Fault Tolerant Control Strategy with Simultaneous Actuator and Sensor Faults for a Quadrotor UAV

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Abstract--This paper considers the control problem for an underactuated quadrotor UAV system in presence of simultaneous actuator and sensor faults. The dynamical model of quadrotor while taking into account various physical phenomena, which can influence the dynamics of a flying structure is presented. Subsequently, a new control strategy based on robust integral backstepping approach using sliding mode taking into account the simultaneous actuator and sensor faults is developed. Lyapunov based stability analysis shows the main advantage of this control strategy which is the stability preservation of the closed loop dynamics of the quadrotor UAV even after the occurrence of actuator and sensor faults at same time. Numerical simulation results are provided to show the good tracking performance of proposed control laws.

Index Terms—Actuator faults, Backstepping control, Dynamic model of Quadrotor, Fault tolerant control (FTC), Robust control, Sensor faults, Unmanned aerial vehicles (UAV).

I. INTRODUCTION

UNMANNED Aerial Vehicles have been developed for performing various missions in the military and civil areas. Quadrotors are one of UAVs which consist of two rods and four actuators as shown in the Fig.1. Even though its structure is simple, the quadrotor is a VTOL (Vertical Task-off and Landing) and can perform most of missions that helicopters can do. In some aspects, the quadrotors have better maneuverability than helicopters because quadrotors have four rotors, which can increase the mobility and loadability.

Even though quadrotors do not have complicated structure, it is not easy to design its controller. The quadrotor is a dynamically unstable and under-actuated nonlinear system which makes the controller design hard. To deal with these difficulties, many researchers have tried to approximate the original dynamic models, or to linearize the systems on certain trim points especially on the hovering attitude.

The quadrotors, has been studied recently by some authors [12], [6], [1], [13], [14], [9], [16], [2], [3], [5], [10], [15], [8], [11], [7], [4]. These systems as many other dynamic systems, present constant or slowly-varying uncertain parameters, but these authors do not take into account the faults affecting the actuators and the sensors of our system, which makes them very limited and induces undesired behavior of quadrotor, or even to instability of the latter after occurrence of actuator or/and sensor faults.

In this paper, the control problem of the quadrotor aircraft in presence of simultaneous actuator and sensor faults is considered. The dynamical model describing the quadrotor aircraft motions and taking into account for various parameters which affect the dynamics of a flying structure such as frictions due to the aerodynamic torques, drag forces along (X,Y,Z) axis and gyroscopic effects is presented. Subsequently, a new control strategy based on robust integral backstepping approach using sliding mode taking into account the simultaneous actuator and sensor faults is proposed. It includes two compensation techniques, the first one is the using an integral action, the second, is to use an another term containing "sign" function. Finally all synthesized control laws are highlighted by simulations which gave fairly satisfactory results despite the presence of actuator and sensor faults.

II. DYNAMICAL MODEL

The quadrotor have four propellers in cross configuration. The two pairs of propellers \{1,3\} and \{2,4\} as described in Fig. 1, turn in opposite directions. By varying the rotor speed, one can change the lift force and create motion. Thus, increasing or decreasing the four propeller’s speeds together generates vertical motion. Changing the 2 and 4 propeller’s speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion; result from 1 and 3 propeller’s speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

Fig. 1 Quadrotor configuration

The quadrotor model (position and orientation dynamic) obtained is given like in [2], [3], [4] by:
\[
\begin{align*}
\dot{\phi} &= \left(\frac{I_y - I_z}{I_z}\right) \dot{\theta} - \frac{J_z}{I_z} \Omega \dot{\theta} - \frac{K_{\phi \theta}}{I_z} \phi^2 + \frac{l}{I_z} u_2 \\
\dot{\theta} &= \left(\frac{I_z - I_x}{I_y}\right) \phi \dot{\psi} + \frac{J_x}{I_y} \Omega \dot{\phi} - \frac{K_{\phi \theta}}{I_y} \theta^2 + \frac{l}{I_y} u_3 \\
\dot{\psi} &= \left(\frac{I_x - I_y}{I_z}\right) \dot{\phi} - \frac{K_{\psi \phi}}{I_z} \psi^2 + \frac{l}{I_z} u_4 \\
\dot{x} &= \frac{K_{fr}}{m} x + \frac{1}{m} (\cos \phi \cos \psi \sin \theta + \sin \phi \sin \psi) u_1 \\
\dot{y} &= -\frac{K_{fr}}{m} y + \frac{1}{m} (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) u_1 \\
\dot{z} &= -\frac{K_{fr}}{m} z - g + \frac{\cos(\phi) \cos(\theta)}{m} u_1.
\end{align*}
\]

The system's inputs are posed \(u_1, u_2, u_3, u_4\) and \(\Omega_r\) a disturbance, obtaining:

\[
\begin{align*}
\dot{u}_1 &= b \left( \alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 \right) \\
\dot{u}_2 &= b \left( \alpha_2^2 - \alpha_3^2 \right) \\
\dot{u}_3 &= b \left( \alpha_3^2 - \alpha_4^2 \right) \\
\dot{u}_4 &= d \left( \alpha_2^2 - \alpha_3^2 + \alpha_3^2 - \alpha_4^2 \right) \\
\Omega_r &= \alpha_1 - \alpha_2 + \alpha_3 - \alpha_4
\end{align*}
\]

The rotors are driven by DC motors with the well known equations [9]:

\[
\begin{align*}
v &= Ri + L \frac{di}{dt} + k_e \omega \\
k_e i &= J_r \frac{d\omega}{dt} + C_e + k_w \omega^2
\end{align*}
\]

As we a small motor with a very low inductance, then we can obtain the voltage to be applied to each motor as follows:

\[
v_i = \frac{1}{\eta} \left( \Omega_i + \mu_i \alpha_i^2 + \mu_\iota \alpha_i + \mu_\iota \alpha_i \right), \quad i \in \{1, \ldots, 4\}
\]

with:

\[
\mu_i = \frac{k_r}{J_r}, \quad \mu_i = \frac{k_k}{J_k}, \quad \mu_\iota = \frac{C_k}{J_k} \quad \text{and} \quad \eta = \frac{k_m}{J_m}
\]

where, \(v_i\) is the motor input, \(\omega_i\) is the angular speed, \(k_e, k_w\) are respectively the electrical and mechanical torque constants, and \(C_e\) is the solid friction.

### III. CONTROL DESIGN OF QUADROTOR WITH SIMULTANEOUS ACTUATOR AND SENSOR FAULTS

The complete model resulting by adding the actuator and sensor faults in the model (1) can be written in the state-space form:

\[
\begin{align*}
\dot{X} &= f(X,U) + E_a F_a \\
Y &= CX + E_s F_s
\end{align*}
\]

with \(X \in \mathbb{R}^n\) is the state vector of the system, \(U \in \mathbb{R}^m\) is the input control vector, \(Y \in \mathbb{R}^p\) is the measured output vector, \(F_a \in \mathbb{R}^u\) is the resultant vector of actuator faults related to the quadrotor motions, \(F_s \in \mathbb{R}^v\) is the sensor faults vector, \(C \in \mathbb{R}^{p \times n}\), \(E_a \in \mathbb{R}^{p \times u}\) and \(E_s \in \mathbb{R}^{p \times v}\) are respectively, the observation matrix, the actuator faults and the sensor faults distributions matrix, such as:

\[
X = [x_1, x_2, \ldots, x_{12}]^T = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}]^T
\]

**Remark 1:** In our contribution, only the velocity sensor faults are considered.

From (1), (6) and considering the actuator and the velocity sensor faults, we obtain the following state representation:

\[
\begin{align*}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8 \\
\dot{x}_9 \\
\dot{x}_{10} \\
\dot{x}_{11} \\
\dot{x}_{12}
\end{bmatrix} &=
\begin{bmatrix}
x_2 \\
a_2 x_1 x_2 + a_3 x_2^2 + a_4 \Omega x_2 + b_2 & + f_{a1} \\
0 & a_4 \\
0 & a_4 \\
a_4 & a_4 \\
0 & a_4 & a_4 \\
0 & a_4 & a_4 \\
a_4 & a_4 & a_4 & a_4 & a_4 & a_4 & a_4 & a_4 & a_4 & a_4 & a_4 & a_4
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12}
\end{bmatrix} + \begin{bmatrix}
0 \\
1/m \dot{u}_i u_1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

with

\[
\begin{align*}
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2 \\
\dot{u}_3 \\
\dot{u}_4
\end{bmatrix} &=
\begin{bmatrix}
\cos(\phi) \cos(\theta) - \sin(\phi) \sin(\psi)
\cos(\phi) \sin(\phi) \sin(\psi) + \cos(\phi) \cos(\theta)
\end{bmatrix}
\end{align*}
\]

and

\[
\begin{align*}
a_i &= \frac{I_z - I_x}{I_z} a_i = -\frac{K_{sa}}{I_z} a_i = \frac{I_y - I_x}{I_y} a_i = -\frac{K_{fa}}{I_y} a_i \\
a_i &= \frac{J_x}{I_x} a_i = \frac{I_y - I_z}{I_y} a_i = -\frac{J_z}{I_z} a_i = \frac{K_{fa}}{I_x} a_i \\
a_i &= \frac{I_z}{I_z} b_i = \frac{J_x}{I_x} b_i = \frac{I_y - I_z}{I_y} b_i = \frac{J_z}{I_z} b_i = \frac{K_{fa}}{I_x} b_i
\end{align*}
\]

The following assumptions are needed for the analysis.

**Assumption 1:** The resultant of actuator faults related to the quadrotor motions and velocity sensor faults are bounded,

\[
[f_a] \leq f_a^* \quad \text{and} \quad [f_{\psi}] \leq f_{\psi}^* \quad \text{where} \quad i \in \{1, \ldots, 4\} \quad \text{and} \quad j \in \{1, \ldots, 6\}
\]

**Assumption 2:** The velocity sensor faults are slowly varying in time, as follows:

\[
f_{\psi} \approx 0 \quad \text{and} \quad i \in \{1, \ldots, 6\}
\]

From (8) it easy to show that:

\[
\begin{align*}
\phi_i &= \arcsin(u_i \sin(\psi_i) - u_i \cos(\psi_i)) \\
\psi_i &= \arcsin(u_i \cos(\psi_i) + u_i \sin(\psi_i))
\end{align*}
\]
The adopted control strategy is based on two loops (internal loop and external loop). The internal loop contains four control laws: control of roll, control of pitch, control of yaw and control of altitude. The external loop includes two control laws of positions x and y. The external control loop generates a desired of roll (ϕd) and pitch (θd) through the correction block. This block corrects the rotation of roll and pitch depending on the desired yaw (ψd) (illustrated by equation (10)). The synoptic scheme below shows this control strategy:

![Synoptic scheme of the proposed control strategy](image)

**Fig. 2 Synoptic scheme of the proposed control strategy**

Basing on backstepping approach, a recursive algorithm is used to synthesize the control laws forcing the system to follow the desired trajectory.

For the first step we consider the tracking-error:

\[ e_1 = x_{id} - x_1 \]  

The corresponding Lyapunov function is given by:

\[ V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} \dot{d}_1^2 \]

It’s time derivative is then:

\[ \dot{V}_1 = e_1 (\dot{x}_{id} - (y_{2} - f_{s1})) + \dot{d}_1 (-\dot{\xi}_1) \]

The stabilization of \( e_1 \) can be obtained by introducing a new virtual control \( y_2 \)

\[ (y_2)_{id} = \alpha_1 + \dot{k}_1 e_1 + \lambda_1 \xi_1 \]

In order to compensate the effect of velocity sensor fault of roll motion, an integral term is introduced. We take:

\[ \xi_i(t) = \int_{0}^{t} e_i(\tau) d\tau \]

It results that:

\[ \dot{V}_1 = e_1 (-k_1 e_1 + \lambda_1 \dot{d}_1) + \dot{d}_1 (-e_1) \]

\[ = (-e_1 \dot{d}_1) \begin{bmatrix} k_1 \\ 1 \end{bmatrix} \begin{bmatrix} -\lambda_1 \\ 0 \end{bmatrix} e_1 \dot{e}_1 \dot{Q} \dot{e}_1 \]

\( k_1 \) and \( \lambda_1 \) are chosen so as to make the definite matrix positive \( Q_1 \), which means that, \( V_1 \leq 0 \)

Let us proceed to a variable change by making:

\[ e_2 = x_{id} + k_1 e_1 + \lambda_1 \xi_1 - y_2 \]

For the second step we consider the augmented Lyapunov function:

\[ V_2 = V_1 + \frac{1}{2} e_2^2 \]

Such as

\[ \dot{\beta}_1 = \dot{x}_d + k_1 (-k_1 e_1 + e_2) + \lambda_1 e_1 - a_1 y_6 - a_1 y_6^2 - a_1 \Omega y_6 + \Delta \beta_1 \]

with: \( (\Delta \beta_1) \) is the unknown part including the resultant of actuator faults related to the roll motion and the velocity sensor faults. It is assumed to be bounded:

\[ |\Delta \beta_1| \leq \lambda_2 \]

The stabilization of \( (e_1, e_2) \) can be obtained by introducing the input control \( u_2 \)

\[ u_2 = \frac{1}{b_1} (\dot{\beta}_1 + e_1 + k_1 e_2 + k_2 \xi_2) \]

It result that

\[ \dot{V}_2 = -e_1^T Q e_1 - k_1 e_2^2 + e_2 (\Delta \beta_1 - \lambda_2 \xi_2) \]

In order to compensate \( (\Delta \beta_1) \), a “sign” function is introduced. We take:

\[ \xi_2(t) = \text{sign} (e_2(t)) \]

The control input \( u_2 \) is then extracted satisfying \( \dot{V}_2 \leq 0 \).

\[ u_2 = \frac{1}{b_1} (\dot{\beta}_1 + k_1 (-k_1 e_1 + e_2) + (1 + \lambda_1) e_1 + k_2 \xi_2) \]

The same steps are followed to extract \( u_3, u_4, u_5, u_6 \) and \( u_1 \).

\[ \begin{align*}
\dot{u}_3 &= \frac{1}{b_1} (\dot{\beta}_1 + k_1 (-k_1 e_1 + e_2) + (1 + \lambda_1) e_1 + k_2 \xi_2) \\
&\quad - a_1 y_6 - a_1 y_6^2 - a_1 \Omega y_6 + \lambda_2 \text{sign} (e_2) \\
\dot{u}_4 &= \frac{1}{b_1} (\dot{\beta}_1 + k_1 (-k_1 e_1 + e_2) + (1 + \lambda_1) e_1 + k_2 \xi_2) \\
&\quad - a_1 y_6 - a_1 y_6^2 + \lambda_2 \text{sign} (e_2) \\
\dot{u}_5 &= \frac{1}{u_1} (\dot{\beta}_1 + k_1 (-k_1 e_1 + e_2) + (1 + \lambda_1) e_1 + k_2 \xi_2) \\
&\quad - a_1 y_6 + \lambda_2 \text{sign} (e_2) \\
\dot{u}_6 &= \frac{1}{u_1} (\dot{\beta}_1 + k_1 (-k_1 e_1 + e_2) + (1 + \lambda_1) e_1 + k_2 \xi_2) \\
&\quad - a_1 y_6 + \lambda_2 \text{sign} (e_2) \\
\end{align*} \]

with

\[ e_1 = x_{id} - x_i, \quad i \in \{3, 5, 7, 9, 11\} \]

and

\[ \xi_i(t) = \int_{0}^{t} e_i(\tau) d\tau, \quad i \in \{3, 5, 7, 9, 11\} \]

\[ \text{sign} (e_i(t)), \quad i \in \{4, 6, 8, 10, 12\} \]
The corresponding lyapunov functions are given by:

$$V_i = \begin{cases} \frac{1}{2} e_i^2 + \frac{1}{2} e_j^2 & i \in [3,5,7,9,11] \text{ and } j \in [2,\ldots,6] \\ \frac{1}{2} e_i^2 & i \in [4,6,8,10,12] \end{cases} \quad (31)$$

such as

$$\tilde{d}_j = \frac{f_j}{\lambda_i} - \dot{\zeta}_i = d_j - \zeta_i \quad i \in [3,5,7,9,11] \text{ and } j \in [2,\ldots,6]$$

$$Q_j = \begin{pmatrix} k_i & -\lambda_i \\ 1 & 0 \end{pmatrix} > 0 \quad i \in [3,5,7,9,11] \text{ and } j \in [2,\ldots,6] \quad (32)$$

$$k_i > 0 \quad i \in [4,6,8,10,12]$$

$$|\Delta \beta_j| = |\beta_j - \beta_{i0}| \leq \lambda_i \quad i \in [4,6,8,10,12] \text{ and } j \in [2,\ldots,6]$$

(A$\beta_j$): represents the unknown parts including the resultants of actuator faults related to the quadrotor motions and the velocity sensor faults.

IV. SIMULATION RESULTS

In order see well the performances of the controller developed in this paper, we introduced four sensor faults \{\$f_{i1}, f_{i2}, f_{i3}, f_{i6}\} added in angular velocities and linear velocity of altitude, with 100% of these maximum values at instants 4s, 6s, 8s and 10s respectively, and four resultants of actuator faults \{\$f_{a1}, f_{a2}, f_{a3}, f_{a6}\} related to the roll, pitch, yaw and altitude motions, with 100% of these maximum values at instants 24s, 26s, 28s and 30s respectively.

The simulation results are obtained based on the real parameters [9] in Table. 1 shown below.

According to Figure. 4 there is a very good tracking of the desired velocities sensor faults and actuator faults, the actual trajectories along roll, pitch, yaw and altitude motions are converged quickly to their desired trajectories after transient deviation caused by the appearance of the corresponding velocity sensor faults. Furthermore, we can see that these actual trajectories are insensitive to actuator faults.

Figure. 3 shows at outset a very good tracking of the desired velocities, but upon the appearance of sensor faults, the measurements of angular velocities and linear velocity of altitude are deviated to their desired velocities, which gives us a wrong information of the velocities of our system.

Fig. 3 Tracking simulation results of angular and linear velocities.

Fig. 4 Tracking simulation results of all trajectories.

Fig. 5 Simulation results of all inputs control
From Figures. 5, it is clear to see that the inputs control corresponding to this control strategy are characterized at outset by a very fast switching caused by the using of the discontinuous compensation term “sign”, with transient peaks during the appearance of velocity sensor faults, but this chattering gone after the occurrence of velocity sensor fault of altitude. we can see also a variation of input control $u_2$ and a very low transient peaks in inputs control $u_2$ and $u_3$ caused by the appearance of the resultant of actuator faults related to the altitude motion.

Fig. 6 Global trajectory of quadrotor in 3D

The simulation results given by Figure. 6 show the efficiency of the control strategy developed in this paper, which clearly shows a good performances and robustness towards stability and tracking even after the occurrence of actuator and velocity sensor faults.

V. CONCLUSION AND FUTURE WORKS

In this paper, we proposed a new strategy of fault tolerant control based on backstepping approach and including the simultaneous actuator and velocity sensor faults. Firstly, we present the dynamical model of quadrotor taking into account the different physics phenomena which can influence the evolution of our system in the space, and secondly by the synthesis of stabilizing control laws in the presence of simultaneous actuator and velocity sensor faults. The simulation results shows a high efficiency of this control strategy, its main advantage is to keep the stability of quadrotor well as these performances during a malfunction of these actuators and velocity sensors. As prospects we hope to develop other fault tolerant control strategies in order to eliminate the chattering phenomenon in the inputs control, while maintaining the stability and the performances of this system with implementation them on a real system.

VI. REFERENCES


