Optimization of ATM cash replenishment with group-demand forecasts

Yeliz Ekinci\textsuperscript{a,b}, Jye-Chyi Lu\textsuperscript{b,*}, Ekrem Duman\textsuperscript{c}

\textsuperscript{a}Industrial Engineering, Istanbul Bilgi University, Istanbul, Turkey
\textsuperscript{b}Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, United States
\textsuperscript{c}Industrial Engineering, Ozyegin University, Istanbul, Turkey

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In ATM cash replenishment banks want to use less resources (e.g., cash kept in ATMs, trucks for loading cash) for meeting fluctuated customer demands. Traditionally, forecasting procedures such as exponentially weighted moving average are applied to daily cash withdraws for individual ATMs. Then, the forecasted results are provided to optimization models for deciding the amount of cash and the trucking logistics schedules for replenishing cash to all ATMs. For some situations where individual ATM withdraws have so much variations (e.g., data collected from Istanbul ATMs) the traditional approaches do not work well. This article proposes grouping ATMs into nearby-location clusters and also optimizing the aggregates of daily cash withdraws (e.g., replenish every week instead of every day) in the forecasting process. Example studies show that this integrated forecasting and optimization procedure performs better for an objective in minimizing costs of replenishing cash, cash-interest charge and potential customer dissatisfaction.

\section*{1. Introduction}

Business analytics is becoming popular in many business practices nowadays. For instance, companies such as Samsung and Coca-Cola have been trying to find ways to utilize available data collected to draw and use the meaningful information for making decisions to improve their manufacturing, logistics or marketing operations. Samsung collects various market data from their retail stores such as customers’ sales quantity, inventory level, order and sales forecast. The data there are synthesized for deciding manufacturing orders and schedules (Ko & Han, 2012). Coca-Cola collects what and how much customers are drinking via RFID. This data helps the company to learn how new drinks are doing in the market, identify differences in regional tastes and help fast-food outlets decide which drinks to serve (Weier, 2009). These instances show that collecting data and using them in an intelligent way for decisions is becoming more and more important in today’s competitive business environment. This manuscript is an attempt to use the collected data to increase the performance of an ATM network.

ATM cash replenishment studies focus on decisions about time schedules that each ATM should be replenished and amount of money that should be loaded. The studies include both forecasting cash demands and optimizing replenishment schedules. This optimization should satisfy both cost constraints of the bank and demand constraints from customers. The ATM cash replenishment is an important real-life problem. One of the challenges in this type of problem is forecasting quality since it includes complicated and noisy data. If a forecasting model has poor quality in predicting future cash demands in ATMs, the replenishment-policy derived based on the forecasted demands will not be effective. Significance of the study stems from the fact that replenishment below the actual demand leads to customer dissatisfaction, while replenishment more than the demand leads to high opportunity cost for the bank. Hence, a methodology, which will optimize the replenishment amount, will be apparently very useful for the banks.

In the literature forecasting and replenishment-policy were studied separately usually by different groups of researchers/practitioners except for the study of Baker, Jayaraman, and Ashley (2013). That is, forecasting studies do not cover the replenishment-policy. The replenishment optimization literature does not include the forecasting process. These isolated approaches cannot solve the above challenges. For instance, when the individual ATM cash demand is forecasted poorly, the optimization model for replenishment will mislead the replenishment decisions.

Consider the vast amount of data variations in each ATM’s demand. In this study, we forecast demand for a group of ATMs, due to the fact that there will be less variability in the aggregated data, the prediction quality would be better. Moreover, since ATMs

* Corresponding author.
E-mail address: jclu@isye.gatech.edu (J.-C. Lu).

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are not generally replenished every day, daily forecasting does not make sense for reducing labor and transportation costs. In our integrated approach a group of nearby ATMs formulates a cluster. The past cash demands in a cluster are used in a replenishment model for deciding the optimal replenishment time interval, i.e., replenishment schedule (using a mathematical model which minimizes the cost). Thus, the demand data are aggregated over ATMs and time intervals to provide a better opportunity of getting a much higher quality forecasting model.

In order to improve the model quality, location variables, which were not considered in the literature are also studied. Since cash is replenished to individual ATMs, group forecast is converted to individual forecasts (using the demand proportions of the ATMs in the group in the past), and finally these individual cash forecasts are loaded to the ATMs. Most business analytics studies deal with the forecasting and optimization tasks separately. This research presents a new venture of integrating informatics (e.g., forecasting) and optimization studies in one framework for solving a difficult real-life problem.

In order to show the potential of the proposed methodology, our procedure is compared against the individual ATM based replenishment-policy by means of forecasting performance and total replenishment cost. Specifically, in the case study, demands for the overlapping time periods that we have data of are studied for one cluster of ATMs for forecasting and optimization comparison and illustrations. It has been seen that forecasting quality of the proposed methodology is higher while the cost is lower. By using our method, necessary amounts are loaded to the ATMs, which will potentially increase customer satisfactions for getting needed cash when they visit ATMs.

The paper includes five more sections. The second section summarizes the previous studies while the third section introduces the proposed methodology. Fourth section illustrates the methodology using the data from a bank in Istanbul, and the fifth section shows comparison details and results. Conclusions and discussions are provided in Section 6.

2. Problem background

2.1. Literature review

The existing literature about ATM cash demand forecasting is divided into two groups. The first group includes studies on demand forecasting at daily level. There are several studies on this topic, which will be summarized below. The second group includes cash replenishment studies. These studies use the forecast values and certain constraints to make a decision about the replenishment policy.

Focus on the first group of literature. After “Forecasting Competition for Artificial Neural Networks and Computational Intelligence” (NNS Competition), the ATM demand forecasting problem became popular. The performance measure was MAPE (mean absolute percentage error) in this competition. The data includes daily cash withdrawals from 111 ATMs in different locations of England. The time frame is two years and the aim is to forecast the next 56 days’ demands (Crone, 2008). The rule in this competition is to use the same model for all ATMs. Andrawis, Atiya, and El-Shishiny (2011) took the first place among the computational intelligence models, where they used some rules such as a simple average to combine forecasts from different models for improving model quality. They also tried to forecast weekly values and then converted them to daily values by a simple linear interpolation scheme. Other work for NNS Competition data includes Coyle, Prasad, and McGimity (2010) who developed a model based on the self-organizing fuzzy neural network, and Wichard (2011) who utilized forecast combination via a simple average idea for improving forecasting quality. Teddy and Ng (2011) reconstructed missing values in the raw data using the weighted average of the last and the next known withdrawal records in the series. Ben Taieb, Bontempi, Atiya, and Sorjamaa (2012) considered the effects of seasonality, input variable selection, and forecast combination.

There are other ATM data used in forecasting studies. For instance, Simutis, Dilijonas, Bastina, and Friman (2007) applied Artificial Neural Networks (ANN) to the three-year data from a bank in Lithuania. Simutis, Dilijonas, and ve Bastina (2008) improved forecasting quality by modeling days of the month by labeling each day from 1 to 31. Brentnall, Crowder, and Hand (2010) studied data collected from 190 ATMs in the United Kingdom over a two-year period. Darwish (2013) applied Interval Type-2 Fuzzy Neural Network (IT2FNN), Zandevakili and Javanmard (2014) used Fuzzy Neural Networks and Arora and Saini (2014) employed Fuzzy ARTMAP Network for simulated data. Gurgul and Suder (2013) modeled withdrawals from selected ATMs of the Euronet network, their idea is the application of Switch ARIMA models to the ATM data.

In most studies cited above, MAPE is usually used as a performance measure in the forecasting studies, and it ranges from 20% to 45%.

There are two important gaps in the previous studies. Firstly, some important days such as festivals and holidays that customers usually need more cash and, can be much unhappy when the money in ATMs is all withdrawn. However, these special days were not examined carefully in the literature. This study uses indicator variables to explore their impact to the cash withdrawn and to improve model quality. Secondly, locations of the ATMs could also be important but only one study mentioned it as a future research topic (Simutis et al., 2007). Note that demography information together with urbanicity of the place where the ATM is located and the points of interests (such as schools, plazas, other banks’ ATMs, shopping centers etc.) in the vicinity can affect the amount of money withdrawn from the ATM significantly. Location variables will also be considered in our studies.

Forecasting daily withdrawals is a very challenging problem for our data collected from a Turkish bank (see Section 4 for details). Since the daily withdraw variations are high, a good forecasting model is difficult to construct, e.g., $R^2$ in predicting individual ATM’s daily withdraw is around 55% and the MAPE is around 28%. Poor forecast might lead to poor replenishment decisions. If ATMs might not be replenished every day, daily forecasting of withdrawals is not needed. This manuscript explores ideas for aggregating daily withdrawals for building forecasting models. The aggregation could be done via time interval or cluster of ATMs. The following provides relevant literature to support these ideas.

Aggregation techniques have been used in demand forecasting. Fisher and Rajaram (2000) presented a methodology for resolving two decisions in designing a merchandise test. One of the decisions was to select the stores to conduct the test, and the other decision was to find a way to create a seasonal forecast for the chain, based on the test results. In order to choose the test stores, they aggregated the stores into clusters that are similar based on historical sales of products, location, size and demographics. Federico, Matteo, Stefano, Roberto, and Guilio (2005) and Kalchschmidt, Verganti, and Zotteri (2006) developed forecasting models using aggregation approaches for handling irregular and/or heterogeneous customer demands. Hadavandi, Shavandi, and Ghanbari (2011) also state that forecast results can be improved by data clustering. Recently, Venkatesh, Ravi, Prinzie, and Van den Poel (2014) has advocated the prediction of cash demand for groups of ATMs with similar day-of-the-week cash demand patterns. They have clustered ATMs into groups having similar withdrawal patterns.
In the second group of the literature the ATM cash replenishment problem has some similarities to the logistics problem. Aggregation of goods in distribution centers is widely used in logistics (Chen, Huang, Chen, & Wu, 2005). Rather than shipping directly from source points to each retail store, retail orders are grouped and shipment decisions are made based on these groupings (Ballou, 1994). Ballou (1994) used zip codes for store clustering. Zarnani, Rahgozar, Lucas, and Taghiyareh (2009) studied the use of spatial clustering. Daganzo (1984) formulated a vehicle routing technique, which is a variant of the classical “cluster-first, route-second” approach. Here, the depot area is first partitioned into districts containing clusters of stops and then vehicle route is constructed to serve each cluster. Gaur and Fisher (2004) and Michel and Vanderbeck (2012) had similar studies.

According to our literature review, it is seen that ATM demand forecast has been always done at daily level. Afterwards, the forecast results were used to decide the replenishment policy (Altunoglu, 2010; Castro, 2009; Simutis et al., 2007). This policy determines the number of days between two replenishments. Van Anholt, Coelho, Laporte, and Vis (2013) solved an inventory-routing problem with pickups and deliveries for replenishing demands of ATMs in a bank at Netherlands. They formulated the problem as a mixed-integer linear programming model, and proposed an exact branch-and-cut algorithm for its resolution. Chotayakul, Charnsathikul, Pichitlamken, and Kobza (2013) determined the amount of money to place in ATMs and cash centers for each period over a given time horizon. They also formulated the problem as a Mixed Integer Program (MIP) and developed an approach based on reformulating the model as a shortest path formulation for finding a near-optimal solution of the problem. Similar to the previous studies, these studies assume the demand as given/known, as well, i.e., it does not perform demand forecasting part. To the best of our knowledge, Baker et al. (2013) is the only study that integrates the cash demand forecast into replenishment decision making. Although this study acknowledges the need of using predicted cash demand, the predictions are performed for each ATM in isolation of the historical data from other ATMs.

2.2. Motivation examples

Many location and indicator variables discussed above have been included to enhance model quality in forecasting demand for optimizing ATM cash replenishment for a Bank at Istanbul. However, the typical R²’s obtained are around 50% and MAPEs are around 30%, which are somewhat unsatisfactory. Based on the above clustering and aggregation ideas, this study expounds ways to improve forecasting quality and integrates it in the replenishment decisions. The following provide modeling results from different aggregations to support our ideas.

2.2.1. ATM grouping

The ATMs that are close to each other have similar cash withdrawal patterns. An example is given in Fig. 1. ATM 1 and ATM 2 are two ATMs in the same county, and they were established on consecutive months: 10/9/2009 and 11/11/2009. ATM 3 is in a different county, and it was established on 4/7/2011. The data is collected from the first day of the ATM’s operation until 4/1/2012. The shapes of the histograms in the same county are similar while the shape of the histogram of the ATM in the other county (which is very far away) is totally different. Moreover, the amounts of withdrawals are also different for ATMs in different counties, probably because of the different characteristics of the vicinities where the ATMs are located.

2.2.2. Aggregation across days

Experience learned from the literature supports aggregating cash withdraws for successive days to improve modeling quality. In Figs. 2a and 2b, the daily and weekly cash withdrawals of a group of ATMs are plotted. In Fig. 2a, there is approximately 900-day data, and it shows a considerable amount of data fluctuations. Additionally, this data has some seasonality but it is hard to model and explain (since the seasonality period is also changing). Fig. 2b shows that the withdrawals in consecutive weeks are more stable. This type of data is easier to model since it is less complicated and less volatile.

We have performed some experiments to compare individual forecasting and group average forecasting performances. The experiments and the findings are given below:

1. Consider grouping nearby ATMs, averaging their daily withdrawals, and further average the resulted data over a seven-day interval, i.e., a week. For a cluster of three ATMs in our pilot study (see details in Section 3), our regression forecasting model has an R² = 0.707. The corresponding MAPE value is 17%. When the cluster forecast is converted to individual ATM forecasts (the method of converting will be explained in Section 3), the R² for them are: 0.616, 0.686, 0.716. The corresponding MAPEs are 25%, 23% and 19%, respectively. These MAPEs are comparable with the MAPEs reported in literature (see Section 2.1).

2. R² of the best model in modeling the average daily withdrawals of the above cluster of three ATMs is 0.665. When this model is applied to the individual ATM data in this group, the R² are: 0.587, 0.677 and 0.511. The corresponding MAPE for each ATM are 27%, 23% and 28%. These R²s and MAPEs are not as good as them in the above studies.

3. R² of the best model from modeling daily withdrawals of the individual ATMs studied above are: 0.530, 0.579 and 0.501. The corresponding MAPEs are 28%, 24% and 31%. These are worse than the two cases studied above.

The results obtained from these experiments show that, group-average forecast has a better performance than the individual forecast, and the forecasting model built from weekly data does a better job than models built from daily data.

3. Methodology

We propose to decide on the replenishment period based on the past data and then to make the forecast for this time period. The demand structure of the ATMs close to each other is similar since the demographics and location properties are the same or very similar for these ATMs. This result motivates us to group the ATMs based on nearest neighborhood. Since the data structures of the ATMs in the same group are similar, we will forecast the average demand of the ATMs in the group. By this way, the data will be less varying and easier to model. The proposed methodology includes six steps which are given below and which will be explained broadly later.

Step 1. Group the ATMs
Step 2. Calculate the daily average cash withdrawals for each group
Step 3. Find the optimal time intervals between two replenishments for each group
Step 4. Based on the previously decided time intervals, develop forecasting models for cash demand
Step 5. Convert group forecast to individual forecast
Step 6. Load individual forecast amount to each ATM in the group
Step 1. The ATMs are grouped using nearest neighborhood approach based on distances. In Section 2.2.1, we have seen that the ATMs that are close to each other have similar cash withdrawal data. Moreover, since the ATMs are close to each other, uploading money to the ATMs in the same group on the same day will decrease the cost of transportation.

Fig. 1. Histograms of three ATMs.

Fig. 2a. Daily average cash withdrawals of a group.
Eq. (1) gives the formula of grouping the ATMs by putting the closest ATMs together. In the equation, the element \( u_{jk} \) is 1 if the \( j \)th ATM is in the same group with \( k \)th ATM, and 0 otherwise. \( ||j, k|| \) is the real (practical) distance between ATM \( j \) and ATM \( k \) and calculated using real maps and roads via Geographic Information Systems (GIS). The same definition applies for \( ||j, i|| \).

\[
u_{jk} = \begin{cases} 1 & \text{if } ||j, k|| \leq ||j, i|| \text{ for each } k \neq i \\ 0 & \text{otherwise} \end{cases}
\] (1)

Using above formula, the groups are generated by putting together the \( j \)th and \( k \)th ATMs if \( u_{jk} \) has the value 1. By using this method, the closest ATMs will be in the same group.

**Step 2.** Daily average cash withdrawals are calculated for each group. This is simply the average of the daily cash withdrawals of all ATMs in the group. In the end, we have one value for each day and each group.

**Step 3.** Using the data prepared in Step 2, the optimal time intervals between two replenishments are found via the following model.

Objective function:

\[
\text{Minimize } \left( \frac{\text{daily effective interest rate}}{365} \right) \times x_i \times i \\
\times \left( \frac{\text{replenished amount}}{100} \right) + \text{cash uploading cost} \\
\times \text{number of cash replenishments}
\]

Constraints:

\[
\sum_{i=1}^{7} x_i = 1, \quad x_i = 0 \text{ or } 1 (i = 1, 2, 3, 4, 5, 6, 7).
\]

In the above linear programming model, the decision variable \( x_i \) is the binary variable denoting if the ATM group is replenished every \( i \) days. \( i \) is decided to take values between 1 and 7 since the bank wants to make replenishments at least once a week in order to check if the ATMs are working. This optimization problem is solved for each group using the past data. The replenished amount is the total demand of \( x_i \) days. The first part of the objective function is the interest cost since there would be an opportunity cost if the bank did not load that amount to the ATMs. The second part gives the replenishment cost. This amount is calculated by a fixed cash uploading cost times the number of cash replenishments. If the ATMs are replenished frequently, this cost will be larger and, smaller otherwise. Cash uploading cost is an amount of money per ATM group. We can also make sensitivity analysis by using different costs.

**Step 4.** In order to develop a forecasting model, the variables should be generated. Based on previous literature and expert opinion the following variables are decided to be included in the model:

- **Dependent variable:** Total amount withdrawn from the ATM group in the decided time period
- **Independent variables:** Time and location related variables

Independent variables include both time and location related variables. The previous literature shows that seasonality is highly examined, therefore we should include time related variables in order to capture the seasonality in the data. Furthermore, using time and location related variables attempt to fill the two important gaps in the previous studies. Some important days such as festivals and holidays were not examined in the literature. Furthermore, location of the ATM was given as a future research by one of the previous studies, while none of them considered the effect of it (Simutis et al., 2007). However, the demographics information together with the urbanicity of the place where the ATM is located and the points of interests (such as schools, plazas, other banks' ATMs, shopping centers etc.) in the vicinity can affect the amount of money withdrawn from the ATM significantly. The dependent variable is the weekly average amount withdrawn from the ATM group.

Some of the independent variables are categorical variables which denote that the week is the week before a religious/national festival or a holiday. There are some other categorical variables which show that the week includes a special day, such as Valentine's day, mothers' day, fathers' day or a religious/national festival. One of the categorical variables denotes that the week includes the first/fifteenth day of the month (this is important since these days...
are the salary days in Turkey). There are also some numeric variables which include the week number (there are 52 weeks in a year and this variable is important to capture the seasonality), the number of points of interests (school, restaurant, shopping center, airport, plaza etc) in 500 m vicinity of the ATM, the number of other ATMs in the street, the population in the street, the number of people who are university graduate in the street. The other variables are given in Appendix A.

Using the above defined variables, forecasting models are developed using linear regression. The formulation is given below:

\[ y_{ij} = a_0 + a_1x_1 + \ldots + a_nx_n + e_{ij} \]

\( y_{ij} \) is the withdrawn amount from \( i \)th group on \( j \)th time point, \( a_0 \) through \( a_n \) are the coefficients, \( x_1 \) through \( x_n \) are the independent variables, and \( e \) is the error and is assumed to be multivariate normally distributed.

Since we are not trying to forecast at daily level, the data is less varying, so we do not have to use complex models. Linear regression will be efficient for our problem. The previous studies, which decide on the optimal replenishment period based on forecast results, use prediction intervals in order to deal with uncertainty. In this study, we make the replenishment period decision in advance by using previous (actual) data, then we make the forecast for this period of time. Therefore we should definitely know how much money to put in the ATM. This is the reason of making point predictions instead of using prediction intervals.

**Step 5.** In order to convert group forecasts to individual forecasts, the proportions of the withdrawn amounts of each ATM in the group is analyzed. Based on these proportions, the group forecast is distributed to each ATM.

**Step 6.** Load individual forecast amount to each ATM in the group.

4. Analysis of the data of a bank

This section gives the demonstration of the idea for a bank which has 152 ATMs in Istanbul. The demand varies significantly between ATMs and between different time points. Additionally, the longest time period that we have data of an ATM is 2.5 years. There are some ATMs which have been recently opened and the time periods for each ATM really differ which makes the problem more difficult.

**Step 1.** Using the latitudes and longitudes of the ATMs in the European side of Istanbul, the ATMs are grouped via Equation 1 given in Section 4. By this way, 27 groups are generated. Fig. 3 shows the locations of the ATMs in Istanbul and the ones in circles represent two examples of the groups.

Istanbul is a very big city with approximately 12 million population and it is a center for trade, industry, business and culture. It is the most crowded city in Turkey since a lot of people immigrate there in order to find a job or to make business. The main characteristics of the districts in Istanbul show big difference. While some of them are more commercial areas including many plazas, big companies, some of them are more residential areas. For example, the right part of the group circled on the right hand side is a highly commercialized area while the vicinity of the left circled group is a more residential area. Furthermore, Istanbul has been the capital of both Byzantine and Ottoman Empires. Due to this fact, it has also touristic areas which are visited by millions of people every year. This area is the southeastern part of the right circle of ATMs. These areas have different demographics and the points of interests located in these areas differ a lot.

**Step 2.** Daily average cash withdrawals and necessary variables are calculated for each group. Since the data has varying time intervals for each ATM due to the fact that the ATMs are established on different dates, it is decided to use the overlapping time period while calculating the variables. To be precise, if there are three ATMs in a cluster, and two of them have two years data and one of them has one year data, then, the recent one year’s data is used for calculations.

**Step 3.** We solved the optimization problem given in the previous section for each group using the past data. The interest rate was taken as 9%. We also made sensitivity analysis for all 27 groups for the following cases:

Case 1: When we take cash uploading cost as 65 TRL, the optimal periods of replenishment for two groups are 6 days, while for others, they are 7 days.

Case 2: When we take uploading cost as 80 TRL, the optimal periods of replenishment for all groups are 7 days.

These results prove that, making the forecast at daily level is unnecessary since it is a very challenging work to do so. Instead, the conclusion we can derive from here is making the forecast basically at weekly level.

**Step 4.** Most of the results in Step 3 show that we have to make the forecasting at weekly level. In order to model the data, many independent variables and one dependent variable are generated.

The regression model is built for every group and the dependent variable is predicted. The modeling results for one of the groups are given below. The \( R^2 \)-squared was found as 0.6595. The fitted versus residual values plot and date versus standardized residuals plot are given in Figs. 4a and 4b. These figures show us that the residuals do not have a pattern and the model explains the data. Most of the independent variables used for model development were found significant at 99.9% significance level. However, the categorical variable indicating that the week includes 15th day of the month (which is a salary day in Turkey) was found significant at 95% significance level. The categorical variable that shows whether the week includes a special day or a weekend before a special day was not found significant.

**Step 5.** The weekly withdrawn amounts of each ATM in the group divided by total withdrawn amount of each group are calculated. The reason for doing so is to find how to distribute the forecasted amounts among individual ATMs in the group. The proportions are analyzed for the last year. If the proportions do not change in time, in other words, if it is stable, the average of these proportions can be used in order to convert the group forecasts into individual ATM forecasts. For the case study that includes a group of 5 ATMs, the average proportions are: 26%, 22%, 17%, 16% and 19%.

**Step 6.** Load individual forecast amount to each ATM in the group.

5. Comparison of individual and grouped forecast and replenishment

5.1. Comparison of forecasting results

The performance measure used in the “Forecasting Competition for Artificial Neural Networks and Computational Intelligence” was MAPE. It expresses accuracy as a percentage and is calculated by the following formula where \( A_i \) is the actual value and \( F_i \) is the forecast value.
MAPE = \( \frac{100\%}{n} \left( \frac{1}{n} \sum_{t=1}^{n} |A_t - F_t| / A_t \right) \)

The difference between \( A_t \) and \( F_t \) is divided by the actual value \( A_t \). The absolute value in this calculation is summed for every fitted or forecasted point in time and divided again by the number of fitted points \( n \). Multiplying by 100 makes it a percentage error.

The data in the competition included daily cash withdrawals from 111 ATMs in different locations of England. The time frame was 2 years and the aim was to forecast the next 56 days’ demand (Crone, 2008). In order to compare our results with the results in that competition, we have decided to use the same experimental setup. In this context, the train set and test set ratio is \( 2 \times 365 \text{ days} / 56 \text{ days} = 13 \). To demonstrate the idea, we used 52 weeks’ data of a group to build the model and forecasted 4 weeks’ demand (so that the ratio is \( 52/4 = 13 \)).

Table 1 shows the results of an example group which consists of 5 ATMs. The table shows the actual withdrawals in 4 weeks of time period, the group forecast results converted to individual ATM forecasts and individual forecast results. For group forecasting, we forecasted the group demand and then converted them to individual ATM demands by using the proportions calculated as explained in Step 5 of the previous section. For individual forecasting, we used the data of one individual ATM to make the forecast for that ATM. The MAPE values for each ATM and each case are calculated. The MAPE results for ATM 1 and ATM 2 are better in indi-
Individual forecasting while for other ATMs proposed methodology does a better job. Finally the average MAPE results show that proposed methodology makes more accurate forecasts.

5.2. Comparison of replenishment plans

In order to compare the proposed procedure and the procedure used in the previous studies, which use daily and individual forecasting; a decision period of 4 weeks and an example group of 5 ATMs are used for demonstration. A comparison methodology based on costs is developed. The formula for the cost is given in Eq. (2).

\[
\text{Initial Cost} = \left(\frac{\text{daily interest rate}}{365}\right) \times \frac{7}{\text{replenished amount}} + \frac{100}{\text{cash uploading cost}} \times \text{number of cash replenishments}
\]

The cost formula given in Eq. (2) is the same with the objective function presented in Step 3 of the proposed methodology. However, here the replenishment period is fixed as 7 days in order to make it precise. Daily interest rate is taken as 9%, cash uploading cost is taken as 80 TL, number of cash replenishments is 4 since we have 4 weeks and we make the replenishment every week, lastly, replenished amount is the forecast value for that week. Three cases are compared in Table 2. In the first case, the amounts converted from group forecasts to individual forecasts are loaded to each ATM. In the second case, the actual amounts are loaded to each ATM. In the third case, the amounts found from individual forecasting are loaded to each ATM. Based on these amounts, the lacking amounts that occur in each case are calculated for each week. These values are zero for the actual case since we upload exactly the same value with the demand. Third case falls short of money more than first case. Then the dissatisfaction costs are added to the initial costs in order to calculate the individual total costs for each ATM. The dissatisfaction cost is given in Eq. (3).

\[
\text{Dissatisfaction Cost} = \frac{\text{lacking amount}}{\text{some percentage}}
\]

Dissatisfaction cost is thought to be a percentage of the lacking amounts since the customers will be dissatisfied when they cannot find money in the ATM. Moreover, their dissatisfaction is expected to be positively correlated with their demand. In other words, the more money they need, the more dissatisfaction they will feel. According to the expert opinion, this percentage is decided to be something between 1% and 3%. The individual total costs show that Case 1 results in lower cost for ATMs 3, 4 and 5 while it results in

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Actual and forecast result comparison.</th>
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<td></td>
<td>ATM 1</td>
</tr>
<tr>
<td>Actual amounts</td>
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<td>51011</td>
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<td>Average MAPE</td>
<td>0.202577</td>
</tr>
<tr>
<td>Individual forecasting</td>
<td></td>
</tr>
<tr>
<td>1. week</td>
<td>58476.58</td>
</tr>
<tr>
<td>2. week</td>
<td>57619.95</td>
</tr>
<tr>
<td>3. week</td>
<td>113728.8</td>
</tr>
<tr>
<td>4. week</td>
<td>84141.88</td>
</tr>
<tr>
<td>MAPE</td>
<td>0.144983</td>
</tr>
<tr>
<td>Average MAPE</td>
<td>0.226924</td>
</tr>
</tbody>
</table>

Fig. 4b. Date versus standardized residuals.
higher cost for ATMs 1 and 2. These results are in parallel with the forecast performance. Therefore this study shows the importance of the forecast accuracy.

The comparison between groups by means of the total cost for a group shows that the first case does better than the third case. Additionally, when the dissatisfaction cost percentage increases, the difference also increases. However, the total cost in the ideal case (Case 2) is much smaller than the other cases, better forecasts will decrease the total costs.

6. Conclusion

ATM cash replenishment problem focuses on the decisions about time schedules that each of a network of a large number of ATMs should be replenished and amount of money should be loaded to ATMs. Previous studies dealing with this problem focused on either the forecasting part or the replenishment policy part of the problem. Moreover there are a number of studies on the ATM cash demand forecasting topic, whereas the literature on the replenishment part is scarce. To the best of our knowledge, no study has solved the forecasting and replenish problems together. Additional challenges in our problem is that the variations in individual ATM demands are large that traditionally used forecasting methods fail to provide a quality prediction useful for replenish optimization.

The main contributions of this study are

(1) integrating forecasting procedure in replenishment decisions;

(2) improving forecasting model quality by developing data aggregation techniques for demands in nearby ATMs and optimal number of successive days;

(3) including both location and time related variables in the forecasting models.

For illustrating and comparing the proposed methods against popular ones in the literature, two and half years of data from 152 ATMs of a bank in Istanbul are analyzed. Both forecasting performance and replenishment costs are compared. The motivation of forecasting withdrawals of a cluster of ATMs was that, when we modeled the data for individual ATMs, the best R² value was found as 0.58 while it was found as 0.71 for a cluster of ATMs. Moreover, it has been seen from the past cash demand forecasting that MAPE ranged from 20% to 45%. Our study has an average MAPE of 20% or lower. In handling as much data demand variations as ours, this shows a reasonable better performance than the ones given by the literature.

In particular, for an example group which consists of five ATMs, the following values are calculated: the actual withdrawals in four weeks, the group forecast results converted to individual ATM forecasts and individual forecast results based on the existing method. The MAPE values for each ATM and each case are calculated. Section 5.1 shows a total of 31% improvement from the five MAPEs from our method over what are obtained from the existing method. In particular, the best improvement (at ATM #5) is 37.9%.

In order to compare the procedures impacting replenish results, a comparison methodology based on costs is developed. In the first case (proposed procedure), the amounts converted from group forecasts to individual forecasts are loaded to each ATM. In the second case, the actual amounts are loaded to each ATM. In the third case, the amounts found from individual forecasting are loaded to each ATM. The comparison of total costs shows that the first case does better than the third case. Additionally, when the dissatisfaction cost (in percentage) increases, the difference also increases. For instance, a total cost of an example group is found as 5449.8
TRL based on our method, while under replenishing individual forecast results it is found as 5527.15 TRL. Moreover, when dissatisfaction penalty rate increases they are 9662.08 TRL and 9960.17 TRL, respectively. In this small example there is an improvement of 100–300 TRL. Extending this cost improvement from one cluster of five ATMs to many clusters of ATMs, following the same rate of improvement, the total weekly cost improvement for 152 ATMs could approach to 3000–9000 TRL.

This study presents a valuable framework on ATM cash replenishment for both researchers and practitioners. The practical implications of the proposed framework are increased customer satisfaction and decreased cost. However, one who wants to apply this methodology should be informed that past data should be used to check whether the data shows similarity for nearby ATMs to create clusters. Moreover, the variables for the forecasting part can be changed or widened based on expert opinions. Expert opinion is also helpful while deciding on the optimal replenishment time intervals.

In the future work, the forecasting performance might be enhanced more by using other methods such as time-series regressions. The objective function for the optimal replenishment might be broadened to include the lost opportunity by letting people use the ATMs of rival banks in the situation where the other ATMs of the same bank are far away.

Acknowledgements

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Appendix A. (list of predictor variables used for forecasting)

Categorical variable which denotes that the week is the week before a special day, such as Valentine’s day, mothers’ day, fathers’ day etc.

Numeric variable which denotes the week number (There are 52 weeks in a year and this variable is important to capture the seasonality)

Numeric variable which denotes the number of points of interests (school, restaurant, shopping center, plaza etc) in 1000 m vicinity of the ATM

Numeric variable which denotes the number of social buildings in the street (such as restaurant, shop, cafeteria etc)

Numeric variable which denotes the number of work related buildings in the street (such as plaza, governmental building, school, etc)

Numeric variable which denotes the number of commercial buildings in the street (such as airport, port, bus terminal)

Numeric variable which denotes the total number of points of interests in the street

Numeric variable which denotes the population in the street

Numeric variable which denotes the number of people who are at the age of between 15 and 19

Numeric variable which denotes the number of people who are at the age of between 20 and 44

Numeric variable which denotes the number of people who are at the age of between 45 and 54

Numeric variable which denotes the number of people who are older than 55

References

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