Non-Stationary Noise Responses of Some Fully Differential On-Chip Readout Circuits Suitable for CMOS Image Sensors

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Abstract—CMOS active-pixel image sensors, as well as charge-coupled devices, generate both white noise and $1/f^\alpha$-noise over several decades depending on biasing current, operating temperature, and the characteristics of the process used, limiting the detector dynamic range. Three readout circuits, based on a fully differential cascode operational transconductance amplifier, designed and realized on a standard CMOS 0.7-μm single polysilicon/double metal process, are proposed for CMOS active-pixel imagers. The first is an uncompensated switched-capacitor (SC) voltage amplifier; the second, an offset-compensated SC amplifier; and the third, a commutable bandpass filter. All three amplifiers allow correlated double sampling and double delta sampling for pixel and column fixed pattern noise suppression, respectively. The amplifiers offer up to 10-Mpixels/s readout rates. A detailed theoretical analysis of the amplifiers response to white noise and low-frequency excess noise is given, considering nonstationary nature of the output signals. An original method based on diffusive Markovian representation of $1/f^\alpha$-noise is used. The theoretical results are compared with experimental data.

I. INTRODUCTION

ACTIVE pixel image sensors implemented using CMOS technology offer many advantages when compared to charge-coupled devices (CCD’s) technology: lower cost, higher pixel readout rates, random accessibility and windowing, lower power consumption, and use of standard power supply or logic levels (+5 V, +3.3 V, ...). The realization of intelligent image sensors (or camera-on-a-chip) including photodetectors, amplifiers, analog-to-digital converters, timing generators, and signal processing circuits on the same chip is also possible [1].

The typical operation of the photogate active-pixel sensor described in [1, Fig. 1] starts with the integration phase (photogate PG at the power supply level VDD) during which incoming photons generate electrons that are integrated and stored under the photogate. The sensing node (floating diffusion) $C_{Q1}$ is then reset, allowing the pixel reference value to be read before the stored photoelectrons are transferred into this high-impedance node, making its potential to decrease to the pixel signal value. All the pixels in a row operate the same way and a scanning process allows the whole array (or a part of it) to be read out. The transfer gate TX separates the photogate and the sensing node.

In order to suppress kTC noise of the floating node capacitance, the correlated double sampling (CDS) readout method is used. First, the reference level ($V_{ref}$) for each pixel of a line of the imager matrix is sampled into the capacitors $C_{ref}$ and memorized using the reference sampling switch SHR (Fig. 1). Then the signal level ($V_{SIG}$) for each pixel is sampled and stored into capacitors $C_{SIG}$, activating the signal sampling switch SHS. The difference $\Delta V$ between these two levels is the useful signal proportional to the amount of incident photons flux into the pixel.

Due to the process nonuniformities, the differences between the threshold voltages of the transistors of analog buffers ($\varepsilon_{T0}$) cause a column FPN (Fig. 2). To compensate this FPN via hardware, the readout chain contains a circuit named double delta sampling (DDS). $V_1$ and $V_2$ signals are sampled firstly. The difference of these signals includes also the $\varepsilon_{T0}$. Then, the DDS switch is activated and the capacitors $C_{ref}$ and $C_{SIG}$ are short-circuited. At this time, the difference between $V_1$ and $V_2$ signals is equal to $\varepsilon_{T0}$ directly. The timing of this process is shown in Fig. 3. Hence, the signal acquisition chain which follows this circuit must carry out the following subtraction and amplify it:

$$\Delta V_0(n + 1) = [V_1(n) - V_2(n)] - [V_1(n + 1) - V_2(n + 1)]$$

$$= (V_{ref} - V_{SIG} + \varepsilon_{T0}) - \varepsilon_{T0}$$

$$= V_{ref} - V_{SIG} = \Delta V.$$ 

All three circuits proposed in this paper carry out this function and eliminate this error.

The low-light detection limit of the sensor is determined by its noise floor. Therefore, it is a common practice for an image sensor to express the noise in electrons by dividing the total output noise by the output-referred conversion gain.

The main noise source in CMOS-active image sensors is the in-pixel source follower MOSFET associated with the floating node capacitance which generate both white and $1/f^\alpha$-noises [2], with $0.6 < \alpha < 2.0$, depending on temperature and biasing conditions [3]. There is also the noise contribution of the column amplifiers [2].

This paper’s aim is to compare both theoretical and experimental noise response of three amplifiers designed on a 0.7-μm standard CMOS technology, available through the EUROPRACTICE MPW service.
Fig. 1. Typical correlated double-sampling circuit including DDS.

Fig. 2. Image taken in darkness with a 256 × 256 CMOS active-pixels array without DDS (presence of column FPN).

Fig. 3. Timing of DDS process.

For these circuits, classical noise calculations methods using Power Spectral Densities (PSD’s) and Wiener–Kinchine’s Theorem are not useful because the output root mean square (rms) noise of these circuits are time-varying and usually, at sampling time, the outputs levels do not reach the stationary values.

Note that the CMOS active-pixel sensor signal readout circuitry temporal noise is outside the scope of this paper and is treated elsewhere [4].

II. DESCRIPTION OF THE CIRCUITS

The op-amp used in readout circuits is a fully differential cascode output operational transconductance amplifier (OTA) (Fig. 4). In this figure, the bias circuits of the input-stage current source transistor (M5) and cascode output-stage transistors (M2–M2’, M3–M3’, and M4–M4’) are not illustrated. The use of a differential op-amp allows to minimize some drawbacks of SC circuits: poor power supply rejection ratio (PSRR), nonideal effects of charge injection of switches, first-order voltage dependencies of capacitors, etc. [5].

This OTA contains a common mode feedback circuit to ensure a high common mode rejection ratio. This circuit compares the two output voltages and fixes the average of them to a common mode voltage \( V_{cm} \) by changing \( V_{in} \).

ELDO circuit simulations using BSIM3v3 transistor models of the OTA give a 70-dB static gain, 65-MHz transition frequency to unity gain, and 60° phase margin with 1.5-pF capacitive charge. It is stable for a given charge capacitance and unstable in open-loop configuration. Supply voltages are 5 V for \( V_{DD} \), 0 V for \( V_{SS} \) and +2.5 V for \( V_{MC} \).

The simplified first-order equivalent model of this OTA for noise calculations is illustrated in Fig. 5. In this model, the OTA is supposed to be symmetrically supplied to simplify the noise calculations. \( g_0 \) is the transconductance. To represent the...
common-mode feedback circuit, the equality block ensuring \( v_{0}^{*} = v_{0}^{*} = v_{0} \) has been added.

### A. Basic SC Voltage Amplifier

The first circuit illustrated in Fig. 6 is a symmetrical basic SC voltage. In phase 1, before the DDS process, the outputs of the CDS circuit (Fig. 1) are sampled on \( C_1-C_2 \) capacitors. At this time, the \( C_2-C_2' \) capacitors are discharged. The outputs of the op-amp are also shorted to \( V_{ss} \) to avoid oscillations of the OTA which would be open-circuited and unstable in this phase.

After the DDS process, the sampling signal passes to 0 V and the difference between the new values and the previous ones are transferred on \( C_2-C_2' \) capacitors. So, the transfer function is

\[
\Delta V_0(n) = \frac{C_1}{C_2} [V_1(n) - V_2(n)] - [V_1(n+1) - V_2(n+1)]
\]

\[
\Delta V_0(n) = \frac{C_1}{C_2} \Delta V
\]

where \( (C_1/C_2) \) is the gain of the amplifier.

One drawback of this circuit concerning the noise is that the intrinsic offset of the op-amp is not eliminated, hence the intrinsic low-frequency (LF) noise, too.

The nonideal effects of switches and op-amps are not considered in this study. Thus, we will not treat here these effects which are already treated largely in literature [6].

### B. Offset-Compensated SC Voltage Amplifier

This amplifier (Fig. 7) eliminates the intrinsic offset voltage of the op-amp, also reducing the intrinsic LF noise [6]. In phase 1, the capacitors \( C_2-C_2' \) are not shorted to \( V_{ss} \), but pre-charged to the offset voltage \( V_{os} \) of the op-amp to cancel it in phase 2.

The transfer function is the same as the basic SC amplifier.

The drawback of this amplifier is its complexity and the difficulty of generating \( S_1-S_2 \) control signals for high speed readout rates. The nonoverlapping of the two phases must be kept as small as possible. During the brief intervals where the control signals are both low, the op-amp is in open-circuit configuration and unstable (Fig. 8).

### C. Commutable Bandpass Filter

It is based on a variable bandpass filter whose center frequency can be shifted from a high frequency (HF) to a LF and vice versa, reducing the noise bandwidth (Fig. 9). The frequency shifting is achieved by activating/disactivating the switches in two phases. At the first phase (HF mode), the switches are activated by doing \( S \) signal high and the \( V_1-V_2 \) signals before DDS process are clamped on \( C_1-C_1' \) capacitors. At this time, the capacitors \( C_2-C_2' \) are shorted to \( V_{ss} \) and discharged. Just before the activation of DDS signal, the \( S \) passes to low level shifting the filter in HF mode.

The transfer function of the filter is

\[
H(s) = \frac{R_0 C_1 s}{(1 + R_a C_1 s)(1 + R_0 C_2 s)}
\]

and the time response

\[
\Delta V_S(t) = \frac{R_0}{R_1} \cdot \frac{\tau_1}{\tau_2} (e^{-t/\tau_1} - e^{-t/\tau_2}) \cdot \Delta V
\]

where \( \tau_1 = R_a C_1 \) and \( \tau_2 = R_0 C_2 \).

By differentiating the above equation and setting it equal to zero, the time where the bandpass filter output has reached its maximum value \( \Delta V_S(t_{\max}) \) and sampled, may be calculated

\[
t_{\max} = \frac{\tau_1 \tau_2}{\tau_1 - \tau_2} \ln \left( \frac{\tau_1}{\tau_2} \right).
\]

In our test circuits, the filter parameters are optimized to have a gain of 6 dB and a \( t_{\max} \) of 50 ns.
A different and nondifferential version of this circuit is proposed and studied in [7] for a Thomson 7895M scientific-grade CCD off-chip signal acquisition systems, with low pixel (25 kpixels/s typically) readout speeds.

III. IMPLEMENTATION OF THE CIRCUITS

The circuits have been implemented on a standard 0.7-μm single polysilicon/double-metal CMOS process with analog options. The layouts, done using CADENCE software, are shown in Fig. 10.

The capacitors are polysilicon/N+ diffusion on p-type substrate and the resistors used in the commutable bandpass filter are lowly doped high ohmic polysilicon resistors. In the process we used the typical matching value for capacitors is 0.11% and for resistors is 0.10%. The switches are made of transmission gates. To reduce the mismatch effects of
transistors, capacitors and resistors, a careful layout must be carried out.

On the test chip, we have added two buffers for each outputs to fix the charge capacitor of the op-amp to 1.5 pF and to drive external loads.

IV. STATE-EQUATIONS OF THE CIRCUITS

In this section, we will develop the state equations of the circuits; they will be used in the next sections to calculate both LF and white-noise responses.

The schematic representations of the circuits are shown in Fig. 11. Note that the parameters in these representations are time varying and take $R_{ON}$ or $R_{OFF}$ values depending of the switches positions for each phases. $R_{1}$ is the output resistance of the noise source.

A. Basic SC Voltage Amplifier

According to the Fig. 11(a), one may write the following differential state equations describing the basic SC amplifier:

\[
\begin{align*}
\frac{du_1}{dt} &= a_1(t) \cdot (e_1 - u_1 - u_2 - v_0) \cdot dt \\
\frac{d^2u_1}{dt^2} &= a_2(t) \cdot (e_1 - u_1) \cdot dt - a_3(t) \cdot (u_2 - v_0) \cdot dt \\
\frac{d^3u_1}{dt^3} &= a_4(t) \cdot (e_1 - u_1 - c_1 + u_2(t) + a_5(t)) \cdot dt \\
\frac{du_0}{dt} &= a_6(t) \cdot (u_2 - u_0) \cdot dt - a_5(t) \cdot (u_2 - u_0) \cdot dt
\end{align*}
\]

with

\[
\begin{align*}
a_1 &= \frac{1}{R_1C_1}, \quad a_2 = \frac{1}{2R_1C_2}, \quad a_3 = \frac{1}{C_2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \\
a_4 &= \frac{1}{2R_1C_3}, \quad a_5 = \frac{1}{2C_3} \left( \frac{1}{R_1} + \frac{1}{R_2} + 2g_0 \right) \\
a_6 &= \frac{1}{C_3} \left( \frac{1}{R_1} + \frac{1}{R_2} + 2g_0 \right).
\end{align*}
\]

To simplify the computations and measurements, we will consider only the case where the second source $e_2$ is equal to zero. In this case, the above equations in matrix form are

\[
\frac{d}{dt} \begin{bmatrix}
u_1 \\ u_1' \\ v_2 \\ u_2' \\ v_0 \\ u_0'
\end{bmatrix} = \begin{bmatrix}
-a_1 & 0 & -a_1 & 0 & -a_1 & 0 \\
0 & -a_1 & 0 & -a_1 & a_1 & 0 \\
-a_2 & 0 & -a_3 & 0 & -a_3 & 0 \\
0 & -a_2 & 0 & -a_3 & a_3 & 0 \\
a_4 & a_5 & a_6 & a_5 & a_6 & a_5 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
u_1 \\ u_1' \\ v_2 \\ u_2' \\ v_0 \\ u_0'
\end{bmatrix} + \begin{bmatrix}
a_1 \\
a_2 \\
0 \\
a_4
\end{bmatrix} \cdot [e_1].
\]

B. Offset-Compensated SC Voltage Amplifier

For the offset-compensated SC amplifier, from Fig. 11(b), the state equations are

\[
\begin{align*}
\frac{du_1}{dt} &= b_1(t) \cdot (e_1 - u_1) \cdot dt - b_2(t) \cdot u_2 - b_3(t) \cdot v_0 \cdot dt \\
\frac{d^2u_1}{dt^2} &= b_2(t) \cdot (e_2 - u_2) \cdot dt - b_2(t) \cdot u_2 \cdot dt + b_3(t) \cdot v_0 \cdot dt \\
\frac{d^3u_1}{dt^3} &= b_3(t) \cdot (e_3 - u_3) \cdot dt - b_3(t) \cdot u_2 \cdot dt - b_3(t) \cdot v_0 \cdot dt \\
\frac{du_0}{dt} &= b_4(t) \cdot (e_1 - u_1 - e_2 + u_2(t) + a_5(t)) \cdot dt + b_5(t) \cdot (u_2 - u_2) \cdot dt \\
&= b_5(t) \cdot (u_2 - v_0) \cdot dt - b_5(t) \cdot v_0 \cdot dt,
\end{align*}
\]

where

\[
\begin{align*}
b_1 &= \frac{k_1}{C_1} \cdot \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) \\
b_2 &= \frac{k_1}{R_3} \cdot \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) \\
b_3 &= \frac{k_1}{C_2} \cdot \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) \\
b_4 &= \frac{k_1}{R_3} \cdot \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) \\
b_5 &= \frac{k_1}{R_2} \cdot \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)
\end{align*}
\]

In matrix form

\[
\frac{d}{dt} \begin{bmatrix}
u_1 \\ u_1' \\ v_2 \\ u_2' \\ v_0 \\ u_0'
\end{bmatrix} = \begin{bmatrix}
-b_1 & 0 & -b_2 & 0 & -b_3 & 0 \\
0 & -b_1 & 0 & -b_2 & b_3 & 0 \\
-b_4 & 0 & -b_5 & 0 & -b_6 & 0 \\
0 & -b_4 & 0 & -b_5 & b_6 & 0 \\
-b_7 & b_7 & b_8 & -b_8 & -b_9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
u_1 \\ u_1' \\ v_2 \\ u_2' \\ v_0 \\ u_0'
\end{bmatrix} + \begin{bmatrix}
b_1 \\
b_4 \\
b_7
\end{bmatrix} \cdot [e_1].
\]
C. Commutable Bandpass Filter

For the commutable bandpass filter, according to the Fig. 11(c), the state equations of the circuit are

\[
\begin{align*}
\frac{du_1}{dt} &= c_1(t) \cdot (e_1 - u_1) \cdot dt - c_2(t) \cdot (u_2 + v_0) \cdot dt \\
\frac{du_2}{dt} &= c_3(t) \cdot (e_2 - u_2) \cdot dt - c_4(t) \cdot v_0 \cdot dt \\
\frac{dv_0}{dt} &= c_5(t) \cdot (e_1 - u_1 - e_2 + u_2') \cdot dt - c_7(t) \cdot (u_2 - u_2') \cdot dt \\
\end{align*}
\]

with

\[
\begin{align*}
c_1 &= \frac{k_2}{C_1} \cdot \left( \frac{1}{R_2} + \frac{1}{R_3} \right), & c_2 &= \frac{k_2}{C_1 R_3} \\
c_3 &= \frac{k_2}{C_2 R_3} \cdot \left( 1 + \frac{R_1}{R_2} \right), & c_4 &= \frac{1}{C_2} \cdot \left[ \frac{1}{R_4} + \frac{k_2}{R_3} \left( 1 + \frac{R_1}{R_2} \right) \right] \\
c_5 &= \frac{k_2}{C_2 R_3} \left( 1 + \frac{R_1}{R_2} \right), & c_6 &= \frac{k_2}{2 C_5 R_3} \\
c_7 &= \frac{1}{2 C_5} \left[ 2g_0 + \frac{1}{R_3} \left( 1 - k_2 R_1 \frac{R_1}{R_3} \right) \right], & c_8 &= \frac{1}{C_5} \left[ 2g_0 + \frac{1}{R_3} + \frac{1}{R_3} \left( 1 - k_2 R_1 \frac{R_1}{R_3} \right) \right] \\
k_2 &= \left[ 1 + R_4 (1/R_2 + 1/R_3) \right]^{-1}.
\end{align*}
\]
In matrix form
\[
\begin{bmatrix}
\frac{d}{dt}u_1 \\
\frac{d}{dt}u_2 \\
\frac{d}{dt}v_0
\end{bmatrix} =
\begin{bmatrix}
-c_4 & 0 & -c_2 & 0 & -c_2 \\
0 & -c_4 & 0 & -c_2 & c_2 \\
0 & 0 & -c_4 & 0 & -c_5 \\
0 & 0 & 0 & -c_4 & c_5 \\
-c_6 & c_6 & -c_7 & c_7 & -c_8
\end{bmatrix} \cdot \begin{bmatrix}
u_1 \\
u_2 \\
v_0
\end{bmatrix}
\]
\[
\begin{bmatrix}
3 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \cdot [c_1] + [v_1].
\]

V. Noise Analysis

In this section, we derive an analytical model to calculate the mean output noise power in terms of variance for a given noise type, white or $1/f^\alpha$

\[
\sigma^2(t) = E\{\Delta v_0^2(t)\}
\]

where $E$ denotes the expected value or mean of $\Delta v_0^2(t)$.

For such circuits, the classical use of PSD and Wiener-Kinchine’s theorem is not applicable for the following reasons:

1) The noise at the end of the last phase is time-varying and depends on the noise voltage stored on the capacitors at each phase;

2) The commutable bandpass filter output never reaches the stationary level at the sampling instant, both for white noise and LF noise inputs. This is also valid for the basic and offset-compensated SC amplifiers with $1/f^\alpha$-noise input for high pixel readout rates.

We will develop only the theoretical noise response of the basic SC amplifier. For the two other circuits, only the simulation and experimental results will be given. The noise response of these circuits may be calculated in the same way using the state equations given in Section IV.

Our measuring system is illustrated in Fig. 12. It gives the standard deviation $\sigma(t)$ of the output signal. The normalized PSD’s of the $1/f^\alpha$-noise sources are shown in Fig. 13 and [8]. These noises sources are constructed from a white-noise source and a bank of low-pass filters, in the same way we will analyze the $1/f^\alpha$-noise responses. A pulse generator generates the sampling signals $S$. The outputs of the amplifier under test are sampled $N$-times for a given time $t$ using a digitizing scope. Then the standard deviation of these samples is computed and displayed in $V_{\text{rms}}$. We used FET-input active probes with 2-pF input capacitance to reduce the load charge. To avoid jitter problems, the scope is synchronized by the signal $S$.

In all of our measurements, we used a high noise-level input so that the noise added by the op-amp, resistors, capacitors, and the measurement system is negligible. Then, in our computations, these noises are not considered.

To carry out noise calculations, we define a generating process in order to obtain the desired noise (band limited white or $1/f^\alpha$) from an ideal white-noise source (Fig. 14). Thus, the study may be reduced to the study of a classical Markovian equivalent diagram where the system is driven by a white-noise source. Then using linearity properties of the system, the noise response of the circuit may easily be developed.

The noise response may be calculated directly by using the stochastic differential equations [7]. But it requires the handling of large matrices and the simulation needs large time computations. For a convenient use, we have preferred to discretize directly the process.

We consider the following stochastic differential equation system under Markovian representation form:

\[
\begin{cases}
\frac{dX}{dt} = A \cdot X + B \cdot \epsilon \cdot dt \\
S = C \cdot X
\end{cases}
\]

where $\epsilon = w$ is the white noise and $\epsilon \cdot dt = d\beta$ is the standard Brownian motion with the following characteristics [9]

\[
E\{d\beta\} = 0, \quad E\{d\beta(t) \cdot d\beta(t')\} = \delta(t-t') \cdot dt \cdot dt'
\]

and

\[
E\{(d\beta)^2\} = dt.
\]

After discretizing (1) with sample time $T$, we obtain the discrete stochastic equation system

\[
\begin{cases}
X(n + 1) = A_d \cdot X(n) + B_d \cdot \epsilon(n) \\
S(n) = C_d \cdot X(n)
\end{cases}
\]

The correlation function of a discrete process $X$ is defined as

\[
R_{XX}(n) = E\{X(n) \cdot X^T(n)\}
\]

where $X^T$ denotes the transposed of $X$.

The correlation matrix of the above discrete stochastic process $X(n+1)$ is

\[
R_{XX}(n+1) = E\{X(n+1) \cdot X^T(n+1)\}.
\]

Using (2), we get

\[
R_{XX}(n+1) = E\{[A_d \cdot X(n) + B_d \cdot \epsilon(n)] \cdot [A_d^T \cdot X(n) + B_d^T \cdot \epsilon(n)]^T\}
\]

\[
= E\{[A_d \cdot X(n) + B_d \cdot \epsilon(n)] \cdot [X^T(n) \cdot A_d^T + \epsilon(n) \cdot B_d^T]\}.
\]

In the sequel

\[
R_{XX}(n+1) = A_d \cdot E\{X(n) \cdot X^T(n)\} \cdot A_d^T
\]

\[
+ E\{B_d \cdot \epsilon(n) \cdot X^T(n) \cdot A_d^T\}
\]

\[
+ E\{A_d \cdot X(n) \cdot \epsilon(n) \cdot B_d^T\}
\]

\[
+ B_d \cdot \epsilon^2(n) \cdot B_d^T.
\]

$X(n)$ only depends on $w(n-1), w(n-2), \cdots, w(n-i)$ and $E\{w(n) \cdot w(n-i)\} = 0$. Then, as $\epsilon(n)$ is a discrete white-noise process $w(n)$ independent from $X(n)$, the second and third terms are zero valued

\[
E\{B_d \cdot \epsilon(n) \cdot X^T(n) \cdot A_d^T\} = 0,
\]

\[
E\{A_d \cdot X(n) \cdot \epsilon(n) \cdot B_d^T\} = 0.
\]

So, if $E\{\epsilon^2(n)\} = \sigma_\epsilon^2$, then

\[
R_{XX}(n+1) = A_d \cdot R_{XX}(n) \cdot A_d^T + B_d \cdot \sigma_\epsilon^2 \cdot B_d^T.
\]
Fig. 11. Schematic representations of amplifiers for noise response analysis. (a) Basic SC amplifier. (b) Offset-compensated SC amplifier. (c) Commutable bandpass filter. $R_1$: output resistance of the noise source; $R_2$: equivalent resistance of $sw_1 - sw'_1$; $R_3$: equivalent resistance of $sw_3 - sw'_3$; $R_4$: equivalent resistance of $sw_2 - sw'_2$; $R_s$: (equivalent resistance of $sw_2 - sw'_2$)/output resistance of the op-amp).

Now, the correlation function of the output may be calculated

\[
S(n) = C_d \cdot X(n) \\
R_{SS}(n) = E\{S(n) \cdot S^T(n)\} \\
R_{SS}(n) = C_d \cdot R_{XX}(n) \cdot C_d^T.
\]

Here, the generating process is a simple first order low-pass filter. Then, the state representation of the system becomes

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} c_1 \\ u_1 \\ u_1' \\ u_2 \\ u_2' \\ v_0 \\ \end{bmatrix} &= \begin{bmatrix} -\xi & 0 & 0 & 0 & 0 & 0 \\ a_1 & -a_3 & 0 & -a_4 & 0 & -a_4 \\ 0 & 0 & -a_1 & 0 & -a_4 & a_1 \\ a_2 & -a_3 & 0 & -a_3 & 0 & -a_3 \\ 0 & 0 & -a_2 & 0 & -a_3 & a_5 \\ a_4 & -a_4 & 0 & -a_5 & a_5 & -a_6 \\ \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ u_1 \\ u_1' \\ u_2 \\ u_2' \\ v_0 \\ \end{bmatrix} \\
&+ \begin{bmatrix} \xi \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \end{bmatrix} \cdot [w]
\end{align*}
\]

A. White-Noise Analysis

For practical reasons, in our models, we limited the band of the input ideal white-noise source by a low-pass filter with a transfer function under its differential form given by

\[
dc_1 = -\xi c_1 \, dt + \xi d\beta
\]

where $c_1$ band limited practical white noise and $\xi/2\pi$ is the cutoff frequency of the filter in Hertz.
or

\[ \frac{d}{dt} X = A \cdot X + B \cdot e \]

where

\[
X = \begin{bmatrix}
\xi_1 \\ u_1 \\
\tilde{u}_1 \\
\xi_2 \\
u_2 \\
\tilde{u}_2 \\
\xi_0
\end{bmatrix}, \quad A = \begin{bmatrix}
-n_1 & 0 & 0 & 0 & 0 & 0 \\
0 & -n_2 & 0 & -n_3 & 0 & -n_4 \\
0 & 0 & -n_2 & 0 & -n_3 & 0 \\
0 & 0 & 0 & -n_2 & 0 & -n_3 \\
0 & 0 & 0 & 0 & -n_2 & 0 \\
0 & 0 & 0 & 0 & 0 & -n_2 \end{bmatrix}, \quad B = \begin{bmatrix}
\xi_1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
and \quad e = [w].

By discretizing this differential equation system and combining with (3) and (4), we obtain the variance of the output \( \sigma_{\Delta u_2}^2(t) \) in Volt\(^2\) (\( C = [0 \ 0 \ 0 \ 0 \ 0 \ 2] \)).

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Fig. 15 shows the theoretical and experimental white-noise response of the three circuits in term of \( \sigma(t) \). The small discrepancy in rise time of the basic SC amplifier is due to the simplified model of the op-amp used. As the rise time is dominated by the time constants \( R_C C_1 \) and \( R_C C_2 \), a better agreement was observed for the variable bandpass filter.

Note that the basic and offset-compensated SC amplifier does not reduce the effects of input white noise. Due to the fact that the input noise is sampled twice, corresponding to the noises of \( V_2(n) \) and \( V_1(n+1) \) signals, the output rms noise power for these circuits is nearly equal to twice the input noise.
level. For the variable bandpass filter, as a large amount of the input noise is filtered out at the second phase, the output noise is smaller.

For the offset-compensated SC amplifier, as our models do not take into account the saturation of the op-amp, there is a mismatch between the simulation and experiments at the nonoverlapping of the two phases.

B. $1/f^\alpha$-Noise Analysis

In this section, we develop the output noise power with a $1/f^\alpha$ input noise PSD where $\alpha$ is equal to 0.6, 1.0, and 1.5. In stochastic processes literature, such noises are known as fractional Brownian motions or fractional noises [10]. The approach for the generating process used here is based on a Markovian input-output representation of fractional noises, elaborated from an infinite-dimension stochastic differential equation developed in [11], [12], and in a slightly different way in [7].

If

$$\begin{align*}
   \frac{dy}{dt} &= -\xi \cdot y \cdot dt + d\beta, \\
   y(0) &= \int_0^\infty y(s) \cdot \mu_\alpha(s) \cdot ds \\
   \mu_\alpha(\xi) &= \left(\frac{2\pi}{\xi}\right)^{\alpha/2} \cdot \frac{\sin(\pi \cdot \alpha/2)}{\pi}
\end{align*}$$

(5)

then $y$ is a $1/f^\alpha$-noise.

It can be seen that the $1/f^\alpha$-noise is the sum of the output signals of an infinite bank of low-pass filters in parallel, driven by the same white-noise source [Fig. 16(a)].

The $\xi$-filter has a transfer function given by $H_\xi(s) = \frac{1}{s + \xi}$.
Due to the linearity of the system (5), the transfer function of the generating process is

\[
H(s) = \int_0^\infty \left( \frac{1}{s + \xi} \right) \mu_\alpha(\xi) \cdot d\xi = \frac{(2\pi)^{\nu/2}}{s^{\nu/2}}
\]

and in the frequency domain \(H(f) = \frac{1}{(2\pi)^{\nu/2}}f^{-\nu/2}\).

Then the spectral density of \(y(t)\) is given by

\[
N_y(f) = |H(f)|^2 \cdot N_w(f) = \frac{1}{f^\nu}.
\]

This shows that the PSD of \(N_y(f)\) is equal to \(1/f^\nu\).

Note that the auto-correlation function of \(y(t)\) is

\[
R_{yy}(t, t') = E\{y(t) \cdot y(t')\}, \quad y > t.
\]

If

\[
y(t) = \int_0^\infty y_\xi \cdot \mu_\alpha(\xi) \cdot d\xi
\]

then \(N_y(f)\) is

\[
N_y(f) = \int_0^\infty N_\eta(\xi) \cdot \varphi_\alpha(\xi) \cdot d\xi
\]

with

\[
\varphi_\alpha(\xi) = 4 \cdot \left( \frac{2\pi}{\xi} \right)^{\alpha-1} \sin \left( \frac{\pi \alpha}{2} \right)
\]

\[
R_{yy}(t, t') = \int_0^\infty R_{\xi\xi}(t, t') \cdot \varphi_\alpha(\xi) \cdot d\xi.
\]

The proof is given in [12].

By using classical integral approximation, we obtain

\[
\begin{align*}
N_y(f) &= \sum_{n=0}^{N} N_\eta(\xi_n) \cdot \varphi_\alpha(\xi_n) \cdot \Delta \xi_n \\
R_{yy}(t, t') &= \sum_{n=0}^{N} R_{\xi\xi}(t, t') \cdot \varphi_\alpha(\xi_n) \cdot \Delta \xi_n
\end{align*}
\]

The set of values for \(\xi_n: \{\xi_0, \ldots, \xi_N\}\) is chosen according to the computational capacities. Note that a geometric distribution \(\xi_{i+1} = \tau\xi_i\) seems to realize a good compromise between the accuracy of the finite dimension of the model and its dimension

\[
\xi_0 \ll 2\pi f_{\text{min}}, \quad \xi_N \gg 2\pi f_{\text{max}}.
\]

To avoid difficulties due to the infinite dimension of the generating process of the \(1/f^\nu\)-noise, and thanks to the linearity, the block diagram in Fig. 16(a) may be transformed into Fig. 16(b).

The state vector

\[
X_{\xi_i} = \begin{bmatrix} y_d \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ \xi_i \end{bmatrix}
\]

is the solution of

\[
\begin{align*}
\frac{dX_{\xi}}{dt} &= A_{\xi} X_{\xi} + B \cdot \omega \cdot dt \\
S_{\xi} &= C \cdot X_{\xi} \\
X_{\xi} &= \int_0^\infty X_{\xi} \mu_\alpha(\xi) \cdot d\xi
\end{align*}
\]

and the output

\[
S = \int_0^\infty S_{\xi}(t) \cdot \mu_\alpha(\xi) \cdot d\xi.
\]

The covariance matrix is

\[
R_{XX}(t, t') = E\{X(t)X^T(t')\}
\]

\[
= \int_0^\infty \int_0^\infty R_{\xi\xi}(t, t') \cdot \mu_\alpha(\xi) \cdot \mu_\alpha(\eta) \cdot d\xi \cdot d\eta
\]

that last expression must be simplified into [12]

\[
R_{XX}(t, t') = E\{X(t)X^T(t')\}
\]

\[
= \int_0^\infty R_{\xi\xi}(t, t') \cdot \varphi_\alpha(\xi) \cdot d\xi
\]

with

\[
\varphi_\alpha(\xi) = 4 \cdot \left( \frac{2\pi}{\xi} \right)^{\alpha-1} \sin \left( \frac{\pi \alpha}{2} \right).
\]

Then

\[
R_{SS}(t, t) = E\{s(t)s(t)\} = \int_0^\infty \sigma_\xi^2 \varphi_\alpha(\xi) \cdot d\xi = \sigma_\xi^2
\]

By discretizing \(R_{XX}(t, t')\) we obtain

\[
R_{XX}(n) = \sigma_\xi^2(n) \approx \sum_{i=0}^{N} \varphi_\alpha(\xi_i) \cdot R_{\xi\xi}(\xi_i; \xi_i) \cdot \Delta \xi_i.
\]

Thus

\[
R_{SS}(n) = \sum_{i=0}^{N} \varphi_\alpha(\xi_i) \cdot R_{\xi\xi}(\xi_i; \xi_i) \cdot \Delta \xi_i.
\]

We write \(R_{SS}(\xi)\) from (4)

\[
R_{SS}(\xi) = C_d \cdot R_{\xi\xi}(\xi) \cdot C_d^T.
\]
Then, for the basic SC amplifier, the state representation of the system becomes

$$
\begin{bmatrix}
\dot{y} \\
\dot{u_1} \\
\dot{u_2} \\
\dot{u_0} \\
\dot{\xi_i}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
a_1 & -a_1 & 0 & -a_1 & 0 & -a_1 \\
0 & 0 & -a_2 & 0 & -a_3 & 0 & -a_3 \\
a_4 & -a_4 & a_4 & -a_5 & a_5 & -a_6
\end{bmatrix}
\begin{bmatrix}
\xi_i \\
u_1 \\
u_2 \\
u_0
\end{bmatrix}
+ 1 \cdot [u]$$

or

$$\frac{d}{dt} X_{\xi_i} = A_{\xi_i} \cdot X_{\xi_i} + B \cdot c$$

where

$$X_{\xi_i} =
\begin{bmatrix}
y \\
u_1 \\
u_2 \\
u_0
\end{bmatrix},
A_{\xi_i} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
a_1 & -a_1 & 0 & -a_1 & 0 & -a_1 \\
0 & 0 & -a_2 & 0 & -a_3 & 0 & -a_3 \\
a_4 & -a_4 & a_4 & -a_5 & a_5 & -a_6
\end{bmatrix},
B =
\begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix},
\text{and } c = [u],$$

To calculate the variance of the output $\sigma_{\Delta y_i}(t)$, similar to the white-noise case, the above set of stochastic differential
equations is discretized and then combined with the formulas (3), (4), and (6).

Plots of theoretical and experimental rms output noise versus sampling instant for $1/f^{\alpha}$ and $1/f^{1.5}$ input noise are given in Figs. 17 and 18, respectively for the three structures studied. The theoretical rms output noise responses versus sampling instant for $1/f^{1.5}$ are given in Fig. 19.

For high pixel readout rates, the two SC circuits have the same output noise level. Despite the offset-compensated SC circuit reduces op-amp noise, it is inefficient on the input $1/f^{0.5}$-noise. Note that between the phase 1 and phase 2, it is unstable and oscillates. The commutable bandpass filter seems to be advantageous to suppress the input noise, especially for low pixel readout rates.

VI. CONCLUSION

Non-stationary noise responses of three fully differential readout circuits for CMOS active-pixel image sensors were studied. Both LF and broadband noises are considered. A nonstandard method has been used for LF noise response using diffusive representation of $1/f^{0.5}$-noise. A good agreement has been observed between theory and experiments. For a given noise-output type (white or $1/f^{0.5}$) of the image sensor and pixel readout rate, the optimum circuit may be chosen.

It should be pointed out that this method can also be used to determine the effects of the LF and white noises of the op-amp and switches on the signal-to-noise ratio of the circuit. In this study, for experimental convenience, we have chosen the signal inputs of the circuits as noise inputs.

The basic SC amplifier is the simplest to realize and the generation of the clock signals needed at high pixel readout rates is easy. Nevertheless, it cannot reduce either the input signal noise nor the intrinsic LF noise of the op-amp used. The op amps and the input signal noises are amplified as well as the input signal.

The offset-compensated SC amplifier is efficient to reduce the intrinsic op-amp noise [6], but not the input LF noise. Another drawback of this circuit is the glitches during the nonoverlapping phase of the clocks which prevent achieving high pixel readout rates (higher than 1 Mpixel/s). Fortunately methods to prevent glitches at the op-amp output are available [13], [14]. Circuit techniques to generate the nonoverlapping clocks from a single clock may be found in literature [15], [16].

For white noise, at low and high pixel readout rates, only the commutable bandpass filter is efficient on input noise and filters out a part of the input noise. When the input noise is dominated by $1/f^{0.5}$-noise, especially for low pixel readout rates (for example, less than 50 frames/s for a 256 × 256 pixels array), the commutable bandpass filter is very suitable. However, it can be pointed out that, while it shows good noise performance for CCD’s [7], with the common CMOS active-pixel image sensor readout architecture [1, Fig. 1], it has no effect on the noise generated by the in-pixel source follower MOSFET, because this noise is already stored on the sampling capacitors $C_{\text{reg}}$ and $C_{\text{Sic}}$. It is advantageous if the column amplifiers give important noise. A drawback of the commutable bandpass filter is that resistance-capacitance ($RC$) values are calculated for a given readout frequency. Another lack of this circuit is that its gain is sensitive to both $RC$ ratios and to its absolute values. Due to the necessity of large capacitor and resistor values, it is also more space consuming in the context of integrated circuit when compared with the two other SC circuits.

With the common readout circuit shown in Fig. 1, the offset-compensated SC amplifier appears as the best trade-off for CMOS active-pixel image sensor in term of noise performance.

REFERENCES


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