ABSTRACT  Linear array SAR can get the 3D image of the target. The geometry with different antenna array angles and look angels will influence the resolution in the height direction. The method of 3D SAR imaging with random antenna array angle is studied. The resolution in the height direction is analyzed, as well as the sampling space of the antenna array. The geometry to reach the best resolution is given. Meanwhile the resolution with horizontal and vertical antenna array are calculated and compared.

Index Terms— linear array SAR, tomography, geometry, best resolution

1. INTRODUCTION

Every antenna in linear array SAR can get a 2D image of the target. The antenna array forms a second synthetic aperture for the target along the height direction, so it has the resolution capability in the height direction, and can generate 3D image.

In the geometry of linear array SAR, the direction of antenna array arrangement can be modeled as the angle away from the horizontal direction. This angle and the look angle will influence the resolution of the 3D image and the sampling space of antenna array.

2. SIGNAL PROCESSING METHOD

In order to get the image along the height direction, the geometry of linear array SAR is shown as Fig 1. \( \phi \) is the angle of antenna array arrangement with horizontal direction. \( \theta \) is the look angle of the referenced antenna, and \( r_0 \) is the referenced slant range. \( L \) is the length of the antenna array, and \( m \) is the position of antenna. \( h \) is the vertical height of the target.

![Fig. 1. The geometry of linear array SAR](image)

We get the spatial spectrum of the 2D focused data for the target from all the antennas, which reflect the image of the target along the high direction.

\[
S_k(k_a,h) = a(r_a,h) \exp \left( j \frac{4\pi}{\lambda} \cdot h \cos \theta \exp \left(-j \frac{2\pi}{\lambda r_0} \left(h \sin \theta \right)^2 \right) \right) L \sin c \left( \frac{2\pi}{\lambda r_0} L \left(h - h_0 \right) \sin \theta \cos (\theta - \phi) \right)
\]

(1)

Where \( a(s,h) \) denotes the reflectivity of the target.

The point spread function in high direction is a \textit{Sinc} function, so we can get the resolution and maximum sampling space of the antennas.

\[
\delta_s = \frac{\lambda r_0}{2 L \sin \theta \cos (\theta - \phi)} \quad , \quad d \leq \frac{\lambda r_0}{2 h_{\text{max}} \sin \theta \cos (\theta - \phi)}
\]

(2)

Where \( h_{\text{max}} \) is the total height of the observed target.

3. ANALYSIS OF THE GEOMETRY
When the length of the antenna array and the total height of the target are fixed on, the resolution in the height direction and sampling space can be analyzed by \( \text{var} = 1/\sin \theta \cos(\theta - \phi) \), with \( \theta \in (0, \pi/2) \) and \( \phi \in [0, \pi/2] \). (3)

We get the minimum value of \( \text{var} \) at \( \phi = \theta \), then \( \text{var}_{\min} = 1/\sin \theta \). (4)

So the highest resolution is achieved in the direction of antenna array vertical to the reference sight direction. And the bigger of the referenced look angle, the higher of the resolution is. Meanwhile when the resolution is high, the space of the antennas must be small.

At \( \phi = 0 \) and \( \phi = \pi/2 \) which can be easily realized in real situation, we can get the conclusions

\[
\begin{align*}
\text{var}_h &= 2/\sin 2\theta, & \phi &= 0 \\
\text{var}_v &= 1/\sin^2 \theta, & \phi &= \pi/2
\end{align*}
\]

and

\[
\begin{align*}
\text{var}_h < \text{var}_v, & \quad \theta < \pi/4 \\
\text{var}_v < \text{var}_v, & \quad \theta > \pi/4
\end{align*}
\]

So the horizontal antenna array has higher resolution when the referenced look angle is lower than 45 degree.

4. EXPERIMENTAL RESULTS

The resolution in the height direction with the different referenced look angles and antenna array angles is simulated. The reference look angle is from 10 degree to 80 degree, and the antenna array angle is from 0 degree to 90 degree.

![Fig.2](image)

**Fig.2.** The resolution in high direction with the different referenced look angles and antenna array angles

The highest resolution is achieved when the two angles are equal and reach 80 degree.

We fixed the referenced look angle on 30 degree, and analyze the resolutions along high direction at different angles of antenna array. The altitude of the antenna is 3000 meters and the length of the antenna array is 25 meters.

![Fig.2](image)

**Fig.2** The point spread function in high direction with different antenna array angles

From the point spread functions, we can see the best resolution (3.6m) is achieved at antenna array angle equal to the referenced look angle. And the resolution with horizontal antenna array (4.2m) is better than vertical antenna array (7.4m).

5. CONCLUSIONS

The point spread function in the height direction is a Sinc function for linear array SAR. The highest resolution is achieved when the antenna array is vertical to the direction of referenced sight. And then the sampling space reaches the smallest value. At the vertical and horizontal antenna array arrangements which can be easily realized, when the referenced look angle is below 45 degree, the horizontal antenna array gets higher resolution, otherwise the vertical antenna array gets.

6. REFERENCES

