Collaborative Nonlinear Transceiver Optimization in Multi-tier MIMO Cognitive Radio Networks with Deterministically Imperfect CSI

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Abstract—The problem of nonlinear transceiver optimization in a multi-tier Multiple-Input Multiple-Output (MIMO) network in Cognitive Radio Network (CR-Net) configuration is studied. The employed transmission schemes are based on the Matrix Decision Feedback Equalizer (DFE) and Tomlinson-Harashima Precoder (THP). It is assumed that the Channel State Information (CSI) is not known perfectly. The performance measure used for optimizing the network is based on the sum Mean Square Error (MSE) of symbol estimation in the system. The design problem is constrained by the transmit power of the Secondary Users (SU’s) as well as the maximum allowed interfering power to the Primary Users (PU’s). The design problem is not jointly convex in its design variables and has infinitely many constraints. To overcome this, a suboptimal iterative procedure is proposed. Based on the chosen model for uncertainty, the two resultant problems are Semidefinite Programs (SDP). These two problems are solved numerically. Finally simulations results are provided to assess the performance of the system.

Index Terms—Cognitive radio network, matrix decision feedback equalizer (DFE) transceiver, Tomlinson-Harashima precoding (THP) scheme, channel uncertainty and imperfect CSI, worst-case design

I. INTRODUCTION

The fixed frequency assignment policy for the spectrum, has led to spectrum scarcity [1]. To reuse the underutilized frequency spectrum, the Cognitive Radio Network (CR-Net) concept was proposed a few years ago and has received an enormous research attention [2]. Combined with Multiple-Input Multiple-Output (MIMO) systems, CR-Nets can provide a reliable and fast network infrastructure in the future.

In this paper an unstructured multi-tier MIMO network is studied which coexists with a Primary Radio Network (PR-Net) in a single frequency band. This setup may be considered in this paper. The objective function and the constraints of the problem formulation have the Second Order Cone (SOC) structure [10]. The aforementioned problem which is based on the minimization of the sum MSE, is not a convex programming problem, because it is not jointly convex in the design variables and also has infinitely many constraints. To overcome these difficulties, a suboptimal iterative solution is provided. It is shown that using the NBE model, the problem would lead to the Semidefinite Program (SDP) [10]. Both problems are solved numerically using YALMIP [11] as the modeler and SDPT3 [12] as the solver.

The remainder of this paper is organized as follows: in Section II the model of the described system for both DFE and THP schemes is defined. The problem formulation is considered in Section III and the iterative solution is proposed in this section. Simulation results are shown and described in Section IV and finally Section V concludes the paper.

Notation: In this manuscript vectors and matrices are denoted using boldface lower-case and upper-case letters respectively. The unit vector, a vector having all its elements equal to one, is displayed using 1. To show the stacking of a set of indexed matrices or vectors, mat \([\{A_i\}_{i=1}^{r}] \triangleq [A_1^T, \ldots, A_r^T]^T\)
is used, and on the other side, the vectorization operator for matrices is vec [ ]. To denote the Frobenius and Euclidean norm of their respective matrix and vector arguments, \( \| \cdot \|_F \) and \( \| \cdot \| \) are used. The transpose and conjugate transpose of the matrices are shown using the superscripts \( (\cdot)^T \) \( and (\cdot)^* \) respectively. The mathematical expectation of the stochastic scalar, vector and matrix quantities is shown using \( E \ [\cdot] \). Finally, \( \mathcal{C} \) is to show the complex number set.

II. SYSTEM MODEL

A multi-tier CR-Net consisting of \( I \) interfering links is assumed (Fig 1). In the \( i \)th link, \( i = 1, \cdots, I \); the SU-Tx has \( T_i \) transmit antennas while the SU-Rx is equipped with \( R_i \) receive antennas, and the SU-Tx is supposed to transmit \( t_i \) symbols at each time instance to its SU-Rx. The transmit symbols, \( s_i \in \mathbb{C}^{t_i} \), are independently drawn and satisfy that: It is assumed that:

\[
E[s_is_i^*] = \begin{cases} \sigma_i^2 I, & i = j, \\ 0, & \text{otherwise}. \end{cases}
\]  

These symbols are precoded to be transmitted over the antenna arrays. In this paper both linear and nonlinear precoding schemes are addressed. The precoded signal, \( x_i \), is transmitted over a set of random channels \( \{H_{i,j} \in \mathbb{C}^{R_i \times T_j}\}_{i,j=1}^I \). The CSI for all important channels is assumed to be imperfectly known, i.e., there are deterministic variations in each realization of channel:

\[
H_{i,j} = \tilde{H}_{i,j} + \Delta_{i,j}, \quad \forall i, j,
\]

where \( \tilde{H}_{i,j} \) shows the nominal values of the channel while \( \Delta_{i,j} \) denotes the uncertainty of the CSI which is assumed to be independent of the transmit party of each link. These variations are characterized either stochastically or deterministically. In the former case, which is known as Stochastic Error (SE) model, the uncertainty follows these properties:

\[
E[\Delta_{i,j}] = 0, \quad \forall j,
\]

\[
E[\text{vec}(\Delta_{i,j})\text{vec}(\Delta_{i,j})^*] = \begin{cases} \sigma_i^2 I, & i = j, \\ 0, & \text{otherwise}, \end{cases}
\]

and in the latter case, also known as Norm Bounded Error (NBE) model, it is assumed that

\[
\|\Delta_{i,j}\|_F \leq \delta_{i,j}, \quad \forall j,
\]

The details of the design problem and its solutions will be revealed later in this and following sections. In this paper, the latter model, i.e., NBE model, is assumed for the uncertainty of the channel. Regardless of the type of uncertainty model and the choice of precoder, the received signal at the receiving peer is:

\[
y_i = H_{i,i}x_i + \sum_{j=1, j \neq i}^I H_{j,i}x_j + n_i, \quad \forall i;
\]

where \( n_i \) is a white Gaussian noise of the receiver, and this noise is independent of the transmit signal.

\[
E[n_i] = 0,
\]

\[
E[n_i n_j^*] = \begin{cases} \sigma_n^2 I, & i = j, \\ 0, & \text{otherwise}. \end{cases}
\]

The aforementioned network is installed within the service range of a primary network having \( K \) PU-Rx’s. The induced signal on the \( k \)th PU-Rx of \( R_k' \) receive antennas, \( u_k \), is

\[
u_k = \sum_{i=1}^I G_{i,k}x_i, \quad \forall k;
\]

where \( G_{i,k} = \tilde{G}_{i,k} + \Lambda_{i,k}, \quad \forall i, k; \)

where using NBE model,

\[
\|\Lambda_{i,k}\|_F \leq \lambda_{i,k}, \quad \forall i.
\]

A. Combination of THP and Linear Equalizer

In this scheme (Fig. 2), the transmit symbol, \( s_i \), is linearly combined with the feedback signals from the other links, i.e., \( \{v_j \in \mathbb{C}^{t_j}\}_{j=1, j \neq i}^I \). The resultant signal would be the feed back to the other links.

\[
v_i = s_i - \sum_{j=1, j \neq i}^I F_{i,j}v_j, \quad \forall i;
\]

where \( F_{i,j} \in \mathbb{C}^{t_i \times t_j} \) is the feedback matrix between the \( i \)th and \( j \)th links. It is possible to rewrite the aforementioned equation to get a straightforward one which states \( s_i \) based on all feedback signals \( \{v_j\}_{j=1}^I \).

\[
s_i = v_i + \sum_{j=1, j \neq i}^I F_{i,j}v_j, \quad \forall i;
\]

To get the transmit symbol, i.e., \( x_i \in \mathbb{C}^{t_i} \); \( v_i \) is linearly precoded by \( P_i \in \mathbb{C}^{t_i \times t_i} \), i.e.,

\[
x_i = P_i v_i, \quad \forall i.
\]
Interestingly, after imposing some mild conditions on the constellation of $s_i$ and its distribution, it is possible to assume that the feedback signals are also zero-mean and independent of each other, with a slightly higher energy relative to the original symbols [13].

$$
E[\mathbf{v}_i\mathbf{v}_j^T] = \begin{cases} 
\sigma_v^2 \mathbf{I}, & i = j, \\
0, & \text{otherwise},
\end{cases}
$$

where by assumption $\sigma_v^2 \geq \sigma_s^2$.

In the receiver, the received signal vector, $\mathbf{y}_i$, is linearly equalized using a matrix $\mathbf{D}_i \in \mathbb{C}^{t_i \times R_i}$ to get the finally estimated vector:

$$
\hat{s}_i \approx D_i \mathbf{y}_i.
$$

### B. Combination of Linear Precoding and DFE

In this scheme (Fig. 3) it is assumed that the precoder has a linear structure. The transmit symbols for $i$th SU-Tx, $\mathbf{x}_i$, is

$$
\mathbf{x}_i = \mathbf{P}_i \mathbf{s}_i,
$$

where $\mathbf{P}_i \in \mathbb{C}^{T_i \times t_i}$ is the linear precoder matrix. The received signal at the receiver is also first linearly equalized using $\mathbf{D}_i \in \mathbb{C}^{t_i \times R_i}$ and then the resultant signal is fed to a non-linear block having a series of feedback filters, $\mathbf{F}_i = [\mathbf{F}_{i,1}, \cdots, \mathbf{F}_{i,l}]^T$, resulting in a temporary signal $\hat{s}_i$, where

$$
\hat{s}_i = D_i y_i + \sum_{j=1 \neq i}^{l} F_{i,j} \hat{s}_j.
$$

As it is clear, to have the feedback signals, some sort of collaboration between users is needed and assumed.

### III. Problem Formulation

To characterize the system, and regardless of its configuration, the following performance measures for each link are required:

$$
\begin{align}
\text{TxP}_i &= E[\mathbf{x}_i^* \mathbf{x}_i], & \forall i; \\
\text{MSE}_i &= E[||\hat{s}_i - s_i||^2], & \text{for DFE}, \\
&= E[||\hat{s}_i - s_i||^2], & \text{for THP}, \\
\text{IP}_k &= E[\mathbf{u}_k^* \mathbf{u}_k], & \forall k;
\end{align}
$$

where all the mathematical expectation are calculated considering the signal domain. In the following proposition, the mathematical expression of these entities are revealed.

**Proposition 1**: For the aforementioned system, the transmit power of $i$th link, its MSE, and the interfering power of $k$th PU-Rx, are represented as

$$
\begin{align}
\text{TxP}_i &= \zeta_i ||\mathbf{P}_i||_F^2 \triangleq ||\mathbf{\pi}_i||^2, & \forall i, \\
\text{IP}_k &= \sum_{i=1}^{l} \zeta_i ||\mathbf{G}_{i,k} \mathbf{P}_i||_F^2, & \forall k, \\
\text{MSE}_i &= \zeta_i ||\mathbf{D}_i \mathbf{H}_i, \mathbf{P}_i - I||_F^2 + \sigma_n^2 ||\mathbf{D}_i||_F^2
\sum_{j=1 \neq i}^{l} \zeta_j ||\mathbf{D}_j \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}_{i,j}||_F^2, & \forall i,
\end{align}
$$

where $\zeta_i = \begin{cases} \sigma_s^2, & \text{for DFE}, \\
\sigma_v^2, & \text{for THP}. \end{cases}$

**Proof**: Please refer to [15].

Using the NBE model for uncertainty, $\{\mathbf{IP}_k\}_{k=1}^{K}$ and $\{\text{MSE}_i\}_{i=1}^{K}$ are not stochastic but not constant as well. The value of these quantities depend on the actual channel realizations.

**Proposition 2**: The above mentioned quantities may be represented as quantities with SOC structure having infinitely many realizations:

$$
\begin{align}
\text{IP}_k &\triangleq ||\mathbf{i}_k||^2 \triangleq ||\mathbf{i}_k + \mathbf{i}_{\Lambda_k} \text{vec} \{\mathbf{\Lambda}_k\}||^2 & (25a) \\
\text{MSE}_i &\triangleq ||\mathbf{m}_i||^2 \triangleq ||\tilde{\mathbf{m}}_i + \mathbf{m}_{\Delta_i} \text{vec} \{\mathbf{\Lambda}_k\}||^2 & (25b)
\end{align}
$$

where

$$
\begin{align}
\mathbf{i}_k &= \text{mat}\left[\left\{\sigma_s \text{vec} \left[\mathbf{G}_{i,k} \mathbf{P}_i\right]\right\}_{i=1}^{K}\right], \\
\mathbf{i}_{\Lambda_k} &= \text{mat}\left[\left\{\sigma_s \text{vec} \left[\mathbf{P}_i^T \otimes \mathbf{I}_{R_i}\right]\right\}_{i=1}^{K}\right], \\
\tilde{\mathbf{m}}_i &= \text{mat}\left[\left\{\sigma_s \text{vec} \left[\mathbf{D}_i \mathbf{H}_{i,j} \mathbf{P}_j - \mathbf{F}_{i,j}\right]\right\}_{j=1 \neq i}\right], \\
\mathbf{m}_{\Delta_i} &= \text{mat}\left[\left\{\sigma_s \mathbf{P}_i^T \otimes \mathbf{D}_i\right\}_{j=1 \neq i}\right].
\end{align}
$$

**Proof**: Please refer to [15].

The design problem, abstractly, would be:

$$
\begin{align}
\text{minimize}_{\mathbf{P}_i,\mathbf{F}_i,\mathbf{D}_i} & \quad \max_{\mathbf{\Delta}_i, ||\mathbf{\Delta}_i||_F \leq \delta_\epsilon} \mathbf{w}^T \text{MSE} \\
\text{subject to} & \quad \text{TxP}_i \leq \rho_i, & \forall i, \\
& \quad \text{IP}_k \leq \gamma_k, & \forall \mathbf{\Lambda}_k : ||\mathbf{\Lambda}_k||_F \leq \lambda_k, & \forall k.
\end{align}
$$

where $\mathbf{w} = [w_1, \cdots, w_l]^T \in \mathbb{R}^l$ is a constant weight vector. By choosing $\mathbf{w} = \mathbf{1}$, the problem could be the Sum MSE.
Using epigraph form this problem may be recast as:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{I} w_i \tau_i \\
\text{subject to} & \quad \|\pi\| \leq \rho_i, \quad \forall i, \\
& \quad \|i_k\|^2 \leq \gamma_k, \quad \forall A_k : \|A_k\|_F \leq \lambda_k, \quad \forall k, \\
& \quad \|m_i\|^2 \leq \tau_i, \quad \forall \Delta_k : \|\Delta_k\|_F \leq \delta_i, \quad \forall i.
\end{align*}
\]

After these preliminaries, it is possible to formulate the design problem for the NBE model as

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{I} w_i \tau_i \\
\text{subject to} & \quad \|\pi\| \leq \rho_i, \quad \forall i, \\
& \quad \|i_k\|^2 \leq \gamma_k, \quad \forall A_k : \|A_k\|_F \leq \lambda_k, \quad \forall k, \\
& \quad \|m_i\|^2 \leq \tau_i, \quad \forall \Delta_k : \|\Delta_k\|_F \leq \delta_i, \quad \forall i.
\end{align*}
\]

It will be shown that this semi-infinite problem can be recast as a SDP. To further simplify these constraints the Schur Complement Lemma is used to express the constraints in an appropriate LMI form.

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{I} w_i \tau_i \\
\text{subject to} & \quad \|\pi\| \leq \rho_i, \quad \forall i, \\
& \quad \|i_k\|^2 \leq \gamma_k, \quad \forall A_k : \|A_k\|_F \leq \lambda_k, \quad \forall k, \\
& \quad \|m_i\|^2 \leq \tau_i, \quad \forall \Delta_k : \|\Delta_k\|_F \leq \delta_i, \quad \forall i.
\end{align*}
\]

It is clear that the aforementioned problem is not a convex problem. To solve this issue, we propose an iterative solution which is a sub-optimum solution. In this iterative solution, first all design variables are initialized randomly, say \(D_0 \), \(P_0 \) and \(F_0 \). Then at each iteration, firstly either the transmit or the receive power is updated fixing the other party. If \(SMSE^n \) and \(SMSE^{n+1} \) are the sum MSE at the \(n\)th and the \((n+1)\)th iteration, respectively, a reasonable stopping criteria is \(\|SMSE^{n+1} - SMSE^n\| \leq \varepsilon \), where \(\varepsilon \) is the tolerance of this algorithm. The choice of \(\varepsilon \) is made in an ad hoc manner and to have fairly good results, be at the order of \(10^{-4} \) or \(10^{-6} \). But it should be noted that the smaller this parameter is, the more iterations are needed and the convergence become slower.

**IV. Simulation Results**

To demonstrate the performance of the proposed methods, a scenario is reported in this section. The system is composed of \(I = 2 \) interfering links each of which is equipped with \(T = 2 \) transmit/receive antennas. In each link at each time instant, \(t = 2 \) symbols are transmitted toward the receiver. It is assumed that the energy of these signals is normalized to be equal to \(\sigma_{s_i} = 1, \forall i \). This system is installed within the operating range of a single user PR-Net in which the sole user is equipped with \(R^T = 2 \) antennas. The power budget of SU’s are set to be equal to \(\rho = 0 \) dBm. The allowed amount of interfering power is set to be equal to the received noise power, \(\forall k, i, \gamma_k = \sigma_{n_i} = -30 \) dBm. Both links are equally weighted, i.e., \(w = 1 \). The algorithm is allowed to reach to 100 iterations maximum but as soon as the difference between two consecutive sum MSE is less than \(\varepsilon = 10^{-4} \) the updating
process is terminated. The CSI matrices for PU’s and SU’s are initialized randomly to be in accordance with a zero-mean unit-variance random process. The results shown are the average of 100 Monte Carlo runs.

In Fig. 4, the transmit power of the the first link is shown. As can be seen, for the case with perfect CSI, the transmit power is approximately 20 dB more than the cases with a fairly small uncertainty size ($\sigma^2_{\Delta} = 0.25$). Interestingly, the gap widens for the system with the largest uncertainty to approximately 30 dB. It seems that for the imperfect CSI cases, the system choses to transmit less power to avoid an unwanted arbitrarily large interference with PU’s, because the provided CSI nominal value is not reliable. It is clear that the transmit power of the DFE-based design is slightly larger than the THP-based design.

In Fig. 5, the sum MSE of the system is plotted. As can be seen, the MSE of the system with perfect CSI is outperforming the others with uncertainty. Intuitively, this is expected and as mentioned previously, it transmits more power which results in a slightly better Signal to Interference plus Noise (SINR) ratio for each symbol ending with a better sum MSE performance. It is also clear that the DFE-Based design is generally performing better than the THP-based design.

V. CONCLUSION

In this paper the problem of joint nonlinear transceiver optimization and design in a multi-tier MIMO CR-Net based on THP and matrix DFE is studied. It is assumed that the CSI is not known perfectly, and is modeled using the NBE model. The original problem formulation is based on the weighted sum MSE and is not a convex mathematical problem and has infinitely many constraints. The proposed algorithms are based on a suboptimum iterative method to update the design parameters. It is shown that the aforementioned problems employing the NBE model, can be cast as SDP’s. These problems are solved numerically and the simulation results are provided.

REFERENCES