ANALOGUE FRACTIONAL-ORDER GENERALIZED MEMRISTIVE DEVICES

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ABSTRACT

Memristor is a new electrical element which has been predicted and described in 1971 by Leon O. Chua and for the first time realized by HP laboratory in 2008. Chua proved that memristor behavior could not be duplicated by any circuit built using only the other three elements (resistor, capacitor, inductor), which is why the memristor is truly fundamental. Memristor is a contraction of memory resistor, because that is exactly its function: to remember its history. The memristor is a two-terminal device whose resistance depends on the magnitude and polarity of the voltage applied to it and the length of time that voltage has been applied. The missing element - the memristor, with memristance M-provides a functional relation between charge and flux, dφ = Mdq.

In this paper, for the first time, the concept of (integer-order) memristive systems is generalized to non-integer order case using fractional calculus. We also show that the memory effect of such devices can be also used for an analogue implementation of the fractional-order operator, namely fractional-order integral and fractional-order derivatives. This kind of operators are useful for realization of the fractional-order controllers. We present theoretical description of such implementation and we proposed the practical realization and did some experiments as well.

Keywords: fractional calculus, fractional-order system, memristive devices, memristor, fractor, fractductor.

1 Introduction

In 1971, professor Leon O. Chua published a paper on the missing basic circuit element - memristor or memory resistor. Professor Leon O. Chua and Dr. Sung–Mo Kang published a paper, in 1976, that described a large class of devices and systems they called memristive devices and systems [6]. Whereas a memristor has mathematically scalar state, a system has vector state. The number of state variables is independent of, and usually greater than, the number of terminals. In that paper, Chua applied the model to empirically observed phenomena, including the Hodgkin-Huxley model of the axon and a thermistor at constant ambient temperature. He also described memristive systems in terms of energy storage and easily observed
electrical characteristics. These characteristics match resistive random-access memory and phase-change memory, relating the theory to active areas of research. Chua extrapolated the conceptual symmetry between the resistor, inductor, and capacitor, and inferred that the memristor is a similarly fundamental device. Other scientists had already used fixed nonlinear flux-charge relationships, but Chua’s theory introduces generality. This relation is illustrated in Fig. 1.

![Connection of four basic electrical elements](image)

Figure 1. Connection of four basic electrical elements (Figure adopted from Ref. [31]).

Thirty-seven years later, on April 30, 2008, Stan Williams and his research group of scientists from HP Labs has finally built real working memristors, thus adding a fourth basic circuit element to electrical circuit theory, one that will join the three better-known ones: the capacitor, resistor and the inductor. They built a two-terminal titanium dioxide nanoscale device that exhibited memristor characteristics [40]. A linear time-invariant memristor is simply a conventional resistor. Important thing is that it is impossible to substitute memristor with combination of the other basic electrical elements and therefore memristor can provide other new functions [14].

Possible applications of memristive systems:
- new memory without access cycle limitations with new memory cells for more energy-efficient computers [34] e.g.: 1 bit = 1 memristor;
- new analog computers that can process and associate information in a manner similar to that of the human brain [30];
- new electronic circuits, e.g. [7, 32]: voltage divider, switcher, compensator, AD – DA converters, etc.;
- new control systems and controllers with memory;

In this paper we present the connection between fractional calculus (fractional order integral and derivative) and behavior of the memristive systems. As we will see, the fundamentals of fractional calculus are based on the memory property of the fractional order integral/derivative and therefore this connection is straightforward. This exceptional property can be used for realization of the fractional order operator as a basic element for implementation of the fractional order controllers.

This paper is organized as follows: Section 1 introduces memristor and memristive devices. In Section 2 is described the fractional calculus. Section 3 is on analogue electrical circuits which exhibit memristive behavior. In Section 4 are described the fractional-order controllers and proposal for their realization with using the memristive system and op-amps. In Section 5 are presented the real measurements. Section 6 concludes this article with some additional remarks.

2 Fractional-Order Derivatives and Integrals
2.1 Definitions

The idea of fractional calculus has been known since the development of the regular calculus, with the first reference probably being associated with letter between Leibniz and L’Hospital in 1695.

Fractional calculus is a generalization of integration and differentiation to non-integer order fundamental operator $a D_t^\alpha f(t)$, where $a$ and $t$ are the limits of the operation. The continuous integro-differential operator is defined as

$$a D_t^\alpha f(t) = \begin{cases} d^{\alpha \tau} & : \alpha > 0, \\ \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{\alpha-n+1} f(\tau) d\tau & : \alpha < 0. \end{cases}$$

The two definitions used for the general fractional differintegral are the Grunwald-Letnikov (GL) definition and the Riemann-Liouville (RL) definition [25], [18]. The GL is given here

$$a D_t^\alpha f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \binom{\alpha}{j} f(t-jh),$$

where $[\cdot]$ means the integer part. The RL definition is given as

$$a D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} d^n \int_a^t \frac{f(\tau)}{(t-\tau)^{n-\alpha+1}} d\tau,$$

for $(n-1 < \alpha < n)$ and where $\Gamma(.)$ is the Gamma function.
The Laplace transform method is used for solving engineering problems. The formula for the Laplace transform of the RL fractional derivative (2) has the form [25]:

\[ \int_{0}^{\infty} e^{-st} D^{\alpha}_{t} f(t) \, dt = s^{\alpha} F(s) - \sum_{k=0}^{n-1} \frac{\dot{f}^{(k)}(0)}{\Gamma(\alpha-k)} s^{\alpha-k-1}, \quad (s > 0), \tag{3} \]

for \((n - 1 < \alpha \leq n)\), where \(s \equiv j\omega\) denotes the Laplace operator.

Some others important properties of the fractional derivatives and integrals we can find out in several works (e.g.: [18, 25], etc.).

### 2.2 Fractional calculus and memristive systems

There are a large number of electric and magnetic phenomena where the fractional calculus can be used [39]. We will consider three of them - capacitor, inductor, and memristor.

Westerlund et al. in 1994 proposed a new linear capacitor model [38]. It is based on Curie’s empirical law of 1889 which states that the current through a capacitor is

\[ I(t) = \frac{V_{0}}{h_{1} t^{\alpha}}, \]

where \(h_{1}\) and \(\alpha\) are constant, \(V_{0}\) is the dc voltage applied at \(t = 0\), and \(0 < \alpha < 1\) (\(\alpha \in \mathbb{R}\)).

For a general input voltage \(V(t)\) the current is

\[ I(t) = C \frac{d^{\alpha} V(t)}{dt^{\alpha}} \equiv C \frac{d^{\alpha}}{dt^{\alpha}} V(t), \tag{4} \]

where \(C\) is capacitance of the capacitor. It is related to the kind of dielectric. Another constant \(\alpha\) (order) is related to the losses of the capacitor. Westerlund provided in his work the table of various capacitor dielectric with appropriated constant \(\alpha\) which has been obtained experimentally by measurements.

For a current in the capacitor the voltage is

\[ V(t) = \frac{1}{C} \int_{0}^{t} I(t) dt^{\alpha} \equiv \frac{1}{C} \int_{0}^{t} D^{\alpha}_{t} I(t). \tag{5} \]

Westerlund in his work also described behavior of real inductor [39]. For a general current in the inductor the voltage is

\[ V(t) = L \frac{d^{\alpha} I(t)}{dt^{\alpha}} \equiv L \frac{d^{\alpha}}{dt^{\alpha}} I(t), \tag{6} \]

where \(L\) is inductance of the inductor and constant \(\alpha\) is related to the "proximity effect". A table of various coils and their real orders \(\alpha\) is described in [29].

As it was already mentioned, Chua in 1971 predicted a new circuit element - called memristor characterized by a relationship between the charge \(q(t)\) and the flux \(\phi(t)\). It is the fourth basic circuit element [6, 23, 24, 34]. The voltage across a charge-controlled memristor is given by

\[ v(t) = M(q(t))i(t), \quad \text{where } M(q(t)) = \frac{d\phi}{dq}. \tag{7} \]

Noting from Faraday’s law of induction that magnetic flux \(\phi(t)\) is simply the time integral of voltage \((d\phi = V(t) dt)\) and charge \(q(t)\) is the time integral of current \((dq = I(t) dt)\), the more convenient form of the current - voltage equation for the memristor is [6]

\[ M(q(t)) \int_{0}^{t} I(t) dt = \int_{0}^{t} V(t) dt, \tag{8} \]

where \(M(q(t))\) is memristance of the memristor. If \(M(q(t))\) is a constant \((M(q(t)) \equiv R(t))\), then we obtain Ohm’s law \(R(t) = V(t)/I(t)\). If \(M(q(t))\) is nontrivial, the equation is not equivalent because \(q(t)\) and \(M(q(t))\) will vary with time.

Similar to capacitor and inductor, the memristor is also not ideal circuit element and we can predict the fractional-order model of such element. Applying the fractional calculus to relation (8), we obtain the following general formula for fractional order memristive systems:

\[ K \frac{D^{\beta}_{t} I(t)}{dt^{\beta}} = \frac{d^{\gamma} \phi}{dt^{\gamma}} \quad (\gamma, \beta \in \mathbb{R}) \tag{9} \]

where \(K\) is the resistance, inductance, capacitance or memristance, respectively.

Applying the Laplace transform technique (3) to equation (9), we get the following relation

\[ K s^{\gamma} I(s) = s^{\beta} \phi(s) \tag{10} \]

and the resulting impedance of the memristive system (MS) is

\[ Z_{MS}(s) = K s^{\gamma-\beta} = K s^{\beta}, \quad (k \in \mathbb{R}) \tag{11} \]

where \(k\) is the real order of the memristive system and for ideal electrical elements has the following particular values, if:

- \(\gamma = 0\) and \(\beta = 0\) then \(k = 0\), we obtain resistor and then \(K = R[\Omega]\);
- \(\gamma = -1\) and \(\beta = 0\) then \(k = -1\), we obtain capacitor and then \(K = 1/C[F]\);
- \(\gamma = 0\) and \(\beta = -1\) then \(k = 1\), we obtain inductor and then \(K = L[H]\);
γ = -1 and β = -1 then k = 0, we obtain memristor and then K = M(t) [Ω];

However, as already has been mentioned, the real electrical element are not ideal and with the help of fractional calculus was shown that the intermediate cases between the known characteristic behaviors of the electrical elements resistor R, capacitor C and inductor L change continuously [32]. By deduction the memristor M, which has storage properties, could be also consider as a real electrical element with the fractional order of its mathematical model. The fractional calculus can help us to described the memory behavior of the memristor. As we can see in the equations (1) and (2), kernels of both definitions consist of the memory term and consider with the history. It is suitable for the memristor description and its applications.

The above concepts of memory devices are not necessarily limited to resistance – memristor but can in fact be generalized to capacitative and inductive systems. If \( x(t) \) denotes a set of \( n \) state variables describing the internal state of the system, \( u(t) \) and \( y(t) \) are any two complementary constitutive variables (current, charge, voltage, or flux) denoting input and output of the system, and \( g \) is a generalized response, we can define a general class of \( n \)th-order \( u \) controlled dynamical systems called memristive systems or devices described by the following equations [7]

\[
\begin{align*}
\frac{dx(t)}{dt} &= f(x, u, t) \\
y(t) &= g(x, u, t)u(t)
\end{align*}
\]

where \( f \) is a continuous \( n \)-dimensional vector function and we assume unique solution for any initial state \( x(t) \) at time \( t = t_0 \).

In Fig. 2 and Fig. 3 are shown current-voltage characteristics of the memristor postulated by professor L. Chua in 1971 (simulation) and the memristor realized by S. Williams in 2008 (experiment).

General fractional-order differential equation (9) can be rewritten to its canonical form and then equations (12) become as follow:

\[
\begin{align*}
0D_t^\alpha x(t) &= f(x, u, t) \\
y(t) &= g(x, u, t)u(t).
\end{align*}
\]

For current controlled memristive system we obtain from Eqs (13) the following equations

\[
\begin{align*}
0D_t^\alpha x(t) &= f(x, I, t) \\
V_M(t) &= M(x, I, t)I(t),
\end{align*}
\]

with \( x \) a vector representing internal variables, \( V_M(t) \) and \( I(t) \) denote the voltage and current across the device, and \( M \) is a scalar, called memristance with the physical unit of Ohm. Similar approach can be used for deriving the description of the memductance and memcapacitance system [35].
3 Analogue Fractional-Order Differentiator/Integrator

We are able to define arbitrary real order $k$ for the memristive system behavior description (11). The amplitude of this impedance function is $A = 20k$ and the phase angle is $\Phi = k(\pi/2)$ for $k \in \mathbb{R}$. Electrical elements (memristive system or fractance) with such property are sometimes called constant phase element for certain frequency range [2]. So far, the constant phase elements (CPE) were approximated by the ladder network constructed form RLC elements, tree network, metal-insulator-liquid interface, etc. [1, 4, 5, 8–12, 17, 19, 26, 33, 36].

We can use an active operating amplifier (op-amps) and its inverting connection with impedance $Z_1$ in direct connection and impedance $Z_2$ in feedback connection. Transfer function of circuit depicted in Fig. 4(b) is:

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)}.$$

Generally, as electrical element with the impedances $Z_1$ and $Z_2$ can be used basic electrical elements (resistor, capacitor, inductor, memristor) or electrical networks (RC ladder, RC tree, RLC grid, CPE, ...). In this way we can obtain various dividers, filters, integrators, differentiators, etc.

![电压divider](image1)

(a) Voltage divider.

![反相连接的OA](image2)

(b) Inverting connection of OA.

Figure 4. Basic connections of two impedances.

Most passive electronic circuits can be reduced to a voltage divider which splits a driving signal between two impedances $Z_1$ and $Z_2$. The impedance of a circuit element is its voltage-to-current ratio at a given frequency. Transfer function of circuit depicted in Fig. 4(a) is simply:

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}.$$

As it was mentioned above we can construct differentiator and integrator as well. For simplifying we assume that inverting
connection of operating amplifier with the memristor and other memristive systems will be used in two type of connections.

The transfer function for above circuit (Type I) can be written as

\[
\frac{V_o(s)}{V_{in}(s)} = -\frac{M}{k} s^{-k}, \quad k \in \mathbb{R} \tag{15}
\]

For \( k = -1 \) (capacitor) we can recognize an ideal differentiator of first order and for \( k < 0 \) (\( k \in \mathbb{R} \)) it is a real differentiator of fractional order. For \( k = 1 \) (inductor) it is an ideal integrator and for \( k > 0 \) it is a real integrator of fractional order.

The transfer function for above circuit (Type II) can be written as

\[
\frac{V_o(s)}{V_{in}(s)} = -\frac{K s^k}{M}, \quad k \in \mathbb{R} \tag{16}
\]

For \( k = -1 \) (capacitor) we can recognize an ideal integrator of first order and for \( k < 0 \) (\( k \in \mathbb{R} \)) it is a real integrator of fractional order. For \( k = 1 \) (inductor) it is an ideal differentiator and for \( k > 0 \) it is a real differentiator of fractional order.

Both type of these connections are useful for the fractional-order controller realization.

4 Fractional-Order Controllers

4.1 Fractional PID controller

Intuitively, with noninteger order controllers for integer order plants, there is a better flexibility in adjusting the gain and phase characteristics than using integer order (IO) controllers. This flexibility makes fractional order (FO) control a powerful tool in designing robust control system with less controller parameters to tune. The key point is that using few tuning knobs, FO controller achieves similar robustness achievable by using very high-order IO controllers.

\( \text{TID} \) controller [27], also known as PI\(^1\)D\(^0\) controller, was studied in time domain in [25] and in frequency domain in [26]. In general form, the transfer function of PI\(^1\)D\(^0\) is given by

\[
C(s) = \frac{U(s)}{E(s)} = K_p + T_i s^{-\lambda} + T_d s^\delta, \tag{17}
\]

where \( \lambda \) and \( \delta \) are positive real numbers; \( K_p \) is the proportional gain, \( T_i \) the integration constant and \( T_d \) the differentiation constant. Clearly, taking \( \lambda = 1 \) and \( \delta = 1 \), we obtain a classical PID controller. If \( \lambda = 0 \) (\( T_i = 0 \)) we obtain a PD\(^0\) controller, etc. All these types of controllers are particular cases of the PI\(^1\)D\(^0\) controller. The time domain formula is that

\[
u(t) = K_p e(t) + T_i D^{-\lambda} e(t) + T_d D^\delta e(t), ~ (D_i^{(\lambda)} \equiv 0, D_d^{(\delta)}). \tag{18}
\]

It can be expected that PI\(^1\)D\(^0\) controller (18) may enhance the systems control performance due to more tuning knobs introduced. Actually, in theory, PI\(^1\)D\(^0\) itself is an infinite dimensional linear filter due to the fractional order in differentiator or integrator.

Similar to the fact that, every year, numerous PI/PID papers have been published, we can foresee that, more and more FO PI/D papers will be published in the future. In general, the following issues should be addressed:

- How to tell there is a need to use FO PI/D controller while integer order PI/D control works well in the existing controlled systems?
- How to predict the performance gain by using FO PI/D controller?
- How to best tune the FO PI/D controller by taking minimum experimental efforts?
- How to best design the experiments to tune FO PI/D controller?
- For a given class of plants to be controlled, how to best design FO PI/D controller?

We comment that since PID control is ubiquitous in industry process control, FO PID control will be also ubiquitous when tuning and implementation techniques are well developed.

4.2 Some others typical fractional order controllers

In this section, three other representative fractional-order controllers in the literature will be briefly introduced, namely, TID (tilted integral derivative) controller, CRONE controller and fractional lead-lag compensator [41].

\textit{TID Controller.} In [13], a feedback control system compensator of the PID type is provided, wherein the proportional component of the compensator is replaced with a tilted component having a transfer function \( s^{-\lambda} \). The resulting transfer function of the entire compensator more closely approximates an optimal transfer function, thereby achieving improved feedback controller. Further, as compared to conventional PID compensators, the TID compensator allows for simpler tuning, better disturbance rejection ratio, and smaller effects of plant parameter variations on closed loop response.

The objective of TID is to provide an improved feedback loop compensator having the advantages of the conventional PID compensator, but providing a response which is closer to the theoretically optimal response. In TID patent [13], an analog circuit using op-amps plus capacitors and resistors is introduced with a detailed component list which is useful in some cases where the computing power to implementing \( T_d(s) \) digitally is not possible. An example is given in [13]
to illustrate the benefits from TID over conventional PID in both time and frequency domain.

**CRONE Controller.** The CRONE control was proposed by Oustaloup in pursuing *fractal robustness* [21, 22]. CRONE is a French abbreviation for “Contrôle Robuste d’Ordre Non Entier” (which means non-integer order robust control). In this section, we shall follow the basic concept of fractal robustness, which motivated the CRONE control, and then mainly focus on the second generation CRONE control scheme and its synthesis based on the desired frequency template which leads to fractional transmittance.

In [20], “fractal robustness” is used to describe the following two characteristics: the iso-damping and the vertical sliding form of frequency template in the Nichols chart. This desired robustness motivated the use of fractional-order controller in classical control systems to enhance their performance.

With a unit negative feedback, for the characteristic equation

\[ 1 + (\tau s)^{\alpha} = 0, \]

the forward path transfer function, or the open-loop transmittance, is that

\[ \beta(s) = \left( \frac{1}{\tau s} \right)^{\alpha} = \left( \frac{\omega_u}{s} \right)^{\alpha}, \tag{19} \]

which is the transmittance of a non integer integrator in which \( \omega_u = 1/\tau \) denotes the unit gain (or transitional) frequency.

In controller design, the objective is to achieve such a similar frequency behavior, in a medium frequency range around \( \omega_u \), knowing that the closed loop dynamic behavior is exclusively linked to the open loop behavior around \( \omega_u \). Synthesizing such a template defines the non-integer approach that the second generation CRONE control uses.

**Fractional Lead-Lag Compensator.** In the above, fractional controllers are directly related to the use of fractional-order differentiator or integrator. It is possible to extend the classical lead-lag compensator to the fractional-order case which was studied in [15, 28]. The fractional lead-lag compensator is given by

\[ C_r(s) = C_0 \left( \frac{1 + s/\omega_b}{1 + s/\omega_h} \right)^r, \tag{20} \]

where \( 0 < \omega_b < \omega_h \), \( C_0 > 0 \) and \( r \in (0, 1) \). The autotuning technique has been presented in [15].

We conclude this section by offering the following remark.

Just like the non-integers are ubiquitous between integers, noninteger order control schemes will be ubiquitous by extending the existing integer order control schemes into their noninteger counterparts. For example, fractional sliding mode control with fractional order sliding surface dynamics; model reference adaptive control using fractional parameter updating law etc. The opportunities for extensions are almost endless. However, the question remains: we need a good reasons for such extensions.

### 4.3 Analogue realization and implementation

In Fig. 6 is shown the proposal for analogue implementation of the fractional-order controller with using the memristive systems as for example memristors and real capacitors and op-amps in inverting connection. The memristive systems could be replaced by any CPE or other electrical RLC networks and instead memristor we can use a usual resistor. Using the suggested circuit is much better because of memory property in the fractional-order controller.

![Figure 6. Analogue fractional-order controller built with memristive systems.](image)

Applying the fractional-order differentiator and integrator described in Section 3 we are able to realize a new type of the fractional-order \( P^{\lambda}I^{\delta}D \) controller.

\[
C(s) = \frac{U(s)}{E(s)} = -\frac{R_2}{R_1} \left( \frac{R_4}{R_3} - \frac{Z_{MS2}(s)}{Z_{MS1}(s)} \right) \frac{M_1(s)}{M_2(s)} = K_p + T_1 s^{-\lambda} + T_2 s^\delta. \tag{21}
\]
Usually we set \( R_2 = R_1 \) and then controller parameters are:

\[
K_p = \frac{R_4}{R_3} \quad (22)
\]

\[
T_i = \frac{Z_{MS2}(s)}{M_2(s)} = \frac{K_2}{M_2} s^{k_1}, \quad \lambda = k_1
\]

\[
T_d = \frac{M_1(s)}{Z_{MS1}(s)} = \frac{M_1}{K_1} s^{k_2}, \quad \delta = k_2
\]

Instead memristive devices or fractance circuit the new electrical element introduced by G. Bohannan which is so called \textit{Fractor} can be used as well [3]. This element - fractor is made of a material with the properties of LiN\(_2\)H\(_5\)SO\(_4\).

5 Experimental results

An alternative version of the connection diagram in Fig. 1 is presented in Fig. 7. By moving parts of the original diagram (moving the resistor symbol \( R \) to the center, for example), and assuming continuous real numerical space for \( \alpha \) and \( \beta \), space is included for the fractor as presented by Bohannan, as well as a new “element,” a so-called fractional inductor, or “\textit{Fractductor}.” This device has a fractional-order coupling between flux and current.

\[
[I = D_t^{\alpha=1} q(t)] \quad [\varphi = D_t^{\beta=-1} V(t)]
\]

Figure 7. Alternative electrical component connection diagram.

Preliminary attempts to construct a fractductor have produced a device using magnetorheological fluid as the core in a transformer-like device. A bode plot (Fig. 8), basic block schematic (Fig. 9), and photo of the experimental device (Fig. 10) are shown.

As we can see in Fig. 9, connection of electrical elements were done according to suggestions described in previous sections. It is practical realizition of the fractional-order memristive systems which can be used for the fractional-order controllers implementation as well.

6 Conclusion

In this brief paper was presented proposal for a new class of the fractional-order controller realization. Described memristive systems are useful for practical implementation of the fractional-order controllers. However this approach gave a good start for detail analysis and design of the analogue fractional order controller. The fractional-order controller gives us an insight into the concept of memory of the suggested fractional operator.

We also proposed a new electrical device, so-called fractductor, which belongs to class of the analogue generalized fractional-order memristive devices. This device has a fractional-order coupling between flux and current. Further work is needed to prove its performance by various simulations and experimental measurements at the circuit. The results may find wide application in new circuits, signal processing and control systems, etc. (e.g.: [16, 37]).

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Figure 8. Bode plot of an experimental fractional flux coupling device, the fractductor.

Figure 9. Simplified schematic diagram of the fractductor and test circuit.

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