“Busy Terminal Problem” and Implications for MAC Protocols in Underwater Acoustic Networks

Yibo Zhu*, Jun-Hong Cui†, Zhong Zhou‡, Zheng Peng*, Huifang Chen†
*Department of Computer Science & Engineering, University of Connecticut, Storrs, CT, USA
†Amazon Inc., Seattle, USA
‡Zhejiang University, Hangzhou, Zhejiang, China
Email: {yibo.zhu, jecui, zhongzhou, zhengpeng}@engr.uconn.edu, chenhf@zju.edu.cn

Last Update: July 2013

Abstract—In half-duplex underwater acoustic networks, MAC protocols usually assume packet transmission can interrupt packet reception so that the exposed terminal problem can be easily handled and system performance is improved. However, in practical an acoustic modem cannot be interrupted at will when it is transmitting or receiving. In other words, the modem cannot transmit packets when it is busy. This problem is referred to as the “busy terminal problem”. In underwater environments, the low transmission rates of acoustic modems result in excessive long transmission times which make the busy terminal problem more severe. As a result, the modem frequently appears to be too busy to transmit newly arrived packets. This would significantly affect packet sending patterns, which may affect collision behaviors in underwater MAC protocols. To better understand the impact of the busy terminal problem, we develop a new theoretical model of successful transmission probability for ALOHA (a basic random access MAC protocol). Extensive simulations show that the proposed model can effectively characterize the collision behaviors in ALOHA. Further, we apply the proposed model in network optimization through a case study on nodal throughput maximization.

Index Terms—busy terminal problem; underwater acoustic networks; medium access control; modeling.

I. INTRODUCTION

Underwater acoustic networks (UANs) have become a very active research area during the past half decade. Compared with terrestrial radio networks, UANs are characterized by low available bandwidth, long propagation delays and high channel dynamics, which pose grand challenges to the design and analysis of almost every core networking problem, including medium access control (MAC), routing, localization, reliable data transfer, just name a few [1]–[6]. In this paper, we focus on a new interesting “terminal” problem, which has been overlooked yet has a great impact on underwater MAC.

In underwater networks, acoustic modems operate in a half-duplex mode, where a node cannot transmit and receive simultaneously. In order to improve the channel utilization, in many MAC protocols, a node is scheduled to transmit packets when it is receiving a packet not for it [7]–[12]. In fact, this is the basic idea used to handle the exposed terminal problem [13], implying that packet reception can be interrupted by packet transmission. However, all real acoustic modems [14]–[17] cannot be interrupted when it is receiving or transmitting. Due to such non-interruptability, a modem is not available even if it is overhearing a packet. As a result, the “terminal” appears to be too “busy” to transmit packets as needed. This problem is referred to as the “busy terminal problem”. Further, because of the low transmission rates of acoustic modems, the busy terminal problem is more acute, exerting a non-negligible impact on underwater MAC protocols.

In this paper, we define the busy terminal problem and study its impact on underwater MAC protocols. For reservation based MAC protocols [7], [9], [18]–[20], it is straightforward to handle the busy terminal problem for data packets by scheduling their sending times subject to not overlapping with any packet reception. However, there is no intuitive method for the packets sent in a random access way, including the control packets in reservation based MAC and all packets in random access MAC protocols. Therefore, it is desirable to understand how the busy terminal problem affects random access MAC protocols.

Since ALOHA is the basis of both random access and reservation based protocols [7], [9], [18]–[22], we study the impact of the busy terminal problem on the performance of ALOHA as the first step. From simulation results, we observe that the classic model of successful transmission probability cannot accurately characterize the collision behaviors in ALOHA underwater when the busy terminal problem is considered. Due to the aforementioned importance of ALOHA, it is critical to accurately model the successful transmission probability for ALOHA. This would provide useful guidelines for future underwater MAC design and analysis.

We propose a modeling framework for the successful transmission probability of random access MAC considering the busy terminal problem. In this framework, we consider four types of conflicts which may result in a failed packet transmission, including the ones caused by the busy terminal problem. Following this framework, we develop a model for ALOHA, which is significantly different from the existing analysis [23]. Particularly, contrary to the conventional point of view, the proposed model implies that the transmission range does affect the successful transmission probability when the number of neighbors is fixed. Extensive simulation results demonstrate that the proposed model can accurately characterize the collision behaviors in ALOHA when the busy terminal problem is considered. Further, as an application of the proposed model, we optimize the packet generation rate.
to maximize the nodal throughput.

Through the discussion and model on BTP in this paper, we also hope we can get modem designers’ attention. Although it is convenient to design a modem featuring BTP and this works well for communication between two peer nodes, it is very critical to allow a modem to interrupt receptions. Otherwise, MAC layer cannot actually control the media access, and thus cannot well avoid collision as expected.

The rest of this paper is organized as follows: Section II defines the busy terminal problem and analyzes its impact on MAC protocols. Section III shows the motivation to model ALOHA with consideration of the busy terminal problem. Section IV presents a new analytical model of the successful transmission probability for ALOHA. Next, this model is validated via simulations in Section V. After that, the new model is applied to maximize nodal throughput in Section VI. Section VII provides the conclusions and future work.

II. BUSY TERMINAL PROBLEM

In this section, we formally define the busy terminal problem and analyze its impact on MAC protocols for UANs.

A. What is “Busy Terminal Problem”? 

In half-duplex UANs, transmission and reception cannot be done simultaneously. In literatures, most existing underwater MAC protocols assume a node can interrupt transmissions and receptions at will to transmit a new packet [7]–[11]. However, to the best of our knowledge, all real acoustic modems, like Teledyne Benthos modem [14], WHOI Micro-Modem [15], LinkQuest modem [16] and OFDM modem [17], are non-interruptible when receiving or transmitting a packet. In other words, an underwater node cannot transmit new packets when it is busy with transmitting or receiving.

For clarity, we take Fig. 1 as example to illustrate the difference between the interruptible and non-interruptible scenarios mentioned above. In both Fig. 1(a) and Fig. 1(b), nodes $N_2$, $N_1$, $N_2$, and $N_3$ sequentially form a string topology. Node $N_4$ is transmitting a packet destined to $N_4$, and $N_2$ can overhear this packet. In order to improve the channel utilization, a MAC protocol may schedule $N_2$ to transmit a packet to $N_3$ during the packet reception. In Fig. 1(a), when the interruptability is supported as commonly assumed in literatures, $N_2$ can switch receiving to transmitting its packet. Then, both $N_3$ and $N_4$ can receive their packets correctly. However, with real acoustic modems, as shown in Fig. 1(b), $N_2$ is unable to transmit its packet at $t$ because it is receiving. In addition, intuitively $N_1$ cannot interrupt the current packet transmission for another one. From these examples, we can see that a modem cannot be interrupted when it is transmitting or receiving, and the modem appears to be too “busy” to transmit newly arrived packets.

**Definition 1.** In half-duplex non-interruptible underwater acoustic networks, a node cannot interrupt reception/transmissions to send a new packet. We call this phenomenon as the busy terminal problem (BTP).

In theory, BTP can occur in any half-duplex non-interruptible networks. In radio networks, BTP is not significant since packet transmission delays are very short so that each packet reception/transmission can only keep a node busy for a very short period. In UANs, in contrast, the long packet transmission times make BTP extremely severe. For example, with Teledyne Benthos modem, each transmitted packet contains a preamble of duration 1.5s. When the modem transmission rate is set to 800bps (the corresponding effective transmission rate is only 667bps [12]), it takes about 7.5s to transmit a packet of size 500B. Such long packet transmission times would make a node busy for an excessively long time and significantly change the actual pattern of the packets injected into the channel, which further affects the performance of MAC protocols.

B. How BTP Affects Underwater MAC?

In this subsection, we analyze the impact of BTP on underwater MAC protocols, which can be classified into two types, slotted and un-slotted MAC protocols.

In slotted MAC protocols, each packet can only be transmitted right at the beginning of a slot and any control packet is received within the same slot. This can be guaranteed by adjusting the slot length. For example, Slotted-FAMA [24] sets slot length to the transmission time of a CTS packet plus the maximum propagation delay such that control packets can be delivered within one slot. Due to the long propagation delays, in slotted MAC protocols, a packet cannot arrive at other nodes immediately after it is transmitted, so a node is not busy with receiving at the beginning of each slot. Therefore, a node can transmit packets at the time points determined by the MAC layer, i.e., BTP does not affect slotted MAC protocols.

On the other hand, in un-slotted MAC, a node may try to transmit at any time, including when it is receiving. However, packet receptions cannot be interrupted because of BTP. Accordingly, packets may not be transmitted as demanded by the MAC layer and thus the patterns of traffic injected into the channel are changed, which would affect the performance of un-slotted MAC protocols. Next, we will categorize un-slotted MAC protocols into random access based and reservation based ones, and analyze the impact of BTP on them in detail respectively.

In random access MAC protocols, nodes are supposed to transmit packets to the channel at will. However, because of BTP, a modem is not always available for transmissions, and this essentially reduces the packets injected into the channel per packet transmission time. On one hand, compared with no BTP scenario, when the traffic is light, most of the reduced packets are supposed to be delivered other than colliding with others. Meanwhile, the negative impact of BTP is insignificant because BTP rarely occurs when the traffic is light. Thus, throughput is only slightly reduced. On the other hand, when the traffic is heavy, most of packets reduced by BTP are supposed to collide with others in no BTP scenario. Therefore,
more packets can be correctly delivered in BTP scenario although there are fewer packets sent to the channel. That is, BTP helps to alleviate collisions and benefits the throughput. In random access MAC protocols, nodes do not know when a packet will arrive, so BTP cannot be avoided.

In reservation based MAC protocols, control packets are sent in a random access manner, so the impact of BTP on them is same as discussed above. As for the packets sent at the reserved/scheduled times, BTP may cause disruptions. Specifically, in order to improve the channel utilization, a node may be scheduled to transmit packets when it is overhearing [7]–[11], [18]. Because of BTP, however, such packets cannot be sent at the scheduled times. This would disturb the schedule and may cause collisions. Therefore, BTP may impair the system performance.

For reservation based MAC protocols, BTP can be handled by ensuring that a scheduled packet is transmitted only when the modem is idle in addition to guaranteeing the conventional requirements for collision avoidance [11]. Nevertheless, this solution only works for the packets whose sending times can be pre-scheduled. The sending patterns of control packets are still affected by BTP because they are sent to the channel following a random access (i.e., ALOHA-like) approach [7], [9], [18], [19].

From the discussion above, we can see that there is no straightforward solution to avoid BTP for packets sent in a random access manner, so it is desirable to analytically understand how BTP affects random access MAC protocols.

III. ALOHA WITH BTP

ALOHA is the most fundamental MAC protocol and has been used by many other advanced ones [7], [9], [18]–[22]. In these protocols, control packets are transmitted using a random access (i.e., ALOHA-like) approach. Thus, it is critical and meaningful to study the impact of BTP on ALOHA first. In this section, we introduce the classic model of successful transmission probability for ALOHA and investigate how BTP affects its accuracy.

A. Classic Successful Packet Transmission Model

ALOHA is originally designed for radio networks, and the model of its successful transmission probability is proposed in Ref. [23]. After considering the high bit error rate in underwater acoustic channel, the model becomes

$$P_s = (1 - BER)^L e^{-2m\lambda T_L}$$

where $\lambda$ is the Poisson traffic rate per node, $m$ is the expected number of neighbors, $T_L$ is the transmission time of a packet containing $L$ bits, $BER$ is the bit error rate, and $(1 - BER)^L$ is the probability of no error in the received packet. Since Eq. 1 has been widely referenced, we call it classic model of successful transmission probability of ALOHA and classic model for short.

Although the classic model is proposed for radio networks, it has been proved that this model can still capture the ALOHA behaviors underwater with long propagation delays [21].

Although BTP is determined by acoustic modems and CSMA is implemented in protocols, ALOHA with BTP behaves similarly to CSMA to some extent. CSMA has been studied and modeled very early in radio networks [25]. However, this model is for one-hop networks only. Recently, there is a model developed for CSMA/CA in multi-hop radio networks [26]. Nevertheless, propagation delays are usually ignored in radio networks and thus the results cannot be applied to long propagation delay characterized UANs. Because of long propagation delays, the effect of a transmission on involved nodes lasts for not only the transmission period (the case in radio networks) but also propagation delay’s longer around the transmission period, which makes the analysis of CSMA or ALOHA with BTP in UANs much more complicated. Although there is a model of CSMA for UANs considering long propagation delays [27], this model only works for single receiver/multiple senders scenario and with a strong assumption that all senders have the same propagation delays to the receiver.

B. Validation of the Classic Model

We conduct simulations to evaluate the performance of ALOHA in BTP and no BTP scenarios respectively. Meanwhile, we compare the classic model with simulation results to validate its accuracy. In the simulations, there are 500 nodes randomly distributed in a $5000m \times 5000m \times 3000m$ area. The transmission range of a node is 600m. As for the transmission rate and preamble, parameters of Teledyne Benthos modem are adopted, i.e., the effective transmission rate is 676 bps and the preamble is 1.5s. Setting Poisson traffic rate $\lambda$ to 0.05 pkt/s and varying packet size from 100B to 1000B produce the results in Fig. 2.

As shown in Fig. 2, the successful transmission probability decreases as the packet size increases for simulation results in both BTP and no BTP scenarios. This is because a longer packet requires a longer transmission time and accordingly collides with other packets with a higher probability. In addition, we can see that the classic model closely matches the simulation results when BTP is ignored. This confirms that the classic model still works in the long propagation delays characterized UANs. However, there is a large gap between the classic model and the simulation results considering BTP. This indicates that BTP significantly changes the collision behaviors in ALOHA and the classic model cannot accurately characterize the collision probability when BTP is considered.

Because ALOHA and its collision model are the bases of the aforementioned works, it is critical to develop an accurate model of successful transmission probability for ALOHA with
consideration of BTP. This would help to guide the future underwater MAC design and analysis.

C. Why not Reuse Model of CSMA for Radio Networks?

Till now, some readers may comment that ALOHA with BTP behaves quite similarly to CSMA! So why do not we reuse the classic model of CSMA? In other words, why do we bother to develop a new model? We examine this question as follows.

CSMA has been extensively studied in the literature. At the beginning, CSMA was modeled in one-hop radio networks [23], [25]. Later on, a multi-hop model for CSMA was developed in [28]. Based on this model, there are many analytical works on CSMA/CA-based networks like 802.11 [26], [29]. These models, however, were developed for radio networks. Thus, they reasonably ignore propagation delays. In other words, the classic CSMA models cannot be applied in UANs, which feature long propagation delays.

Further, in UANs, long propagation delays also affect the transmissions in CSMA. Particularly, the propagation delays between a sender and its neighbors are different, so even the same transmission from a sender arrives at its neighbors at different times. In addition, due to long propagation delays, not like radio networks, in UANs a successful initiated transmission mission does not guarantee correct reception. These differences make the analysis of CSMA in UANs much more complicated than in radio networks.

IV. MODELING ALOHA WITH BTP

In this section, we model the successful transmission probability of ALOHA considering BTP in UANs. First, we introduce the network model and notations. Then, we present the modeling framework. Following this framework, we show the model derivation step by step.

A. Network Model and Notations

Throughout the analysis in this paper, we use the following network model and notations.

- Similar to many underwater MAC analysis works [21], [30], [31], nodes are randomly distributed in a multi-hop network area with density \( \rho \). The transmission range of a node is \( R \). Thus, each node has an average of \( m = \frac{4\pi R^2}{3} \rho \) neighbors.
- The traffic generated by each node follows an independent identical Poisson process with the same traffic rate \( \lambda \). The modem transmission rate is \( B \), and each generated packet contains a preamble of duration \( T_p \). Each packet has \( L \) bits and its transmission time is \( T_L = T_p + L/B \).
- The sound speed in water is \( v \). For a given pair of nodes \( N_1 \) and \( N_2 \), the propagation delay between them is \( T_{N_1N_2} \), and their distance is \( d_{N_1N_2} = v T_{N_1N_2} \).
- Bit error rate is \( BER \), and then the probability \( P_{nc} \) of no packet error caused by channel is \( (1 - BER)^L \).

In the proposed model, whenever a packet arrives at the MAC layer, it will be transmitted immediately. In addition, retransmission is not considered. These are consistent with the assumptions of the classic model such that the proposed model can specifically show how BTP affects the collision behaviors in ALOHA. Although it is more practical to consider the adjacent channel interference like that in Ref. [32], due to the same reason, we assume any overlapped packet receptions can corrupt all involved packets, which is similar to that in the classic model. In addition, because there is no retransmission, a packet will be dropped if failed to be transmitted due to collisions or BTP.

B. Modeling Framework

To model the successful transmission probability of ALOHA, we need to consider all possible conflicts. In contrast with no BTP scenario, there are more types of conflicts causing a failed transmission when BTP is considered. Particularly, common neighbors affect packet collisions in a different and much more complicated behavior. Next, we will refer to Fig. 3 to illustrate these conflicts.

As shown in Fig. 3(a), node \( N_N \) is a neighbor of \( N_S \), and \( N_C \) is a common neighbor of \( N_S \) and \( N_R \). \( N_H \) is a neighbor of \( N_R \) while a hidden terminal of \( N_S \). For a given packet from \( N_S \) to \( N_R \), there are totally four \(^1\) types of conflicts as described below.

1) Rx/Tx and Tx/Tx Conflicts at the sender: As shown in Fig. 3(b), node \( N_S \) cannot transmit at time \( t_2 \) because it is busy with receiving a packet from its neighbor \( N_N \), which is a Rx/Tx conflict. This is a new issue caused by BTP. In addition, \( N_S \) cannot send a packet to the channel at \( t_4 \) because it is still transmitting another one. This is a Tx/Tx conflict.

2) Rx/Rx Conflict caused by hidden terminals at the receiver: Still referring to Fig. 3(c), we can see that at receiver \( N_R \), the packet sent by \( N_S \) at time \( t_2 \) collides with the packet sent by \( N_S \)'s hidden terminal \( N_H \) at time \( t_1 \). The collision behavior

\[^1\] Strictly speaking, there are five possible conflicts considered. Because Rx/Tx and Tx/Tx Conflicts at the sender cause failed transmissions in a similar way, we put them in the same category and analyze them together.
here is similar to that in the conventional analyses of hidden terminals.

3) Rx/Rx Conflict caused by common neighbors at the receiver: In Fig. 3(d), $N_S$ transmits a packet at time $t_1$. Unfortunately, at $N_R$, this packet collides with the packet sent by $N_C$ at time $t_2$, where $N_C$ is a common neighbor of $N_S$ and $N_R$. This case is similar to the conflicts caused by hidden terminals, but note that BTP may alleviate the conflicts caused by common neighbors. For example, $N_C$ is unable to transmit its packet at $t_2$ because it is receiving the packet from $N_S$. If $N_C$ can send this packet out like in no BTP scenario, it would collide with the packet from $N_S$ at $N_R$. Thus, common neighbors cause collisions in different behaviors from hidden terminals. This is also determined by BTP and has not been studied yet.

4) Tx/Rx conflict at the receiver: Due to half-duplexity, a node cannot receive when it is transmitting, and this would result in a Tx/Rx conflict. In Fig. 3(e), for instance, the receiver $N_R$ cannot receive $N_S$’s packet sent at $t_2$ because it is transmitting. Similar to the case in Fig. 3(d), the behavior of this kind of conflict is also affected by BTP and different from that considered in conventional analyses.

If all these conflicts can be avoided and there is no error caused by the channel, a successful packet transmission can be guaranteed. We have introduced $P_b$ to be the probability of no packet error caused by the channel. Here we also introduce $P_{b_{Rx}}$ to denote the probability that a node is busy with transmitting or receiving, and then $P_{b_{Tx}}$, which is defined as $1 - P_{b_{Rx}}$, indicates the probability of no Rx/Tx or Tx/Rx conflict at the sender, i.e., the probability that the sender can transmit. We also define $P_{b_{Rx}Rx}$ as the probability of no Rx/Rx conflict caused by hidden terminals, $P_{b_{Rx}Rx}$ as the probability of no Rx/Rx conflict caused by common neighbors, and $P_{b_{Tx}Rx}$ as the probability of no Tx/Rx conflict at the receiver. Because these conflicts are mutually independent, the successful transmission probability can be obtained as $P_n = P_{b_{Rx}Rx} P_{b_{Rx}Rx} P_{b_{Rx}Rx}$.

Till now our discussion is based on a given pair of sender $N_S$ and receiver $N_R$, i.e., their distance $d_{N_S, N_R}$ is fixed. In order to get the expected transmission success probability $P_a$, we need to consider the distribution of $d_{N_S, N_R}$, whose probability density function is $-\frac{3d_{N_S, N_R}^2}{R^3}$ for $0 < d_{N_S, N_R} \leq R$. Then we can have

$$P_a = \int_{d_{N_S, N_R}} P_{b_{Rx}Rx} P_{b_{Rx}Rx} P_{b_{Rx}Rx} \times P_n \, d_{N_S, N_R}$$

Next we will derive $P_{b_{Rx}Rx}$, $P_{b_{Rx}Rx}$, and $P_{b_{Rx}Rx}$, respectively. Then, $P_a$ can be obtained via Eq. 2.

C. Derivation of $P_{b_{Rx}Rx}$

First, we give the derivation of $P_{b_{Rx}Rx}$, which is the probability of no Rx/Tx or Tx/Rx conflict at the sender. As shown in Fig. 4, after the sender $N_S$ starts to receive a packet, $N_S$ cannot transmit during the coming $T_L$ time period because of BTP. In addition to the packets from the $m$ neighbors of $N_S$, $N_S$ could also make itself busy for $T_L$ time period because of the Tx/Tx conflict. Thus, there are totally $m+1$ nodes which may render the sender busy. We assume the propagation delays between the sender and these $m+1$ nodes are $T_i$ ($1 \leq i \leq m+1$) respectively. For all these $m+1$ nodes, if the $i^{th}$ node starts to transmit at time $t$. Thus, the corresponding probability is

$$P_n = \left(1 - e^{-\lambda T_L}\right)^m$$

If all these $n$ packets cannot be sent out due to BTP for all $0 \leq n < \infty$, the $i^{th}$ node will not make the sender busy at time $t$. The corresponding probability is

$$P_n = \left(1 - e^{-\lambda T_L}\right)^m$$

Note that each node generates traffic following a Poisson process, and we can have the probability $P_n$ that the $i^{th}$ node tries to transmit $n$ packets during the interval $[t - T_i - T_L, t - T_i]$ as below

$$P_n = \left(\frac{\lambda T_L}{n!}\right)^n e^{-\lambda T_L}$$

With probability $(\sum_{n=0}^{\infty} P_n n n)\left(\sum_{n=0}^{\infty} P_n n n\right)^{m+1}$, all these $m+1$ nodes do so and the sender can transmit at time $t$. Recall that this probability has been defined as $P_{b_{Rx}Rx}$, so we have

$$P_{b_{Rx}Rx} = \left(\sum_{n=0}^{\infty} P_n n n\right)^{m+1}$$

Substituting Eq. 3 into Eq. 5 and applying the conclusion in Appendix A yield

$$P_{b_{Rx}Rx} = e^{-\lambda T_L(1 - P_n(m+1))}$$

Recall that $P_{b_{Rx}Rx} = 1 - P_n$. Then we can solve Eq. 6 and get

$$P_n = \frac{W(\lambda T_L(m+1))}{\lambda T_L(m+1)}, \quad P_{b_{Rx}Rx} = \frac{W(\lambda T_L(m+1))}{\lambda T_L(m+1)}$$

where $W(*)$ is Lambert W function [33].

D. Derivation of $P_{b_{Rx}Rx}$

In this subsection, we derive $P_{b_{Rx}Rx}$, the probability that there is no Rx/Rx conflict with the packets from hidden terminals at the receiver. We will first analyze the collision window of a hidden terminal, which helps to straightforwardly obtain $P_{b_{Rx}Rx}$.

As demonstrated in Fig. 5, $N_H$ starts to transmit a packet to $N_R$ at time $t$ and $N_R$ will receive this packet during the interval $[t + T_{H}, t + T_{H} + T_L]$. If $N_R$, a hidden terminal of $N_S$, transmits any packet during the interval $[t + T_{H}, t + T_{H} + T_L]$, it will cause a collision at $N_R$. This interval is called collision window. Note that $N_H$ is an arbitrary hidden terminal, and
thus we have that the duration of a hidden terminal’s collision window is $2T_L$, which is same to that in no BTP scenario.

Let $n_h$ be the number of hidden terminals, which locate in the vertical area in Fig. 6(a). Given $d_{NSNR}$, $n_h$ is

$$n_h = \rho \pi \left( 4R^2/3 - 2(R - d_{NSNR}/2)^2 \right) \times (2R/3 + d_{NSNR}/6) \tag{8}$$

If all these $n_h$ hidden terminals cannot transmit during the collision window of size $2T_L$ because of BTP, $N_S$’s packet will not collide with any packet from hidden terminals. Hence,

$$P_{R_xR_z}^{N_h} = \left( \sum_{n=0}^{\infty} P_n^h \frac{(2T_L \lambda)^n e^{-2T_L \lambda}}{n!} \right)^{n_h} \tag{9}$$

Further, applying the conclusion in Appendix to Eq. 9 gives

$$P_{R_xR_z}^{N_h} = e^{-2T_L \lambda (1-P_b)n_h} \tag{10}$$

E. Derivation of $P_{R_xR_z}^{N_C}$

In this subsection, we will present the model of $P_{R_xR_z}^{N_C}$, the probability of no Rx/Rx conflict caused by common neighbors at the receiver. As shown in Fig. 6(a), node $N_C$ is a common neighbor of $N_S$ and $N_R$, and $N_S$ starts to transmit a packet destined to $N_R$ at $t$. We will refer to this example to study the collision window of a common neighbor.

When BTP is ignored, the collision window is illustrated in Fig. 6(b). Similar to the analysis in Section IV-D, we can get that $N_C$’s collision window is $[t + T_{NSNC} - T_{NCNR}, t + T_{NSNR} - T_{NCNR} + T_L]$. When BTP is considered, however, the collision window changes. As demonstrated in Fig. 6(c), $N_S$’s packet passes $N_C$ during the time interval $[t + T_{NSNC}, t + T_{NSNR} + T_L]$. Because of BTP, $N_C$ cannot start a transmission during this $T_L$ period. Accordingly, this interval should be excluded from the collision window of $N_C$. Note that $N_S$, $N_C$ and $N_R$ form either a triangle or a straight line in Euclidean space, so $\min(T_{NSNR} - T_{NCNR}, T_{NSNC})$ holds. Based on this inequality, we have $t + T_{NSNR} - T_{NCNR} + T_L \leq t + T_{NSNC} < t + T_{NSNR} < T_{NSNC}$. Then, the collision window of $N_C$ becomes $[t + T_{NSNR} - T_{NCNR} - T_L, \min(t + T_{NSNR} - T_{NCNR} + T_L, t + T_{NSNC})]$. Additionally, $N_S$ can transmit at time $t$, so it is not busy at that time. Because $N_C$ is $N_S$’s neighbor, $N_C$ does not send any packet out during interval $[t - T_{NSNC} - T_L, t - T_{NSNC}].$ Otherwise, because of BTP, $N_S$ must be receiving the packet from $N_C$ instead of transmitting its own packet at time $t$. Therefore, this interval should also be excluded from the collision window of $N_C$. By applying the triangle inequalities to the triangle formed by $N_C$, $N_S$ and $N_R$ again, we have $t - T_{NSNC} - T_L \leq t + T_{NSNR} - T_{NCNR} - T_L$ and $t - T_{NSNC} < t + T_{NSNR} - T_{NCNR} + T_L$. Also, intuitively we have $t - T_{NSNC} \leq t + T_{NSNC}$. Then, $N_C$’s collision window becomes $\lceil \min(t - T_{NSNC}, t + T_{NSNR} - T_{NCNR} - T_L), \min(t + T_{NSNR} - T_{NCNR} + T_L, t + T_{NSNC}) \rceil$. In order to obtain $P_{R_xR_z}^{N_C}$, we need to calculate the size $T_{CW}$ of the collision window. $T_{CW}$ is determined by $T_L$ and the relative positions among $N_S$, $N_R$ and $N_C$. Based on their relationships, we can obtain $T_{CW}$ under different constraints as shown in Appendix B.

Then we can start to calculate $P_{R_xR_z}^{N_C}$. For clarity, we introduce a coordinate system as depicted in Fig. 6(a), in which the nodes $N_S$ and $N_R$ are on the $x$ axis and their midpoint is the origin. Assuming that $N_C$ is at $(x, y)$ subject to $y \geq 0$ on the X-Y plane, it is straightforward to get that there are $\lceil \min(t - T_{NSNC}, t + T_{NSNR} - T_{NCNR} - T_L), \min(t + T_{NSNR} - T_{NCNR} + T_L, t + T_{NSNC}) \rceil$. By applying the triangle inequality, we need to calculate the size $2T_L$ of the collision window of $N_C$. Based on their relationships, we can obtain $T_{CW}$ under different constraints as shown in Appendix B.

F. Derivation of $P_{R_xR_z}^{N_R}$

In addition to the collisions caused by hidden terminals and common neighbors, the receiver $N_R$ may cause collisions due to Tx/Rx conflicts. In this subsection, we model the corresponding probability $P_{R_xR_z}^{N_R}$.

Since $N_R$ is in the common neighbor region, the conclusion derived in section IV-E is also applicable here. Specifically, by
assumining \( N_C \) is at the same position as \( N_R \), we can get the interval that \( N_R \) may cause a conflict is \( \max \{ t - T_{NSNR}, t + T_{NSNR} - T_L, t + T_{NSNR} \} \). Then, the size \( T_{CR} \) of the collision window is given as

\[
T_{CR} = \begin{cases} 
T_L & T_{NSNR} \geq T_L/2 \\
2T_{NSNR} & 0 < T_{NSNR} < T_L/2 
\end{cases}
\] (15)

If \( N_R \) does not transmit any packet during its collision window of size \( T_{CR} \), there will be no Tx/Rx conflict at \( N_R \). Accordingly, \( P_{TxxR} \) is

\[
P_{TxxR} = \sum_{n=0}^{\infty} P_b n \frac{(\lambda T_{CR})^n e^{-\lambda T_{CR}}}{n!}
\] (16)

Further, applying the conclusion in Appendix A gives

\[
P_{TxxR} = e^{-\lambda T_{CR}(1-P_b)}
\] (17)

V. MODEL VALIDATION

In this section, we validate the proposed model with AquaSim [34], a NS-2 based simulator for UANs. We demonstrate and compare the simulation results of the classic model and the proposed model in both BTP and no BTP scenarios. We also investigate the impact of edge effects [35], which cause gaps between the proposed model and simulation results. At last, we discuss the connection to the networks in the real world, whose network sizes may be very small and suffer severe edge effects.

A. Simulation Settings

Unless specified otherwise, the simulations adopt the following settings: there are 500 nodes randomly distributed in a 3-Dimensional space of 5000m x 5000m x 3000m. BER is 10^{-5}. Node’s transmission range is 600m. Every node generates packets of length 500B following an independent identical Poisson process with traffic rate \( \lambda = 0.05 \) pkt/s. As for the modem transmission rate and preamble, we refer to the parameters of both Teledyne Benthos modem and OFDM modem. With the former one, the effective transmission rate can reach 667bps in the horizontal channel and the packet preamble is about 1.5s. In the same channel environment, an OFDM modem can transmit at 3045bps with a packet preamble of duration 0.486s [36]. We run simulations with each set of modem parameters respectively, which helps to sufficiently validate the proposed model.

B. Effects of Packet Size

In this part, we evaluate the effects of packet size by increasing it from 100B to 1000B by step 100B. Applying the Teledyne Benthos modem settings and OFDM settings produces the results in Fig. 7(a) and Fig. 7(b) respectively.

From both Fig. 7(a) and Fig. 7(b), we can see the classic model coincides with the simulation results in no BTP scenario. This confirms that the classic model is valid in UANs when BTP is ignored. We can also see there is a large gap between it and the simulation results with BTP. This is because the classic model does not consider BTP. As the packet size increases, the gap becomes much larger. The essential reason is that a longer packet results in a longer transmission time, which makes BTP more severe and more significantly affect the system performance.

As for the proposed model, delightfully, both Fig. 7(a) and Fig. 7(b) show that it can well match the simulation results with BTP. This indicates that the proposed model can well capture the collision behaviors in ALOHA.

Compared with the simulation without BTP, as shown in Fig. 7, there are fewer collisions when BTP is considered, especially with a large packet size. As discussed in Section II-B, this is because BTP reduces the collisions. Specifically, when a packet is short, like the beginning part in Fig. 7(a), its transmission time is also short. In this case, most of packets reduced by BTP are supposed to be delivered successfully in no BTP scenario. That is, ALOHA with consideration of BTP delivers fewer packets than that when BTP is ignored. Since BTP rarely occurs when packets are short, these two sets of simulation results are close at the beginning. However, when packets get longer, which result in longer transmission times, nodes are busy more frequently and most packets reduced by BTP are supposed to cause collisions in no BTP scenario. From this perspective, BTP helps ALOHA achieve a higher \( P_s \).
C. Effects of Packet Generation Rate

The proposed model is also validated by increasing the packet generation rate $\lambda$ from 0.01 to 0.2 by step 0.01 pkt/s. The results are demonstrated in Fig. 8.

From Fig. 8, we can see that the classic model closely matches the simulation results in no BTP scenario while significantly differs from the simulation results in BTP scenario. Because BTP is the only difference between these two sets of simulations, BTP must be the root reason changing the collision behaviors in ALOHA. Unfortunately, the classic model does not consider this issue and thus is not accurate. As for the proposed model, it well coincides with the simulation results with BTP in both sub-figures. Again, these results justify that the proposed model can well characterize the collision behaviors in ALOHA when BTP is considered.

Specifically, as $\lambda$ increases, a node becomes busy more frequently and thus BTP affects the collision behaviors more significantly. Therefore, the gap between these two sets of simulation results becomes larger as $\lambda$ increases. In addition, we can see $P_s$ drops as $\lambda$ increases for all curves in Fig. 8. This is because more packets are sent in the unit time and leads to more collisions with a higher $\lambda$. According to the discussion in Section II-B, it is also reasonable that BTP reduces the traffic when $\lambda$ is relatively low at the beginning part in Fig. 8(a) and helps to alleviate collisions when $\lambda$ is high in both Fig. 8(a) and Fig. 8(b) comparing with no BTP scenario.

D. Effects of Transmission Range

According to the classic model, node’s transmission range does not affect $P_s$ when the expected number of neighbors is fixed. On the contrary, the proposed model indicates that node’s transmission range does affect. To investigate its effect, we increase the transmission range from 400m to 1000m by step 100m. Meanwhile, we proportionally scale the network size to make sure the average number of neighbors does not change. The simulation results are given in Fig. 9.

From both Fig. 9(a) and Fig. 9(b), we can see the simulation results do not consider BTP has a constant $P_s$. This is consistent with the conclusion based on the classic model, i.e., node’s transmission range $R$ does not affect $P_s$.

When BTP is considered, however, we can observe that $P_s$ drops as $R$ increases. This is because $R$ changes $T_{CW}$. When packet transmission time $T_L$ is fixed, the longer $R$ is, the larger $T_{CW}$ is. Therefore, a longer $R$ results in a lower $P_s$. But note that $T_{CW}$ is upper bounded by $2T_L$, so $P_s$ in BTP scenario is lower bounded by that in no BTP scenario. In addition, we can see the proposed model can accurately characterize the effect of $R$. Particularly, in both subfigures, the gap between the proposed model and simulation without BTP is very small and only up to 0.02.

E. Effects of Modem Transmission Rate

In order to sufficiently validate the proposed model with different modem transmission rates, we vary it from 500 bps to 6000 bps. For solely showing the impact of transmission rate, the preamble is always set to 0.3s. The simulation results are shown in Fig. 10.

From Fig. 10, we can observe that the classic model matches the simulation results without consideration of BTP but its trend is significantly different from that of simulation results considering BTP. By contrast, we can see that the proposed model is very close to the simulation results considering BTP and they always follow the same trend. Based on Fig. 10, we can conclude that the proposed model can characterize the impact of BTP accurately.

From Fig. 10, we can also observe that the gap between these two sets of simulations become larger as the transmission rate increases. This is because a higher transmission rate results in a shorter packet transmission time. When the packet transmission time is shorter, a node is accordingly busy with receiving a packet for a shorter period and BTP occurs less frequently. As a result, the simulation results considering BTP are closer to that ignoring BTP as the transmission rate goes up.

F. Understanding Edge Effects

Through the simulation results above, we can see there are small gaps between the proposed model and simulation results without consideration of BTP. After extensive investigations,
we notice edge effects exert [35] influence on the gap between the simulation and the proposed model. Specifically, the nodes near the network edge exhibit a different behavior than those towards the center because they have fewer neighbors. This is referred to as edge effects. In order to observe the impact of edge effects, we simulate the scenario of 960 nodes in a 3-Dimensional area of 6000m×6000m×4000m, which can ensure the same node density but a smaller portion of nodes at the edge, i.e., less severe edge effects. By varying the packet size, we get the results shown in Fig. 11.

From Fig. 11, we can see that $P_s$ becomes lower with a larger network size. This is because there is a smaller portion of nodes near the edge. Specifically, the successful transmission probabilities of nodes near the edge are higher than others because they have fewer neighbors. Thus, $P_s$ is lower when there are smaller portion of nodes near the edge. In addition, we can see the proposed model can better match the simulation results when the network area is larger. This is reasonable. Specifically, in the network of size 6000m×6000m×4000m, there is larger portion of nodes having equal expected number of neighbors. This can better match the assumption of equal number of neighbors in the proposed model, and thus the proposed model can better coincide with the simulation results. If the network size approaches to infinite, almost all nodes would have equal number of neighbors and the proposed model will perfectly match the simulation results.

G. Connection to the Real World

From the simulation results above, we can see edge effects impair the accuracy of the proposed model and its impact cannot be neglected in small networks. In the real world, however, sometimes the network size is relatively small and most nodes have different numbers of neighbors. In this case, edge effects are severe and we cannot apply the proposed model directly. But fortunately the proposed modeling framework still works and we can model the successful transmission probability $P_s$ of no Tx/Tx or Rx/Tx conflict. Then, based on nodes’ positions and $P_{s}$’s of no Tx/Tx or Rx/Tx conflict, we can calculate $P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}$ and $P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}$ for each sender/receiver pair following the corresponding derivations in Section IV. The only difference is that we need to change the continuous node distribution to the discrete one gotten based on the network topology. Finally, $P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}$ can yield the $P_s$ between this sender/receiver pair. Taking average of $P_s$ between one node (sender) and all its neighbors (receivers) would give the expected $P_s$ of this node.

VI. CASE STUDY

In this section, we will use a case study to show why an accurate model of successful transmission probability is important and how it helps to solve other network problems. Specifically, we will develop a nodal throughput model and optimize packet generation rate $\lambda$ to maximize nodal throughput $\Lambda$.

On average, each node transmits $\lambda$ packets per second with a successful transmission probability $P_s$. Thus, one node can successfully deliver $\Lambda P_s$ packets per second. Because each packet carries $L$ bits data, $\Lambda$ is

$$\Lambda = \lambda P_s$$ (18)

If $R$ and $T_L$ are given, intuitively $\Lambda$ goes up as $\lambda$ increases from 0 at the beginning. This is because more data packets are successfully delivered with a low packet collision probability. As $\lambda$ continuously increases, the collision probability is so high that most packets collide and $\Lambda$ drops. Therefore $\Lambda$ is convex with respect to $\lambda$. Then, the optimum $\lambda$ to maximize $\Lambda$ must satisfy Eq. 19.

$$\frac{d\Lambda}{d\lambda} = 0$$ (19)

Substituting Eq. 2 for $P_s$ and converting the expression to iteration format with respect to $\lambda$ yield Eq. 20.

$$\lambda = \frac{\int_{0}^{R \frac{3d_{NS}N_{R}^{2}}{R^2}P_{N_{R}N_{R}}^{N_{R}N_{R}}P_{N_{R}N_{R}}^{N_{R}N_{R}}P_{N_{R}N_{R}}^{N_{R}N_{R}}dN_{S}N_{R}}{\int_{0}^{R \frac{3d_{NS}N_{R}^{2}}{R^2}dN_{S}N_{R}}}}$$ (20)

Recall that $\Lambda$ is convex with respect to $\lambda$. Thus, using the numerical approach (iteration) we can get the optimum $\lambda$ to maximize $\Lambda$ via Eq. 20.

Similarly, based Eq. 1 and Eq. 19, we can get the optimum $\lambda$ given by the classic model is

$$\lambda = (2nT_L)^{-1}$$ (21)

We conduct simulations to validate the optimum $\lambda$ obtained via both models. The simulation settings are same to that in section V-C and Teledyne Benthos modem parameters are used. According to the simulation settings, for the proposed model, $\bar{R}$ is less than $vT_L/2$. Then, based on the integral domains illustrated in Appendix C, we can get $P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}$ as below.

$$P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}} = e^{-2\lambda T_{CR}R_{N_{Rx}N_{Rx}}}$$ (22)

Similarly, we have $T_{CR}=RT_{N_{S}N_{R}}$ according to Eq. 15. Then, based on Eq. 17, $P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}}$ is

$$P_{N_{Rx}N_{Rx}}^{N_{Rx}N_{Rx}} = e^{-2\lambda T_{CR}R_{N_{Rx}N_{Rx}}}$$ (23)

After substituting Eqs. 7, 10, 22, and 23 into Eq. 20, we can calculate the optimum $\lambda$ via the iteration approach.

Fig. 12 shows the nodal throughput obtained via simulation with consideration of BTP, the optimum $\lambda$ as given by Eq. 20 and 21. From Fig. 12, we can observe that the peak nodal throughput 24.39bps appears when $\lambda = 0.046 pkt/s$. Delightedly, the optimum $\lambda$ given by Eq. 20 is 0.061_pkt/s and the corresponding nodal throughput is 23.49bps. The absolute error is only 0.9bps and the relative error is 3.7%. The error is delivered by $P_s$. As shown in Fig. 8(b), the $P_s$ for $\lambda = 0.06 pkt/s$ given by the proposed model is almost same to the $P_s$ for $\lambda = 0.05 pkt/s$ in simulation without BTP. The error of $P_s$ finally determines the error of the optimum $\lambda$. As analyzed in Section V-F, the error of $P_s$ is essentially caused by edge effects, and the optimum $\lambda$ is expected to be much closer to the simulation results as network size becomes larger.
We can also see the optimal $\lambda$ given by Eq. 21 is 0.0111bps, whose error is intolerable. The root reason is that the classic model cannot capture the collision behavior in ALOHA when BTP is considered.

From this case study, we can learn that it is critical to accurately model the probability of successful transmission because it is not only a key metric of a MAC protocol but also the fundamental to solve many other MAC problems.

Fig. 12. Optimum $\lambda$

VII. CONCLUSION AND FUTURE WORK

In this paper, we have formally defined a newly identified issue, the busy terminal problem, in UANs. This new problem significantly changes the collision behaviors and affects performance of MAC protocols. In order to analytically study the impact, we have further developed a theoretical model to analyze the successful transmission probability of ALOHA in underwater acoustic networks. Through extensive simulations, we find the proposed model can well characterize the collision behaviors in ALOHA. In addition, based on the proposed model, we develop a nodal throughput model and optimize $\lambda$ to maximize nodal throughput. The obtained optimum $\lambda$ is also confirmed by the simulation results.

Future Work Since control packets usually randomly access the channel in reservation based MAC protocols, the work in this paper would help to provide a better understanding of collision behaviors in reservation based MAC protocols for UANs. Along this line, we plan to investigate the impact of the busy terminal problem on reservation based MAC protocols and explore new practical solutions correspondingly.

REFERENCES

Note that by subtracting the left endpoint from the right one, we can evaluate the expression shown in Fig. 13 depending on the relative positions of the integral domains.

**A. Simplification Method**

The simplification of $\sum_{N=0}^{\infty} P_{b}^{N} K^{N} e^{-K} N!$ like expression is frequently introduced in this paper, where $K$ can be any positive coefficient. For clarity, here we show how to simplify this general case. Then, the conclusion in Eq. 24 can be applied directly as needed.

$$\sum_{N=0}^{\infty} P_{b}^{N} K^{N} e^{-K} N! = \sum_{N=0}^{\infty} e^{-K(1-P_{b})} (P_{b}K)^{N} e^{-P_{b}K} N!$$

$$= e^{-K(1-P_{b})} \sum_{N=0}^{\infty} (P_{b}K)^{N} e^{-P_{b}K} N!$$

$$= e^{-K(1-P_{b})}$$

**B. $T_{CW}$ Calculation**

As listed in Eq. 25, there are four cases of interference window subject to constraints $C.1$: $T_{\text{NC}} < T_{\text{NS}}$, $T_{\text{NS}} < T_{\text{NR}}$; $T_{\text{L}}$, and $T_{\text{NR}} = T_{\text{L}}$; and $T_{\text{NS}} < T_{\text{NR}} = T_{\text{L}}$ and $T_{\text{NS}} < T_{\text{NR}} > T_{\text{L}}$, respectively.

$$\begin{bmatrix}
[t+T_{\text{NS}}-T_{\text{NC}}-T_{\text{NR}}-T_{\text{L}}, t+T_{\text{NS}}-T_{\text{NC}}+T_{\text{NR}}+T_{\text{L}}] & C.1 \\
[t+T_{\text{NS}}-T_{\text{NC}}-T_{\text{NR}}-T_{\text{L}}, t+T_{\text{NS}}] & C.2 \\
[t-T_{\text{NS}}-T_{\text{NC}}-T_{\text{NR}}+T_{\text{L}}] & C.3 \\
[t-T_{\text{NS}}-T_{\text{NC}}+T_{\text{NR}}+T_{\text{L}}] & C.4 
\end{bmatrix}$$

By subtracting the left endpoint from the right one, we can have $T_{CW}$ as below:

$$T_{CW} = \begin{cases}
2T_{L} & C.1 \\
T_{L}+T_{\text{NS}}+T_{\text{NR}}-T_{\text{L}}, T_{L}+T_{\text{NS}}+T_{\text{NR}}-T_{\text{L}}+T_{\text{NR}} & C.2 \\
T_{L}+T_{\text{NS}}-T_{\text{NR}}+T_{\text{L}} & C.3 \\
2T_{\text{NS}} & C.4
\end{cases}$$

**C. Integral Domains of $P_{\text{RCR}}$ Calculation**

In this subsection, we divide the integral domain of Eq. 14. Note that $T_{\text{NS}}$ is fixed for a given pair of nodes $N_{R}$ and $N_{S}$. In addition, we assume that all packets have the same length, so $T_{L}$ is constant. Therefore, $T_{\text{NC}} - T_{\text{NS}} = T_{\text{NS}}$, and $T_{\text{NS}}$ is the equation of a hyperbola with focus points $N_{S}$ and $N_{R}$ subject to $T_{\text{NS}}$ subject to $T_{\text{NC}} > T_{\text{NS}}$, $T_{\text{NS}} > T_{\text{L}} > 0$, and $T_{\text{NC}} - T_{\text{NS}} > T_{\text{L}}$ and $T_{\text{NS}} < T_{\text{L}}$. Similarly, $T_{\text{NC}} + T_{\text{NS}} = T_{\text{NS}} + T_{\text{L}}$ is the equation of an ellipse with focus points $N_{S}$ and $N_{R}$ subject to $T_{\text{NC}} + T_{\text{NS}} > T_{\text{NS}} + T_{\text{L}}$. When $T_{\text{NC}} - T_{\text{NS}}$, the integral domain has the subcases shown in Fig. 13 depending on the relative positions of the intersections among $x$ axis, $y$ axis, the ellipse, the hyperbola (may not exist), and the circle. Since some subcases do not satisfy the restrictions imposed by the physical meanings, they are not included here.

In the subfigures of Fig. 13, the horizontal region, vertical region, $15^\circ$ hatched region and the $45^\circ$ hatched region satisfy $C.1$, $C.2$, $C.3$, and $C.4$, respectively. In other words, the $T_{\text{CW}}$ is $2T_{L}$, $T_{\text{L}} + T_{\text{NS}} + T_{\text{NC}} > T_{\text{L}}$, and $T_{\text{L}} + T_{\text{NS}} + T_{\text{NC}} > T_{\text{L}}$, respectively in these four kinds of regions.

When $T_{\text{NC}} - T_{\text{NS}} < T_{\text{NS}}$, we reform Eq. 25 as below to meet the definition of hyperbola.

$$\begin{bmatrix}
[t+T_{\text{NS}}-T_{\text{NC}}-T_{\text{LR}}-T_{\text{L}}, t+T_{\text{NS}}-T_{\text{NC}}+T_{\text{LR}}+T_{\text{L}}] & C.5 \\
[t+T_{\text{NS}}-T_{\text{NC}}-T_{\text{LR}}-T_{\text{L}}, t+T_{\text{NS}}] & C.6 \\
[t-T_{\text{NS}}-T_{\text{NC}}-T_{\text{LR}}+T_{\text{L}}] & C.7 \\
[t-T_{\text{NS}}-T_{\text{NC}}+T_{\text{LR}}+T_{\text{L}}] & C.8
\end{bmatrix}$$
where $C.5$ is $T_{N_5N_C} - T_{N_5N_R} \geq T_L - T_{N_5N_R}$ and $T_{N_5N_C} + T_{N_5N_R} \geq T_{N_5N_R} + T_L$, $C.6$ is $T_{N_5N_C} - T_{N_5N_R} \geq T_L - T_{N_5N_R}$ and $T_{N_5N_C} + T_{N_5N_R} \geq T_{N_5N_R} + T_L$, $C.7$ is $T_{N_5N_C} - T_{N_5N_R} \leq T_{N_5N_R} - T_L$, $T_{N_5N_C} + T_{N_5N_R} \geq T_{N_5N_R} + T_L$, and $C.8$ is $T_{N_5N_C} - T_{N_5N_R} \leq T_L - T_{N_5N_R}$ and $T_{N_5N_C} + T_{N_5N_R} \leq T_{N_5N_R} + T_L$.

In the case of $T_{N_5C_N} < T_{N_5C_N}$, the possible subcases are shown in Fig. 14. In this set of figures, $N_R$ is on the positive $x$-axis and $N_S$ is on the negative $x$-axis, which are different from that in Fig. 13. In Fig. 14, the horizontal region, vertical region, 135° hatched region, and the 45° hatched region satisfy $C.7$, $C.8$, $C.5$, and $C.6$, respectively. That is, in these four kinds of regions, $T_{CW} = T_L + T_{N_5N_C} - T_{N_5N_R} + T_{N_5N_R}$, $2T_{N_5N_C}$, $2T_L$, $T_L + T_{N_5N_C} + T_{N_5N_R} = T_{N_5N_R}$ respectively.

Based on the relative positions of the intersections among the $x$-axis, $y$-axis, the ellipse, the hyperbola, and the circle, we can get the constraints to each subcase shown in Fig. 13 and Fig. 14, respectively. Then by grouping the subcases by the relationship between $R$ and $\nu T_L$ in their constraints, we can get the integral domains as below. For clarity, $d_L$ is used to denote $\nu T_L$.

1) If $0 < R \leq d_L/2$, integrate in the domains shown in Fig. 13(b) for $0 < d_{NSNR} \leq R$, Fig. 14(g) for $0 < d_{NSNR} \leq R$.

2) If $d_L/2 < R \leq 3d_L/4$, integrate in the domains shown in Fig. 13(d) for $0 < d_{NSNR} \leq R - d_L/2$, Fig. 13(c) for $R - d_L/2 < d_{NSNR} \leq 2R - d_L$, Fig. 13(b) for $2R - d_L < d_{NSNR} \leq R$, Fig. 14(i) for $0 < d_{NSNR} \leq R - d_L/2$, Fig. 14(h) for $R - d_L/2 < d_{NSNR} \leq 2R - d_L/2$, Fig. 14(g) for $2R - d_L/2 < d_{NSNR} \leq d_L/2$, Fig. 14(d) for $d_L/2 < d_{NSNR} \leq R$.

3) If $3d_L/4 < R \leq d_L$, integrate in the domains shown in Fig. 13(d) for $0 < d_{NSNR} \leq R - d_L/2$, Fig. 13(c) for $R - d_L/2 < d_{NSNR} \leq 2R - d_L$, Fig. 13(b) for $2R - d_L < d_{NSNR} \leq R$, Fig. 14(i) for $0 < d_{NSNR} \leq R - d_L/2$, Fig. 14(h) for $R - d_L/2 < d_{NSNR} \leq d_L/2$, Fig. 14(a) for $d_L/2 < d_{NSNR} \leq 2R - d_L$, Fig. 14(d) for $2R - d_L < d_{NSNR} < d_{NSNR} \leq R$.

4) If $d_L < R \leq 3d_L/2$, integrate in the domains shown in Fig. 13(d) for $0 < d_{NSNR} \leq R - d_L/2$, Fig. 13(c) for $R - d_L/2 < d_{NSNR} \leq d_L$, Fig. 13(a) for $d_L < d_{NSNR} \leq R$, Fig. 14(i) for $0 < d_{NSNR} \leq d_L/2$, Fig. 14(c) for $d_L/2 < d_{NSNR} \leq R - d_L/2$, Fig. 14(b) for $R - d_L/2 < d_{NSNR} \leq d_L$, Fig. 14(e) for $d_L < d_{NSNR} \leq R$.

5) If $3d_L/2 < R$, integrate in the domains shown in Fig. 13(d) for $0 < d_{NSNR} \leq d_L$, Fig. 13(a) for $d_L < d_{NSNR} \leq R$, Fig. 14(i) for $0 < d_{NSNR} \leq d_L/2$, Fig. 14(c) for $d_L/2 < d_{NSNR} \leq d_L$, Fig. 14(f) for $d_L < d_{NSNR} \leq R - d_L/2$, Fig. 14(e) for $R - d_L/2 < d_{NSNR} \leq R$. 