcal{A}_{j(n+1)k}\). \(E(W_{imk})\) is equal to the triple integral of \(W_{imk}\) multiplying by the joint possibility distribution function of \(\mathcal{A}_{imk}\), \(\mathcal{A}_{jnk}\) and \(\mathcal{A}_{j(n+1)k}\), which is formulated as Eq. (11).

\[ E(W_{imk}) = \int_{-\infty}^{\mathcal{A}_{imk}} \int_{-\infty}^{\mathcal{A}_{jnk}} \int_{-\infty}^{\mathcal{A}_{j(n+1)k}} W_{imk}(\mathcal{A}_{imk}, \mathcal{A}_{jnk}, \mathcal{A}_{j(n+1)k})d\mathcal{A}_{imk}d\mathcal{A}_{jnk}d\mathcal{A}_{j(n+1)k} \]  

(11)

We can further obtain Eq. (12) from Eqs. (1)–(4) and (11).

\[ E(W_{imk}) = \int_{\mathcal{A}_k} \int_{\mathcal{A}_k} \int_{\mathcal{A}_k} W_{imk}(T_{imk})f(T_{imk})f(T_{j(n+1)k})dT_{imk}dT_{jnk}dT_{j(n+1)k} \]  

(12)

3.3. Stochastic integer programming model

As previously stated, the purpose of this paper is to develop a stochastic programming model for the BTP-STT. The objective function of the model is the total expected waiting time cost for three types of passengers, which is denoted as \(E(C)\). The proposed model for the BTP-STT can be formulated in Eqs. (13)–(18) as below.

\[ E(C) = \min \sum_{i \in L} \sum_{k \in K} \sum_{m-1} \sum_{j \in J_k} \alpha PA_{imk}E(W_{imk}) + \sum_{i \in L} \sum_{k \in K} \sum_{m-1} \beta PB_{imk}E(WB_{imk}) + \sum_{i \in L} \sum_{k \in K} \sum_{m-1} \gamma PB_{imk}E(WT_{imk}) \]  

(13)

s.t.

\[ 0 \leq x_i \leq H_i, \quad \forall i \in L \]  

(14)

\[ \sum_{k \in K_i} \tau_{ik} \leq t_i, \quad \forall i \in L \]  

(15)

Eqs. (7), (9) and (10)  

(16)

\[ x_i \in \{0, 1, 2, \ldots, H_i\}, \quad \forall i \in L \]  

(17)

\[ \tau_{ik} \in \{0, 1, 2, \ldots, t_i\}, \quad \forall i \in L, \quad k \in K_i \]  

(18)

where Eq. (13) states the objective to minimize the total expected waiting time cost. The first term, the second term and the third term in the objective function are the waiting time cost for transferring passengers, the waiting time cost for boarding passengers and the waiting time cost for through passengers, respectively. Constraint (14) forces the first trip departure time of each line at the beginning of the planning period \(T\) and guarantees the practicality of results. Constraint (15) ensures that the total slack time allocated to each trip of each line does not exceed a threshold value. Constraint (16) states that the expressions of three terms, i.e., \(E(W_{imk})\), \(E(WB_{imk})\) and \(E(WT_{imk})\) in the objective. Constraints (17) and (18) represent the domain of decision variables.

3.4. Model analysis and combinatorial complexity

In this section, we explore the relationship between the terms in the objective of OPM and decision variables, and analyze the solution space of the BTP-STT and the combinatorial complexity.
Property 1. The expected term \( E(W_{imk}) \) in the objective is a decreasing function of slack time \( \tau_{ik} \), and it is not relevant to \( x_i \).

Proof. According to Eqs. (1)–(5) and (7), we calculate the integral expression of \( E(W_{imk}) \), and easily obtain the result as below.

\[
E(W_{imk}) = \rho \exp(-\lambda(b_{ik} - a_{ik})) \cdot (-1/\lambda + E(T_{imk}) + \tau_{ik} - \delta_{ik} - b_{ik}) + \rho/\lambda \exp(-\lambda(E(T_{imk}) + \tau_{ik} - \delta_{ik} - a_{ik}))
\]

The first-order derivative of \( E(W_{imk}) \) with regard to \( \tau_{ik} \) is derived as follows.

\[
\frac{dE(W_{imk})}{d\tau_{ik}} = \rho \left( \exp(-\lambda(b_{ik} - a_{ik})) - \exp(-\lambda(E(T_{imk}) + \tau_{ik} - \delta_{ik} - a_{ik})) \right)
\]

As \( b_{ik} > E(T_{imk}) + \tau_{ik} - \delta_{ik} \) holds, we have \( \frac{dE(W_{imk})}{d\tau_{ik}} < 0 \). Hence, \( E(W_{imk}) \) is monotonically decreasing with regard to \( \tau_{ik} \), and the expression of function \( E(W_{imk}) \) does not include \( x_i \). □

Property 2. The expected term \( E(W_{imk}) \) is an increasing function of slack time \( \tau_{ik} \), and it is also not relevant to \( x_i \).

Proof. According to Eqs. (1)–(5) and (9), we calculate the integral expression of \( E(W_{imk}) \), and easily obtain the result as below.

\[
E(W_{imk}) = \rho (-1/\lambda + E(T_{imk}) + \tau_{ik} - \delta_{ik} - a_{ik}) + \rho/\lambda \exp(-\lambda(E(T_{imk}) + \tau_{ik} - \delta_{ik} - a_{ik}))
\]

The first-order derivative of \( E(W_{imk}) \) with regard to \( \tau_{ik} \) is derived as follows.

\[
\frac{dE(W_{imk})}{d\tau_{ik}} = \rho \left( 1 - \exp(-\lambda(E(T_{imk}) + \tau_{ik} - \delta_{ik} - a_{ik})) \right)
\]

As \( E(T_{imk}) + \tau_{ik} - \delta_{ik} > a_{ik} \) holds, we have \( \frac{dE(W_{imk})}{d\tau_{ik}} > 0 \). Consequently, \( E(W_{imk}) \) is monotonically increasing with regard to \( \tau_{ik} \), and the expression of function \( E(W_{imk}) \) does not include \( x_i \). □

Property 3. The expected term \( E(W_{jnk}) \) depends on the difference between \( S_{jnk} \) and \( AT_{imk} \), and the difference between \( S_{jnk} \) and \( AT_{jnk} \).

Proof. According to Eqs. (1)–(5), (10) and (12), we calculate the integral expression of \( E(W_{jnk}) \), and obtain the approximate result as below.

\[
E(W_{jnk}) = \rho^2 \left\{ (S_{jnk} - AT_{imk} - w_{ijk}) + \rho H_j \cdot \exp(-\lambda_1 (S_{jnk} - AT_{imk} - w_{ijk})) - (0.95 - \exp(-\lambda_2 (S_{jnk} - AT_{jnk}))) / \lambda_1 \right\} + o(\cdot)
\]

where \( \lambda_1 = 3/(b_{ik} - a_{ik}) \) and \( \lambda_2 = 3/(b_{jk} - a_{jk}) \). \( AT_{imk} \) denotes the earliest arrival time of the \( m \)-th trip line \( i \) at transfer node \( k \), which is calculated by Eq. (24). \( AT_{jnk} \) denotes the earliest arrival time of the \( n \)-th trip of line \( j \) at transfer node \( k \), which is calculated by Eq. (25). \( o(\cdot) \) is a negligible positive value. According to the definition of transfer waiting time, the expression \( S_{jnk} - AT_{imk} - w_{ijk} \) is greater than zero. Passengers are at great risk of missing the connecting bus if the difference of \( S_{jnk} \) and \( AT_{imk} \) is small. In this case, the second term in the curly brace plays a crucial role in the value of \( E(W_{jnk}) \). On the other hand, the first term obtains the leading power of the value of \( E(W_{jnk}) \) if the difference of \( S_{jnk} \) and \( AT_{imk} \) is large. The third term in the curly brace is decided by slack time \( \tau_{ik} \). The slack time improves the probability of successful transfer.

\[
AT_{imk} = x_i + (m - 1)H_i + \sum_{k' \in K_{ik}} \tau_{ik'} + a_{ik}
\]

\[
AT_{jnk} = x_j + (n - 1)H_j + \sum_{k' \in K_{jk}} \tau_{jk'} + a_{jk} \quad \Box
\]

Property 4. The BTP-STT is a combination optimization problem, and is one of complicated variations of the BTP.

Proof. The stochastic integer programming model for the BTP-STT shows that there are a great number of alternative combinations of \( x_i \) and \( \tau_{ik} \) for a given bus network. The objective value depends on the selection of such combinations. The BTP-STT extends the BTP under deterministic setting to the case where stochastic travel times are considered. Therefore, the BTP-STT is one of variations of the BTP. The BTP-STT determines both the departure time of each trip of all lines and the amount of slack time allocated to each transfer node. The solution space of the BTP-STT is enlarged compared with that of the
BTP, which makes the BTP-STT more complicated than the BTP. Odijk (1996) studied a special case of BTP, in which the bus arrival and departure occur periodically, and proved the problem is NP-complete by reduction from vertex-coloring problem. The BTP can also be modeled as a quadratic semi-assignment problem, which is showed to be NP-complete (Désilets, 1989). Therefore, the BTP-STT is too intractable to obtain exact solution in a reasonable time frame.

Constraints (14), (15) and (17), (18) define the feasible solution space of OPM. For a certain line \( i \), the range of values for \( x_i \) is zero to \( H_i \). The number of alternative combinations \( \tau_{ik} \) can be considered as the general case of ordered partition of an integer. The number of the ordered partition of an integer \( r \) into \( k \) parts is \( \binom{r}{k} \) (Andrews and Eriksson, 2004). The option of slack time allocated to selected transfer nodes along line \( i \) is one of combinational results of the ordered partition of an integer \( r \). From the constraints (17) and (18), the range of values for \( r \) is 1 to \( t_i \). The total number of the ordered partitions of \( r (r = 1, 2, \ldots, t_i) \) into \( k (k = 1, 2, \ldots, |K_i|) \) parts can be calculated by Eq. (26)

\[
\sum_{k=1}^{|K_i|} \binom{|K_i|}{k} \sum_{r=1}^{t_i} \binom{r-1}{k-1}
\]

For line \( i \), the number of alternative combinations of \( x_i \) and \( \tau_{ik} \) can be calculated by Eq. (27)

\[
(1 + H_i) \sum_{k=1}^{|K_i|} \binom{|K_i|}{k} \sum_{r=1}^{t_i} \binom{r-1}{k-1}
\]

The complexity of the BTP-STT can be expressed by the number of combinational solutions. Suppose that, for a bus network with \(|L|\) lines, each line has \(|K_i|\) transfer nodes, and the total slack time allocated to each trip of line \( i \) is \( t_i \). The number of combinational solutions for the bus network can be represented as follows.

\[
\prod_{i=1}^{|L|} \left(1 + H_i\right) \sum_{k=1}^{|K_i|} \binom{|K_i|}{k} \sum_{r=1}^{t_i} \binom{r-1}{k-1}
\]

Even for a simple bus network, for example \(|L| = 2, |K_i| = 2, H_i = 10\) and \( t_i = 3 \) (\( i \in L \)), the number of feasible combinational solutions is 9801.

Property 4 and the complexity of the BTP-STT show that OPM in this paper is intractable to solve, especially for large-scale instances. In addition, the three expected terms in the objective of OPM are difficult to be written in linear forms. The term \( E(W_{sym}) \) is a triple integral, and is difficult to find an analytic form. Even for small-scale instances, traditional approaches (e.g., branch and bound method) are not effective to solve the proposed model. Eq. (28) states that the number of feasible solutions increases exponentially with the number of lines, the number of transfer nodes and the total slack time allocated to each line. There is no efficient algorithm for optimally solving large-scale problems in a reasonable time frame. Based on the above discussion, finding the exact solution to the BTP-STT is very difficult. In Section 4, we design an improved genetic algorithm with local search (GALS) to solve the BTP-STT.

4. An improved genetic algorithm with local search

Genetic algorithm (GA) is a well-established meta-heuristic algorithm that emulates the evolution theory of nature (Beasley et al., 1993). The common operations of GA are described by Reeves (1997). Due to its simplicity, minimal problem restrictions and global perspective, GA has been successfully applied to a wide variety of complex optimization problems, such as transit network design (Chakroborty and Dwivedi, 2002; Cipriani et al., 2012; Nayeem et al., 2014), and timetable generation (Chakroborty et al., 1995, 1997, 2001; Cevallos and Zhao, 2006; Shafahi and Khani, 2010). Chakroborty (2003) stressed the effectiveness of GA in solving the BTP. However, the classical version of GA and its completely probabilistic search strategy cause time-consuming in converging to global optimum, even if it can easily find the area of good fitness. Combing a local search strategy into GA overcomes the weakness of GA, and enhances the intensification search on a special part of the solution space (Arroyo and Armentano, 2005; Yu et al., 2013). To tailor the GA to effectively solve the BTP-STT, we design the GALS. The encoding of chromosome, crossover and mutation operation, and local search strategy are described as below.

![Fig. 2. An example of encoded chromosome.](image)
4.1. Encoding of chromosome

Integer encoding method is adopted for chromosome. A chromosome is composed of \(|L|\) line sections. The chromosome is represented by a vector with \(|L| + \sum_{i=1}^{L} |K_i|\) elements. Each line section includes the first trip departure time of the line and the amount of slack time allocated to each transfer node of the line. Fig. 2 shows an example of an encoded chromosome that has three line sections and two parts for each line section. According to the coding rule of line section, the gene value 5 of the first line section represents that the first trip departure time of line 1 is the 5-th minute in the panning period, and the gene values 2, 2 and 3 are the amount of slack time allocated to the first transfer node, the second transfer node and the third transfer node of line 1, respectively.

For line 1 section, \(x_1\) is generated at random in the range of \([0, H_1]\). \(s_{11}\), \(s_{12}\) and \(s_{13}\) are generated by one of the ordered partitions of integer \(r\) into \(k\) parts. The values of \(r\) and \(k\) are generated at random in the range of \(\frac{1}{2} \in \mathbb{N}; t_1/c_{138}\) and \(\frac{1}{2} \in \mathbb{N}; K_1/c_{138}\), respectively. In line 1 section, \(r = 7, k = 3\). The rest of line sections have a similar encoding mechanism.

4.2. Fitness evaluation

Fitness evaluation is used to measure the goodness of candidate individuals. The fitness value in the GALS is calculated by Eq. (29). The genetic search prefers individuals with higher fitness.

\[
g(d_p) = \left\{ \max_{p \in P} \left\{ f(d_p) \right\} - f(d_p) + \mu \right\} \div \left\{ \max_{p \in P} \left\{ f(d_p) \right\} - \min_{p \in P} \left\{ f(d_p) \right\} + \mu \right\}
\]

where \(d_p\) denotes the \(p\)-th individual of current population, \(g(d_p)\) is the fitness value of \(d_p\), \(f(d_p)\) is the objective value of the individual \(d_p\), \(\mu\) is defined as a positive real number in the range of \([0, 1]\) to make sure that the value of denominator in Eq. (29) is not zero. In the GALS, the value of \(\mu\) is 0.5.

Because of the integral functions (i.e., the integral expressions of \(E(W_{i,m,k})\), \(E(W_{B_{i,m,k}})\) and \(E(W_{T_{i,m,k}})\) in the objective function of OPM, it is difficult to calculate the objective function value. \(E(W_{B_{i,m,k}})\) and \(E(W_{T_{i,m,k}})\) can be calculated by Eqs. (19) and (21), respectively. The three integral expression of \(E(W_{i,m,k})\) is calculated using Monte Carlo method introduced by Li et al. (2010).

4.3. Selection

Tournament selection approach is adopted in our designed GALS. In the tournament we select individuals based on the fitness value. This selection approach is sensitive enough to distinguish the individuals of nearly same fitness values (Nayeem et al., 2014; Yu et al., 2014).

4.4. Genetic operators

A uniform crossover method is used for crossover operation. Fig. 3 shows an example of the uniform crossover method. A crossover mask with binary values is generated at random for the corresponding genes of a chromosome. In this example, the crossover mask is 011011. The genes of the parents swap if the corresponding crossover mask value is 1. In the example, the second part of line 1 section, the first part of line 2 section and two parts of line 3 section are swapped, respectively. Two children generated after the crossover operation are also shown in Fig. 3.

A mutation mask with binary values is generated before mutation operator. For each line section, only one part is selected to mutate. An example of the mutation operator is shown in Fig. 4. In the example, the mutation mask is 100101. If the corresponding mask value is 1, the genes of parent need mutating. Valid genes are generated at random to replace the original ones.

![Fig. 3. An example of crossover operation.](image-url)
4.5. Local search strategy

We first select individuals from the mutated individuals. Then, a local search mask with binary values is generated for the second part of each line section of each chromosome. There is only one line section is selected for local search. The neighborhoods of the gens of the selected part are generated. For line $i$ section, there are $\sum_{k=1}^{K_i} \left( \binom{K_i}{k} \sum_{r=1}^{n_i} \left( \binom{r-1}{k-1} \right) \right) - 1$ neighborhoods for the gens of the selected part. Fig. 5 shows an example of the local search. In the example, the gens of line 3 section are selected. If the total available slack time for each trip of line 3 is 4 min, the neighborhoods of the selected gens {2,2} are \{4,0\}, \{0,4\}, \{3,1\}, \{1,3\}, \{3,0\}, \{0,3\}, \{2,1\}, \{1,2\}, \{2,0\}, \{0,2\}, \{1,1\}, \{1,0\}, \{0,1\}, \{0,0\}. The neighborhoods of the offspring chromosome are generated by replacing the gens {2,2} with its neighborhoods.

4.6. The algorithm

In the GALS, we use the maximum number of iterations as the stop criterion. In addition, some high quality individuals of the current generation are copied to the next generation. The number of elite individuals to be appended is predefined and is denoted by Elite_size. With the operators described above, the pseudo code of the GALS algorithm for the BTP-STT is given as below.

**Algorithm 1. Genetic algorithm with local search (GALS)**

**Input**: BTP-STT instance  
- Pop_size $\leftarrow$ population size  
- Max_iterator $\leftarrow$ maximum number of iterations  
- Crossover_rate $\leftarrow$ crossover rate  
- Mutation_rate $\leftarrow$ mutation rate  
- Elite_size $\leftarrow$ number of elite individuals  
- $P \leftarrow \{\}$  

**Output**: The best solution and its objective value

for (iterator = 1 to iterator = Pop_size) do  
    $P \leftarrow P \cup$ Initial_individual  
end for  

Iter = 0;  
while (iter < Max_iterator) do  
    for (each individual $P_i \in P$) do  
        Fitness_value ($P_i$)  
        if (Fitness_value ($P_i$) > Fitness_value (Best)) then  
            Best $\leftarrow P_i$  
        end if  
    end for  

    Elite $\leftarrow$ the Elite_size number of fittest individuals in $P$  
    $Q \leftarrow$ Elite  
    for (Pop_size $-$ Elite_size)/2 times do  
        Parent $P_a$ $\leftarrow$ Tournament_Selection($P$)  
        Parent $P_b$ $\leftarrow$ Tournament_Selection($P$)  
        Offspring $P_c$, $P_d$ $\leftarrow$ Crossover($P_a$, $P_b$)  
        $Q \leftarrow Q \cup \{\text{Mutation} (P_c), \text{Mutation} (P_d)\}$  
    end for  
    for (each individual $P_i \in Q$) do  
        $P_i$ $\leftarrow$ Best_fitness_individual ($P_i$, Best_neighborhood ($P_i$))  
    end for  

    Updating $Q$  
    $P \leftarrow Q$  
    iter++  
end while  

return Best solution and its objective value
5. Computational experiments

In this section, we first use a small bus network to illustrate the effectiveness of OPM in comparison with DTM proposed in Shafahi and Khani (2010). The objective function in the DTM is multiplied by the value of a unit waiting time in order to be consistent with OPM. The computational experiments are carried out, and the timetables obtained from OPM and the DTM are analyzed. Then, ten types of instances randomly generated in a practical setting are tested to verify the effectiveness of OPM, and to analyze the performance of our designed GALS. We set the parameters of GALS as Table 1.

5.1. Small instance and results

We consider a small bus transit network with eight lines and three transfer nodes as shown in Fig. 6. In the rest of this section, unless otherwise specified, parameters in the instance are as follows: \( T = 120 \) min, \( H_i = 15 \) min, \( t_i = 5 \) min, \( \alpha = 0.3 \) $/pass./min, \( \beta = 0.3 \) $/pass./min, \( \gamma = 0.08 \) $/pass./min, \( \delta_{ik} = 1 \) min and \( w_{ijk} = 1 \) min. The other parameters of the instance are provided by an expert from a local bus company. These parameters are collected by the bus company from historical data. Therefore, we cannot provide by ourselves the information of the data collection and analysis. The travel spread for each travel time is defined in range of [3,5]. The pairs of the earliest and latest possible travel times from the starting nodes of lines to each transfer node are shown in Table 2. The average number of transferring passengers is one unit per transfer for each pair of connecting lines. The ratio of the average number of three types of passengers is denoted by \( PT_{imk} \), \( PB_{imk} \) and \( PR_{imk} \), and \( PB_{imk} \) and \( PR_{imk} \) are generated in range of [5,10] and [5,10], respectively. The ratios of the average number of three types of passengers are shown in Table 3. Following Ceder et al. (2001), transfers between two lines in opposite directions (or sharing the same route segment and intersect at a common node) are not taken into account.

In Tables 2 and 3, the symbol “–” indicates that the line does not pass through the transfer node, and it has the same mean in following tables. The values in each bracket in Table 2 denote the earliest and latest possible travel times from the starting node of the line to the transfer node, respectively.

In this paper, we denote \( C_1 \), \( C_2 \) and \( C_3 \) as the waiting time cost for transferring passengers, the waiting time cost for boarding passengers and the waiting time cost for through passengers, respectively.

Table 4 shows the obtained solutions from OPM and DTM. As previously described, the other departure times are calculated by adding the headway and slack times to the first trip departure times. The bus travel times of all trips of lines used in DTM are their mean travel times. The scheduled departure times at transfer nodes under OPM are slightly greater than that under DTM on account of adding slack times. For example, the slack times allocated to transfer node 1, 2 and 3 along line 1 are 2, 1, 2 min, respectively. Similarly, the scheduled travel times under OPM are all greater than that under DTM. The last four rows show the waiting time cost for each type of passengers and the total waiting time cost. The waiting time cost for transferring passengers under OPM is slightly less than that under DTM. Inversely, the waiting time cost for boarding passengers and the waiting time cost for through passengers under OPM are both larger than that under DTM. Because the assumption (3) mentioned in Section 3 and constant travel times are considered in DTM, the waiting time cost for boarding passengers under OPM is larger than that under DTM.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max_iterator</td>
<td>500</td>
</tr>
<tr>
<td>Pop_size</td>
<td>100</td>
</tr>
<tr>
<td>Elite_size</td>
<td>20</td>
</tr>
<tr>
<td>Crossover_rate</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation_rate</td>
<td>0.05</td>
</tr>
</tbody>
</table>
passengers and the waiting time cost for through passengers are both zero. The total waiting time cost for OPM is higher than that of DTM at the planning stage. In next section, the two timetables obtained from OPM and DTM are tested in different scenarios (i.e., operational stages) where bus travel times are stochastic.

5.2. Effectiveness analysis of OPM in comparison with DTM

To show the effectiveness of OPM in comparison with DTM, the bus travel times in 20 scenarios are randomly generated following a shifted truncated exponential distribution. We calculate the related cost of the timetables generated by OPM and DTM. Fig. 7(a), (b), (c) and (d) plots the waiting time cost for three types of passengers and the total waiting time cost, respectively. For each scenario, Fig. 7(a), (b) and (d) shows that OPM performs better than DTM in terms of the waiting time cost for transferring passengers, the waiting time cost for boarding passengers and the total waiting time cost, respectively. However, Fig. 7(c) shows that adding slack time into timetable increases the waiting time cost for through passengers.

Table 5 presents the values of indexes evaluating the effectiveness of OPM. In Table 5, Mean is the average value of related cost. DMEAN is the difference between the average value of related cost under OPM and that under DTM. RCS is the cost-saving of OPM for each passenger in comparison with DTM.

One can observe from Table 5 that reduces the total waiting time cost by an average of 9.5% (i.e., \((1342.50 - 1215.46)/1342.50 \times 100\%\)). It can be seen that the average waiting time cost for transferring passengers decreases by 171.25. The average waiting time cost for boarding passengers decreases by a smaller magnitude of 75.42. The average cost-saving of OPM for each transfer passenger reaches 1.19. The average waiting time cost for each boarding passenger decreases 0.081 while the average waiting time cost for each through passenger increases 0.118. The transferring passengers are the main beneficiaries while the interest of through passengers is affected by adding slack time into timetable.
Based on the above discussion, although the total waiting time cost under DTM is less than that under OPM at planning stage, the performance of timetable obtained from DTM gets worse when the timetable is applied to real operational stage. It can be concluded that OPM is more reasonable, and saves passenger waiting time cost, especially for transferring passengers. In other words, the service quality for transferring passengers is largely affected by the service reliability.

5.3. Influence of the number of impacted passengers on the effectiveness

The optimal solution for timetabling also depends on the number of impacted passengers. Note that the effectiveness of OPM becomes stronger as the ratio of the number of transferring and boarding passengers increases. However, OPM may be not effective when the number of through passengers is extremely large compared to the number of boarding and transferring passengers. Therefore, we analyze the influence of the number of impacted passengers on the effectiveness of OPM in this section. The ratio of the number of transferring passengers and the number of boarding passengers for each line can be estimated, which is assumed to be 1:5. We assume $PT_{lmk} : PB_{lmk} : PR_{lmk} = 1:5:0$. The average values of total waiting time cost for OPM and DTM under different values of $\theta$ from 5 to 12 are shown in Fig. 8. For each value of $\theta$, 100 scenarios are tested in the timetables generated by OPM and DTM, respectively.

One can observe from Fig. 8 that both the average values of total waiting time cost for OPM and DTM increase as the value of $\theta$ increases. When $\theta$ varies from 5 to 10, OPM performs better than DTM, and the gap between them becomes small. Theoretically, the average value of total waiting time cost for OPM is always less than that of DTM as $\theta$ increases. In short, the effectiveness of OPM gradually decreases as $\theta$ increases.

5.4. Effectiveness of OPM for different scale instances

To verify the effectiveness of OPM, we design ten instances with different sizes based on the information of parameters introduced in Section 5.1. In addition, the earliest travel times from the starting node to each transfer node for each line are within [5, 50]. The number of different pairs of lines to synchronize at each transfer node is within [1, 5]. The instance size is represented by the number of lines $|I|$, the number of transfer nodes in the bus network $|K|$ and the total available slack time for each line $|t_i|$. The details for these ten instances are shown in Table 6.

For each instance, we obtain two timetables generated by OPM and DTM. The timetables are tested in 100 scenarios that simulate the bus travel times at the operation stage. The travel times for each scenario are randomly generated following the shifted truncated exponential distribution as Eq. (1) shows.

Table 7 describes the results of the average total waiting time cost and the transferring failure rates for each instance that are tested in 100 scenarios. In Table 7, $Av C$ represents the average total cost for the timetable used in 100 scenarios of each instance.

$Av CS$ is the average cost-saving percentage of OPM in comparison with DTM. One can see from Table 7 that the $Av C$ under OPM is less than that under DTM for each instance. The values of $Av CS$ show that the timetable obtained from OPM for large instances is more cost-saving.

$Av RTF$ is the average rate of transferring failure of the bus network. In the definition of transferring failure, we mention the case where one passenger misses the planned connecting bus even if the passenger can transfer to the following bus, to transferring failure. $Av RTF$ can be calculated by the equation: $Av RTF = \frac{1}{100|K|} \sum_{s=1}^{100} \sum_{k=1}^{|K|} RTF_{sk}$. $RTF_{sk}$ is denoted as the rate of transferring failure at transfer node $k$ in scenario $s$. $RTF_{sk}$ is equal to the percentage of the number of transferring failure versus the number of planned transfers at transfer node $k$. The rates of transferring failure demonstrate the significant better
results for OPM considering the stochastic travel times and adding slack time into timetable than for DTM. This can be also reflected by the cost-saving for each transferring passenger in Table 5, which illustrates the effect of OPM on transferring passengers. OPM reduces the rates of transferring failure especially for small-scale instances. We can conclude that adding slack time into timetable greatly benefits transferring passengers by reducing the rate of transferring failure.

5.5. Performance analysis of GALS

For each instance, we repeatedly run GALS 100 times to solve OPM. The performance results of GALS in comparison with GA are shown in Table 8. In the table, MIN, MAX and AVG represent the best solution, the worst solution and the average value of the objective function among 100 run times. Av Gap is defined as \((AVG - MIN) / MIN \times 100\%\) to analyze the stability of GALS and GA. Av Time is the average computing time.

It can be seen from Table 8 that the values of MIN, MAX and AVG obtained by GALS for each instance are better than that obtained by GA. The value of Av Gap under GALS is less than that under GA, which shows that GALS is more stable than GA. Although the Av Time for each instance solved by GALS is larger than that by GA, the increasing of computation time is acceptable.

Based on the above analysis, we can conclude that the performance (except the computational time) of our designed GALS is much better than that of general GA for solving the BTP-STT.

Table 5
Effectiveness analysis on related costs of OPM.

<table>
<thead>
<tr>
<th></th>
<th>OPM</th>
<th>DTM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
</tr>
<tr>
<td>Mean</td>
<td>685.62</td>
<td>235.98</td>
</tr>
<tr>
<td>DMEAN</td>
<td>-171.25</td>
<td>-75.42</td>
</tr>
<tr>
<td>RCS</td>
<td>1.19</td>
<td>0.081</td>
</tr>
</tbody>
</table>

Fig. 7. Comparison of various cost components under OPM and DTM.
Fig. 8. The average values of total waiting time cost under OPM and DTM for different $\theta$ values.

Table 6
Sizes of ten instances for the BTP-STT.

<table>
<thead>
<tr>
<th>Instance</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>A6</th>
<th>A7</th>
<th>A8</th>
<th>A9</th>
<th>A10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>I</td>
<td>$</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>$</td>
<td>K</td>
<td>$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>8</td>
<td>8</td>
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<tr>
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<td>t</td>
<td>_i</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 7
Average total waiting time costs and rates of transferring failure of the timetables under OPM and DTM.

<table>
<thead>
<tr>
<th>Instance</th>
<th>OPM</th>
<th>DTM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Av$</td>
<td>$Av$</td>
</tr>
<tr>
<td></td>
<td>$C$</td>
<td>$RTF$ (%)</td>
</tr>
<tr>
<td>A1</td>
<td>689.3</td>
<td>14.1</td>
</tr>
<tr>
<td>A2</td>
<td>1625.6</td>
<td>23.8</td>
</tr>
<tr>
<td>A3</td>
<td>2108.5</td>
<td>38.4</td>
</tr>
<tr>
<td>A4</td>
<td>3418.8</td>
<td>31.2</td>
</tr>
<tr>
<td>A5</td>
<td>4008.6</td>
<td>33.8</td>
</tr>
<tr>
<td>A6</td>
<td>5128.5</td>
<td>34.0</td>
</tr>
<tr>
<td>A7</td>
<td>7214.5</td>
<td>29.6</td>
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<tr>
<td>A8</td>
<td>8254.8</td>
<td>32.6</td>
</tr>
<tr>
<td>A9</td>
<td>11546.5</td>
<td>29.5</td>
</tr>
<tr>
<td>A10</td>
<td>18654.5</td>
<td>34.0</td>
</tr>
</tbody>
</table>

Table 8
Performance results of GALS in comparison with GA.

<table>
<thead>
<tr>
<th>Instance</th>
<th>GALS</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>$AVG$</td>
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6. Conclusions and future studies

In this paper, we consider one of variations of the BTP (i.e., the BTP-STT) in which travel times are random and adding slack time is considered. A stochastic programming model is developed for the BTP-STT. The mathematical properties of the model and the computational complexity are analyzed. Based on this analysis, a GALS is designed to solve the BTP-STT. The numerical results based on different scale instances demonstrate that the proposed model and our designed GALS are effective, and suggest that adding slack time into timetable greatly benefits transferring passengers by reducing the rate of transferring failure.

Some important extensions should be made in future studies. These extensions include, but not limited to: One can consider that the headway between consecutive trips of each line varies in certain range instead of being constant. It is meaningful to take into account the change in passengers’ travel behavior when the bus network timetable is redesigned.

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