MMSE Combining Scheme with Subblock Noise Covariance Matrix for Fractional Sampling-OFDM Receivers

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***Abstract*—A diversity scheme with fractional sampling (FS) in OFDM receivers is investigated recently. When a sharp filter is employed, the correlation of noise samples among the adjacent subcarriers increases, and the performance of this scheme is deteriorated. Therefore, low-complexity subcarrier based MMSE combining scheme with the subblock of the noise covariance matrix is proposed in this paper. Numerical results through computer simulation show that the MMSE combining scheme proposed in this paper can outperforms a conventional MRC scheme and a subcarrier based MMSE combining scheme, when the sharp filter is employed.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been used for many wireless applications such as digital terrestrial broadcasting, high-speed WLANs, or wireless Broad band communications because of its high spectral efficiency and robustness to multipath channels [1].

Many kinds of diversity schemes have been investigated for OFDM systems in order to improve the performance over multipath fading channels. One of them is a fractional sampling (FS) scheme [2].

This scheme can achieve path diversity using a single antenna with the sampling rate higher than the Nyquist rate [2]. However, FS requires excess bandwidth of the channel to achieve diversity [2]. If transmitting and receiving filters have sharp frequency response, the diversity gain through FS decreases.

A subcarrier based noise whitening and maximal ratio combining (MRC) scheme has been investigated in [2]. It is relatively low complexity and can achieve suboptimal performance. However when the sharp filter is employed, the performance of this scheme is deteriorated significantly. To achieve further diversity gain through FS, maximum likelihood (ML) detection is the optimum. Nevertheless, the ML scheme requires to calculate and minimize the Euclidean distance for the symbol on each subcarrier. This is prohibitive complexity if the number of subcarriers or the constellation size increases.

Another suboptimum combining scheme is a minimum mean squared error (MMSE) algorithm [4][5]. However, MMSE combining over the whole subcarriers also demands large computational complexity. On the other hand, the correlation of the noise samples are relatively large only among the adjacent subcarriers. Thus, a novel MMSE combining scheme that uses the subblock of the noise covariance matrix is proposed. The proposed scheme can reduce the complexity of the MMSE combining. The paper is organized as follows, In Section 2, a system model is described. The proposed MMSE combining scheme is described in Section 3. Numerical results through computer simulation are presented in Section 4. Section 5 concludes this paper.

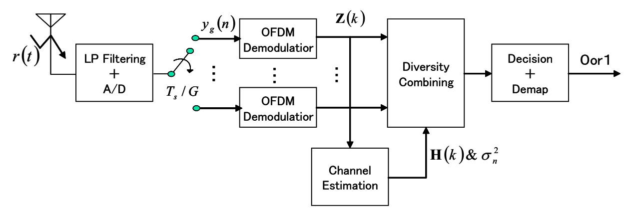

Fig. 1. FS-OFDM Receiver.

II. System Model

A. FS-OFDM System

Fig. 1 shows the block diagram of a receiver with FS. Suppose the data symbol on the kth subcarrier is $s[k](k = 0, ..., N-1)$, the OFDM symbol with the guard interval (GI) can be expressed as

$$u[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s[k] e^{j\frac{2\pi nk}{N}}, \tag{1}$$

where $n(n = 0, 1, ..., N+L-1)$ is the time index, N is the IDFT length and L is the GI length. The received signal after down conversion can be expressed as

$$y(t) = \sum_{n=0}^{P-1} u[n] h(t - nT_s) + v(t), \tag{2}$$

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where $h(t)$ is the complex gain of the composite channel and is given by $h(t) := p(t) \star c(t) \star p(-t)$, $p(t)$ is the impulse response of the baseband filter and T_s is the baud rate, $c(t)$ is the impulse response of the physical channel, and $v(t)$ is the additive Gaussian noise filtered at the receiver. In FS, $y(t)$ is sampled at the rate of T_s/G, and $y_g[n]$ is expressed by

$$y_g[n] = \sum_{l=0}^{P-1} u[l]h_g[n-l] + v_g[n], \quad g = 0, ..., G-1, \tag{3}$$

where G is fractional sampling ratio, and $y_g[n]$, $h_g[n]$, and $v_g[n]$ are polynomials of sampled $y(t)$, $h(t)$, and $v(t)$, respectively, and are expressed as

$$\begin{aligned} y_g[n] &:= y(nT_s + gT_s/G), & (4)\\ h_g[n] &:= h(nT_s + gT_s/G), & (5)\\ v_g[n] &:= v(nT_s + gT_s/G). & (6) \end{aligned}$$

After removing the GI and taking DFT, the symbol at the kth subcarrier is given by

$$\mathbf{z}[k] = \mathbf{H}[k]s[k] + \mathbf{w}[k], \tag{7}$$

where $\mathbf{z}[k] = [z_0...z_{G-1}]^T$, $\mathbf{w}[k] = [w_0...w_{G-1}]^T$, and $\mathbf{H}[k] = [H_0...H_{G-1}]^T$ are $G \times 1$ column vectors, and the gth components are given as

$$\begin{aligned} [\mathbf{z}[k]]_g &:= z_g[k] = \sum_{n=0}^{N-1} y_g[n]e^{-j\frac{2\pi kn}{N}}, & (8)\\ [\mathbf{w}[k]]_g &:= v_g[k] = \sum_{n=0}^{N-1} v_g[n]e^{-j\frac{2\pi kn}{N}}, & (9)\\ [\mathbf{H}[k]]_g &:= H_g[k] = \sum_{n=0}^{N-1} h_g[n]e^{-j\frac{2\pi kn}{N}}, & (10) \end{aligned}$$

respectively.

In FS, the noise samples are correlated over the subcarriers. The covariance matrix of the noise samples are expressed as

$$\mathbf{R}_w = E[\mathbf{w}\mathbf{w}^H], \tag{11}$$

where $\mathbf{R}_w$ is the $GN \times GN$ matrix and has N by N of $G \times G$ subblocks. Suppose that $\mathbf{R}_w[k_1, k_2]$ is the (k_1, k_2)th subblock, the (g_1, g_2) element of $\mathbf{R}_w[k_1, k_2]$ is given by

$$\begin{aligned} &E[w_{g1}[k_1]w_{g2}^*[k_2]]\\ &= \sigma_v^2 \frac{1}{N} \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} p_2((n_2 - n_1 + (g_2 - g_1)/G)T_s),\\ &\qquad\qquad \times e^{j\frac{2\pi(k_1 n_2 - k_2 n_1)}{N}}. \qquad (12) \end{aligned}$$

Where σ_v^2 means the noise variance, and $p_2(t) := \int p(t')p(t' + t)dt'$ is the deterministic correlation of $p(t)$.

III. Subcarrier based MMSE Combining

A. MMSE Combining

The coefficients of the MMSE combining, $\mathbf{W}_{mmse}$, can be designed to minimize the following criterion.

$$MSE = E||\mathbf{W}_{mmse}\mathbf{z} - \mathbf{s}||^2, \tag{13}$$

where $\mathbf{W}_{mmse}$ is the $GN \times 1$ coefficient vector, $\mathbf{z} = [\mathbf{z}^T[0]...\mathbf{z}^T[N-1]]^T$, $\mathbf{s} = [\mathbf{s}^T[0]...\mathbf{s}^T[N-1]]^T$. Then,

$$\mathbf{W}_{mmse} = \mathbf{H}^H[\mathbf{H}\mathbf{H}^H + \mathbf{R}_w]^{-1}, \tag{14}$$

where $\mathbf{H} = diag[\mathbf{H}[0], ..., \mathbf{H}[k], ..., \mathbf{H}[N-1]]$, and the estimated symbol is expressed as

$$\hat{\mathbf{s}}_{mmse} = \mathbf{W}_{mmse}\mathbf{z}. \tag{15}$$

Calculation of $\mathbf{W}_{mmse}$ in Eq. (13) requires the inverse of $GN \times GN$ matrix. This is a large amount of computation as G or N increases. In order to reduce the complexity, a subcarrier based MMSE algorithm is investigated. The combining coefficients for the k th sucarrier, $\mathbf{W}_{mmse}[k]$, is given as

$$\mathbf{W}_{mmse}[k] = \mathbf{H}^H[k][\mathbf{H}[k]\mathbf{H}^H[k] + \mathbf{R}_w[k]]^{-1}, \tag{16}$$

where $\mathbf{R}_w[k] = E[\mathbf{w}[k]\mathbf{w}[k]^H]$. The estimate of $s[k]$ is given by

$$\hat{\mathbf{s}}_{mmse}[k] = \mathbf{W}_{mmse}[k]\mathbf{z}[k]. \tag{17}$$

B. Subcarrier based MMSE Combining with Subblock Noise Covariance Matrix

Subcarrier based MMSE combining scheme in Eq. (17) does not take the amount of noise correlation over the subcarriers into account. The correlation of the noise samples among the adjacent subcarriers is relatively higher than the rests if the sharp filter is employed. A MMSE combining scheme with the subblock of the noise covariance matrix is proposed in this paper. Suppose the subblock size is $LG \times LG$ with the expression in Eqs. (16) and (17), the estimate of $s[k]$ in the proposed scheme is given by

$$\hat{\mathbf{s}}_B = \mathbf{H}_B^H \left[\mathbf{H}_B\mathbf{H}_B^H + \mathbf{R}_{wB}\right]^{-1} \mathbf{z}_B, \tag{18}$$

where,

$$\hat{\mathbf{s}}_B = \begin{bmatrix} \hat{s}[k - \frac{L-1}{2}] \\ \vdots \\ \hat{s}[k] \\ \vdots \\ \hat{s}[k + \frac{L-1}{2}] \end{bmatrix}, \mathbf{z}_B = \begin{bmatrix} \mathbf{z}[k - \frac{L-1}{2}] \\ \vdots \\ \mathbf{z}[k] \\ \vdots \\ \mathbf{z}[k + \frac{L-1}{2}] \end{bmatrix},$$

$$\mathbf{H}_B = \begin{bmatrix} \mathbf{H}[k - \frac{L-1}{2}] & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & \mathbf{H}[k] & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \mathbf{H}[k + \frac{L-1}{2}] \end{bmatrix},$$

$\mathbf{R}_{wB} =$

$$E\begin{bmatrix} \mathbf{w}[k-\frac{L-1}{2}]\mathbf{w}^H[k-\frac{L-1}{2}] & \cdots & \mathbf{w}[k+\frac{L-1}{2}]\mathbf{w}^H[k-\frac{L-1}{2}] \\ \vdots & \ddots & \vdots \\ \mathbf{w}[k-\frac{L-1}{2}]\mathbf{w}^H[k+\frac{L-1}{2}] & \cdots & \mathbf{w}[k+\frac{L-1}{2}]\mathbf{w}^H[k+\frac{L-1}{2}] \end{bmatrix}.$$

IV. Numerical Results

A. Simulation Conditions

To evaluate the performance of the proposed MMSE combining scheme with the subblock noise covariance matrix, numerical results through computer simulation are presented. The simulation conditions are presented in Table I. Relevant to the IEEE802.11a standard, the number of subcarriers is 64. 48 subcarriers are used for data transmission, and 4 subcarriers are for pilot symbols. The symbol duration is $4\mu s$, the length of the GI is $0.8\mu s$, ideal channel estimation at the receiver is assumed. The response of the pulse shaping filter, $p_2(t)$, is assumed to be a truncated sinc pulse, with the duration of $4T_s$, or $8T_s$. Fig. 2 shows the relation between the pulse duration and the frequency response of the filter. It is clear that as the pulse duration of $p_2(t)$ spreads, the excess bandwidth decreases. As the multipath channel model, 32 path Rayleigh fading model with uniform delay profile, and Indoor Residential A model are employed [6] .

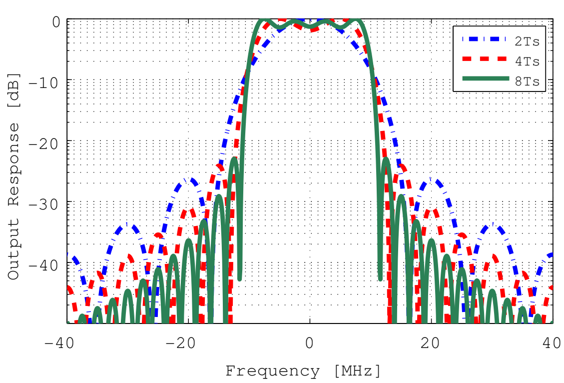

Fig. 2. Frequency response of truncated sinc pulse.

TABLE I
SIMULATION CONDITIONS

Modulation scheme	QPSK+OFDM
FFT length	64
Number of subcarrier	64
Number of data subcarrier	52
OFDM symbol length	$0.8 + 3.2[\mu s]$
Multipath channel model	32 path Rayleigh fading Indoor Residential A
Fading	Quasi-static
Channel Estimation	Ideal

B. BER Performance and E_b/N_0

Fig. 3 shows the average BER performance of the proposed scheme and the conventional scheme on Rayleigh fading channel where $p_2(t)$ is the truncated sinc pulse with the duration of $8T_s$. It is clear that the proposed scheme improves the performance by 1.5 dB at the BER of about 10^{-4}.

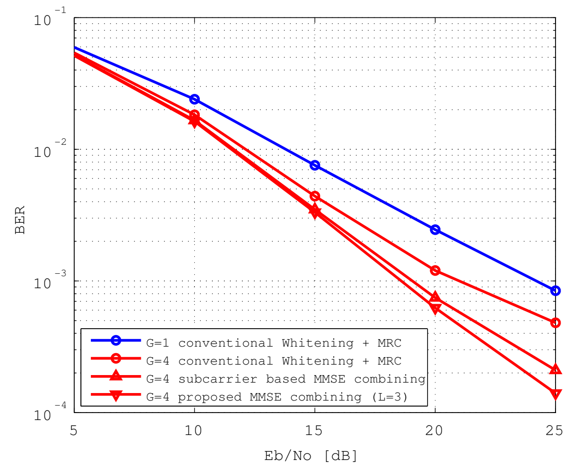

Fig. 3. Relation between BER performance and E_b/N_0 on Rayleigh fading channel ($p_2(t)$ duration is $8T_s$).

C. BER Performance and Subblock Size

Fig. 4 shows the BER performance versus the subblock size for the proposed scheme. As the subblock size increases, the improvement on the BER performance diminishes and the computational cost increases (shown in Sec. II). $L = 3$ is selected as the size of the subblock noise covariance matrix for these reasons.

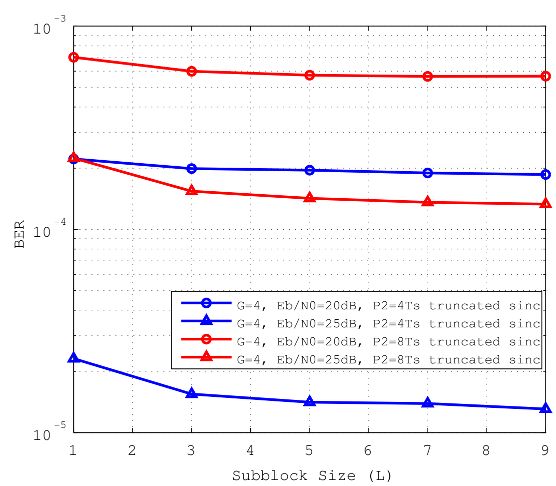

Fig. 4. Relation between the BER performance and subblock size on Rayleigh fading channel.

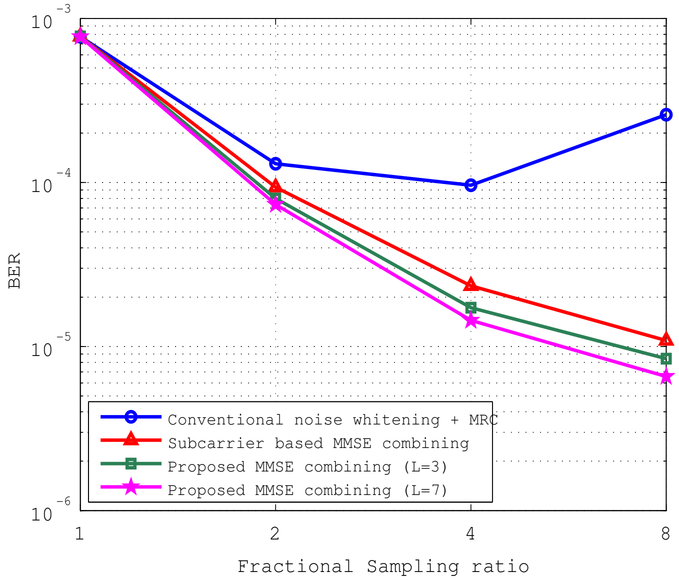

Fig. 5. BER vs FS ratio on 32path Rayleigh Fading Channel ($p_2(t)$ duration is $4T_s$, E_b/N_0=25dB).

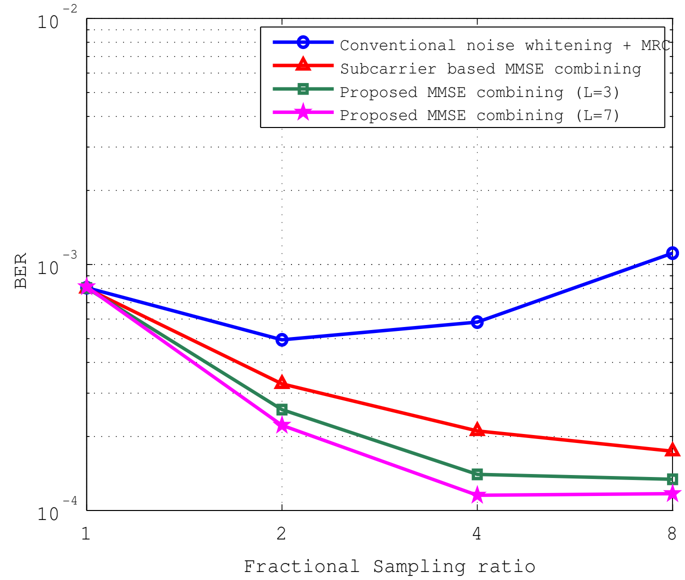

Fig. 6. BER vs FS ratio on 32path Rayleigh Fading Channel ($p_2(t)$ duration is $8T_s$, E_b/N_0=25dB).

D. Pulse Shaping Filter and BER Performance

1) 32 Path Rayleigh Fading: Figs. 5 and 6 show the average BER versus the oversampling ratio when E_b/N_0=25dB on the 32 path Rayleigh fading model. Compared to Fig. 5, the diversity gain through FS in Fig. 6 is small. This is because the diversity gain depends on the excess bandwidth of $p_2(t)$ [2]. It can be seen that the subcarrier based MMSE combining scheme and the proposed combining scheme can reduce the BER performance as compared to the conventional subcarrier based noise whitening and MRC scheme. This is because the condition numbers of the matrix $\mathbf{R}_w^{-\frac{1}{2}}$ used in the conventional subcarrier based noise whitening scheme are high at the particular subcarriers. When the sharp filter is employed, the noise correlation among the adjacent subcarriers increase and the matrix $\mathbf{R}_w^{-\frac{1}{2}}$ emphasize the noise. Especially, as the oversampling ratio increases, the proposed scheme can reduce the BER performance while the MRC scheme actually deteriorates the performance. Moreover, the proposed scheme reduces the BER by half as compared to the subcarrier based MMSE scheme when $G = 8$ and the duration of $p_2(t)$ is $4T_s$, and when $G = 4$ and the duration of $p_2(t)$ is $8T_s$.

2) Indoor Residential A: Figs. 7 and 8 show the average BER versus the oversampling ratio when E_b/N_0=25dB on the Indoor Residential A model. Diversity gain on the Indoor Residential A model is smaller as compared to the Rayleigh fading model because of its short delay spread. However, it is shown that the BER improvement by the proposed scheme is unaffected by the length of the delay spread.

E. Complexity and Subblock Size

Table. II shows the complexity of the combining schemes. For the comparison of the complexity, the computational

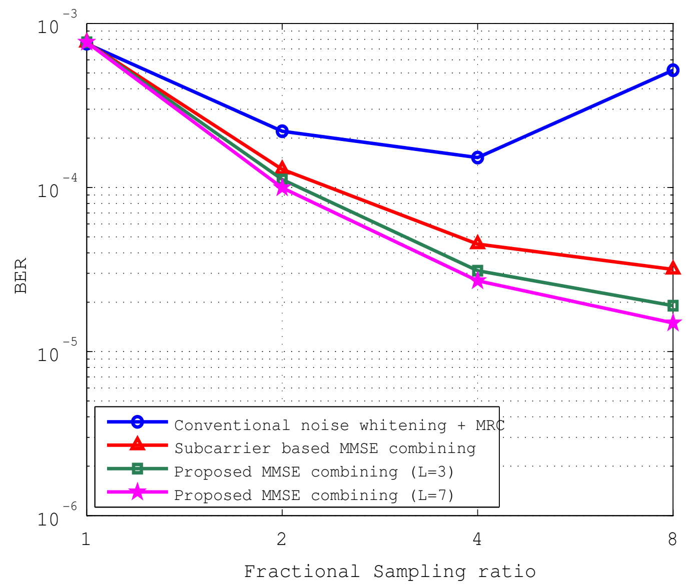

Fig. 7. BER vs FS ratio on Indoor Residential A model ($p_2(t)$ duration is $4T_s$, E_b/N_0=25dB).

complexity of the inverse matrix is compared in this paper. The calculation of inverse matrix requires the complexity which is proportional to the cube of the matrix size. The subcarrier based noise whitening and MRC scheme and the subcarrier based MMSE scheme are the least complex schemes. It is clear that the proposed MMSE combining scheme in Eq. (18) can reduce the complexity of the MMSE combining in Eq. (15). However, as L increases the complexity of the proposed scheme also drastically increases.

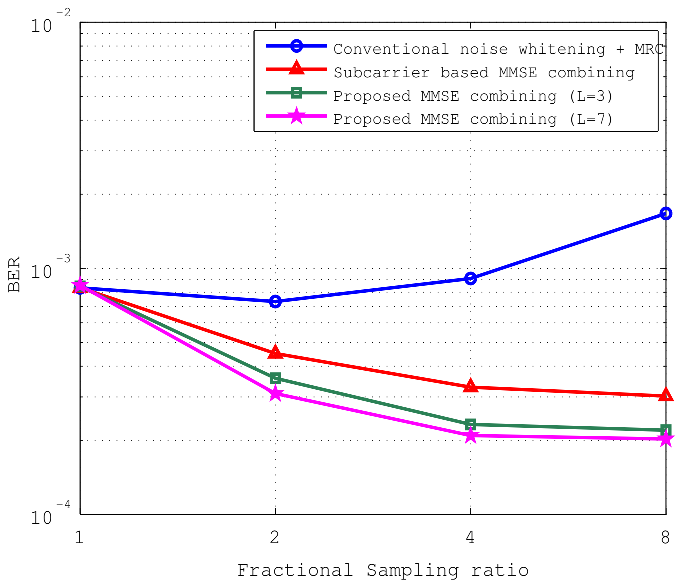

Fig. 8. BER vs FS ratio on Indoor Residential A model ($p_2(t)$ duration is $8T_s$, E_b/N_0=25dB).

TABLE II
RELATION BETWEEN COMPLEXITY AND SUBBLOCK SIZE

	Computational cost	Proportion
Subcarrier based noise whitening + MRC	$G^3 \times 52$	1
Subcarrier based MMSE combining	$G^3 \times 52$	1
Proposed MMSE combining	$(G \times L)^3 \times 52$	L^3 $27@L = 3$
MMSE combining (L=64)	$(G \times 64)^3$	5041

V. CONCLUSIONS

In this paper, the MMSE combining scheme with the subblock noise covariance matrix for FS-OFDM receivers is proposed. Conventional subcarrier based noise whitening and MRC scheme, the subcarrier based MMSE combining scheme and the proposed MMSE combining scheme are compared, and the proposed scheme has evaluated with the BER performance and the complexity. Through the computer simulation, it is shown that the proposed MMSE combining scheme can outperforms the conventional schemes, especially as the FS ratio increases or the response of the filter becomes sharp.

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