

2006-1084: USING LEVER ANALOGY DIAGRAMS IN TEACHING COMPOUND PLANETARY GEAR TRAINS

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Using Lever Analogy Diagrams in Teaching Compound Planetary Gear Trains

Abstract

The planetary gear trains are widely used in many automotive, aerospace and marine applications. The planetary gear trains are introduced to undergraduate mechanical engineering students in the course of Kinematics and Dynamics of Machines. Traditional methods of analyzing planetary gear trains, by means of torque and speed calculations, tend to be slow and cumbersome. The complexity involved has, no doubt, kept many students from becoming familiar with the capability of planetary gearing. It is our intent in this paper to describe the lever analogy method of analysis and to present a miniature 'cookbook' of levers for various planetary arrangements.

1. Introduction

The planetary gear trains, also known as epicyclic gear drives, are widely used in many automotive, aerospace and marine applications. The planetary type mechanism is the most used mechanical mechanism in conventional automatic transmissions for the majority of passenger cars. As an application in automotive automatic transmission, the number of speed ratios is determined by the kinematic structure and corresponding clutching sequence of its planetary gear train.

The planetary gear trains are introduced to undergraduate mechanical engineering students in the course of Kinematics and Dynamics of Machines. Traditional methods of analyzing planetary gear trains, by means of torque and speed calculations, tend to be slow and cumbersome. The complexity involved has, no doubt, kept many students from becoming familiar with the capability of planetary gearing. It seems appropriate in the Kinematics and Dynamics of Machines course to present a simpler method of analyzing and characterizing gear trains. The method is called the Lever Analogy Diagrams which are commonly utilized in industry¹⁻⁴.

The lever analogy diagram is very useful in analyzing gear train that has more than two connected planetary gear sets. For a single planetary gear set, it is no need to add a level of abstraction. The lever analogy is a translational-system representation of the rotating parts for the planetary gear. In the lever analogy, an entire compound planetary gear train can usually be

represented by a single vertical lever. The input, output and reaction torques are represented by horizontal forces on the lever. The lever motion, relative to the reaction point, represents rotational velocities. By using the lever, for an example, one can easily visualize the essential functions of the transmission without addressing the complexities of planetary gear kinematics.

Most of the kinematics and dynamics textbooks utilize Train Value Formulation Method or Tabular (Superposition) Method for analyzing of planetary gear trains. Using computer simulation software, such as Working Model and Visual Nastran, to improve students understanding of the mechanisms has rapidly increased in recent years^{5,6,7,8,9}. However, it takes time and effort to construct demonstration models for instructional purpose. It is our intent in this paper to describe the lever analogy method of analysis and to present a miniature ‘cookbook’ of levers for various planetary arrangements. It has been our instructional experience that the use of this tool not only makes torque and speed calculations easy, but also improves students’ ability to visualize the results and understand the effect of gear tooth ratios.

2. Modeling Procedure

The procedure of setting up a lever system analogous for planetary gear sets is: (1) replace each gear set by a vertical lever; (2) rescale, interconnect, and/or combine levers according to the gear sets’ interconnections; and (3) identify the connections to the lever(s), according to the gear sets’ connections. The lever is a basic building block of the analogy which replaces the planetary gear set. The lever proportions are determined by the numbers of teeth on the sun and ring gears. And then the next step is combining the levers and identifying the connections in the gear sets.

Figure 1 shows the free body diagrams of two meshing gears. The pitch diameter or teeth per inch along the circumference is the same for two meshing gears, as illustrated in Equation (1). The ratios of speeds and torques are shown in Equations (5) and (6).

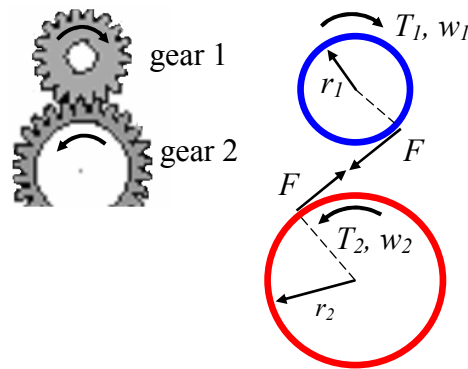


Figure 1. Free body diagram of two meshing gears

$$2\pi r_1 / N_1 = 2\pi r_2 / N_2 \quad (1)$$

$$V = w_1 * r_1 = w_2 * r_2 \quad (2)$$

$$T_1 = F * r_1 \quad (3)$$

$$T_2 = F * r_2 \quad (4)$$

$$\therefore w_1 / w_2 = r_2 / r_1 = N_2 / N_1 \quad (5)$$

$$T_1 / T_2 = r_1 / r_2 = N_1 / N_2 \quad (6)$$

where V is tangential velocity at contact point; r_1 and r_2 are radii; w_1 and w_2 are angular velocity; T_1 and T_2 are torques; N_1 and N_2 are number of teeth on gear 1 and 2 respectively.

The free body diagram of a planetary gear is shown Figure 2, where N , r , T , and w represent the teeth number, radius, torque, and angular velocity of gear respectively. The subscripts p , pc , s and r are respectively for planet, planet carrier, sun, and ring gears. The sum of torques at pinion is shown in Equation (9). Equation 10 illustrates the torque ratio between sun and ring gears.

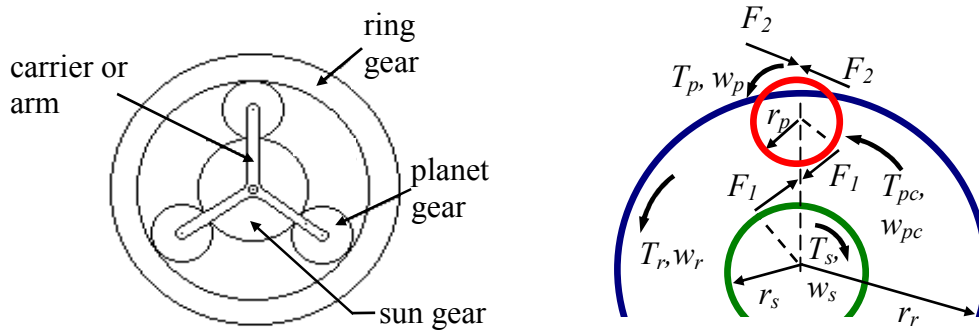


Figure 2. Free body diagram of planetary gear

$$F_1 * r_p - F_2 * r_p = 0, \quad \therefore F_1 = F_2 = F \quad (7)$$

$$T_s = F * r_s, T_r = F * r_r, \quad \therefore T_s / T_r = N_s / N_r \quad (8)$$

$$T_s + T_r - T_{pc} = 0 \quad (9)$$

Therefore,

$$T_s = T_{pc} * \frac{N_s}{N_r + N_s}, \quad T_r = T_{pc} * \frac{N_r}{N_r + N_s} \quad (10)$$

The tangential velocities at ring and planet gears are V , and

$$V = w_r * r_p, \quad \text{also } V = w_p * r_p + w_{pc} * r_r \quad (11)$$

$$\frac{2 * \pi * r_p}{N_p} = \frac{2 * \pi * r_r}{N_r} \quad (12)$$

$$w_r * N_r = w_p * N_p + w_{pc} * N_r \quad (13)$$

Using common tangential velocities at sun and planet gears gives:

$$w_s * N_s = -w_p * N_p + w_{pc} * N_s \quad (14)$$

Sum of the Equations (13) and (14), it gives:

$$w_r * N_r + w_s * N_s = w_{pc} * (N_r + N_s) \quad (15)$$

2.1 Single Planetary Gear Set

The stick diagram for a single planetary gear set is illustrated in Figure 3(a) and the lever replacement is shown in Figure 3(b). Using lever diagram, the torque equations are derived from Equations (16) to (18), and angular speeds in Equations (19) and (20). The justification for these substitutions may not be obvious, but it can show that the horizontal force and velocity relationship of the lever are identical to the torque and rotational velocity relationship of the gear set. For an example shown in Figure 4, when the carrier of a simple gear set is grounded, the ring and sun rotate in opposite directions at relatively speeds inversely proportional to their numbers of teeth; and the corresponding points on the analogous lever behave the same.

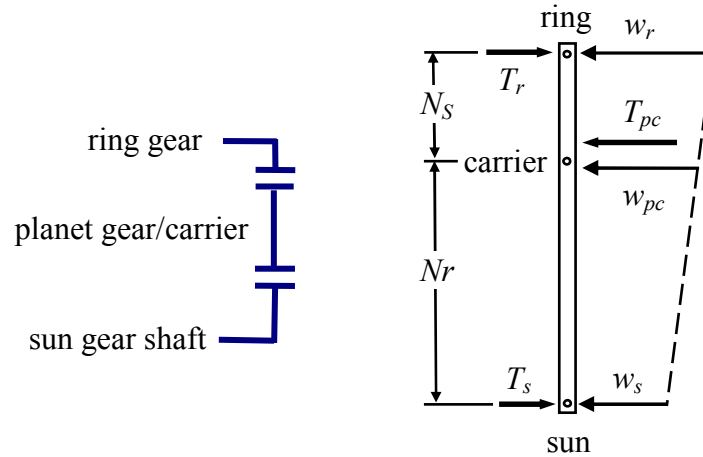


Figure 3. Lever representations of single planetary gear

$$T_r = T_{pc} * \frac{N_r}{N_r + N_s} \quad (16)$$

$$T_{pc} = T_r + T_s \quad (17)$$

$$T_s = T_{pc} * \frac{N_s}{N_r + N_s} \quad (18)$$

$$\frac{w_r - w_s}{N_s + N_r} = \frac{w_{pc} - w_s}{N_r} \quad (19)$$

$$\therefore w_{pc} = \frac{N_r}{N_s + N_r} w_r + \frac{N_s}{N_s + N_r} w_s \quad (20)$$

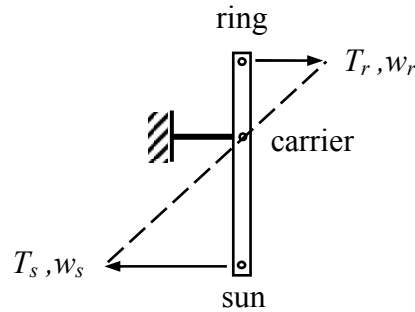
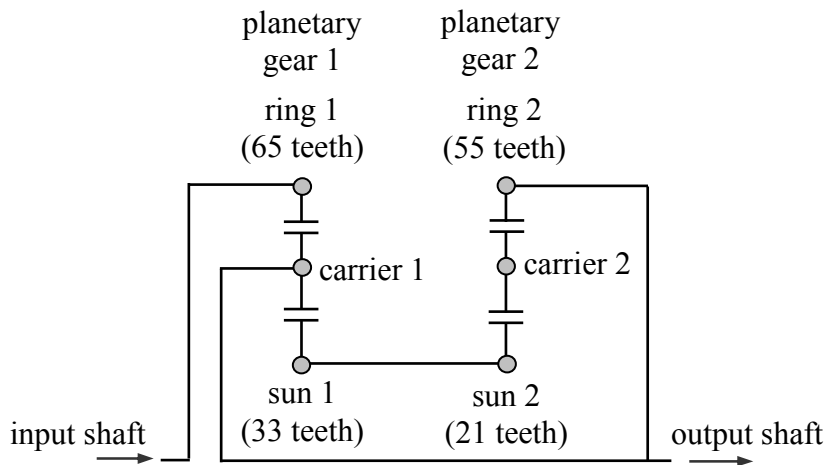


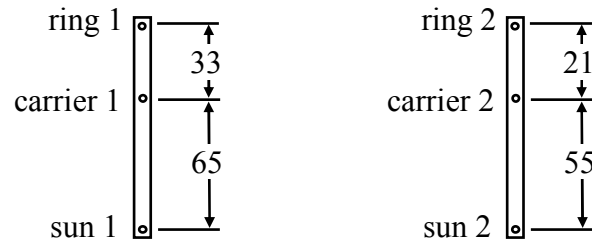
Figure 4. Example of lever representations of single planetary gear

2.2 Interconnections Between Planetary Gear Sets

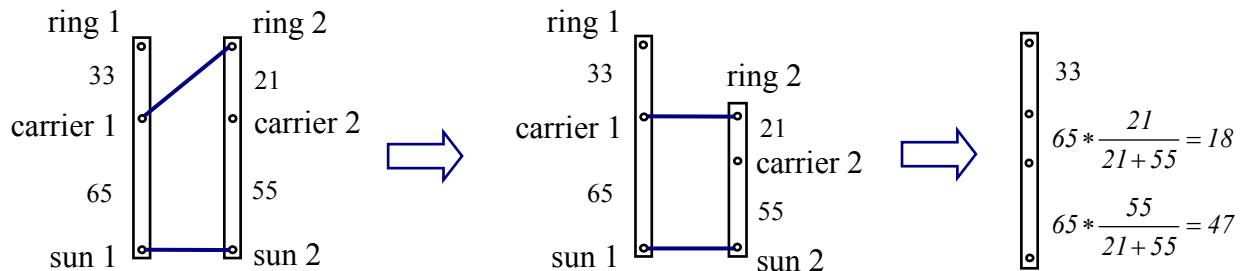
The interconnections between gear sets are replaced by horizontal links connected to the appropriate places on the levers. Whenever two gear sets have a pair of interconnections, the relative scale constants and placement of their analogous levers must be such that the interconnecting links are horizontal. Levers connected by a pair of horizontal links remains parallel, and therefore can be replaced functionally by a single lever having the same vertical dimension between points. This is illustrated using an example shown in Figure 5 along with the levers representing the two simple planetary gear sets. Let teeth numbers at planetary gear 1 are 65 and 33 teeth for ring and sun gear respectively; and at planetary gear 2 are 55 and 21 teeth for ring and sun gear respectively. Figure 5(a) is a stick diagram of two planetary gear sets, 5(b) is two levers representations, and 5(c) is the process of combining two levers.



(a) A stick diagram of two connected planetary gear sets



(b) Lever representation of each gear set



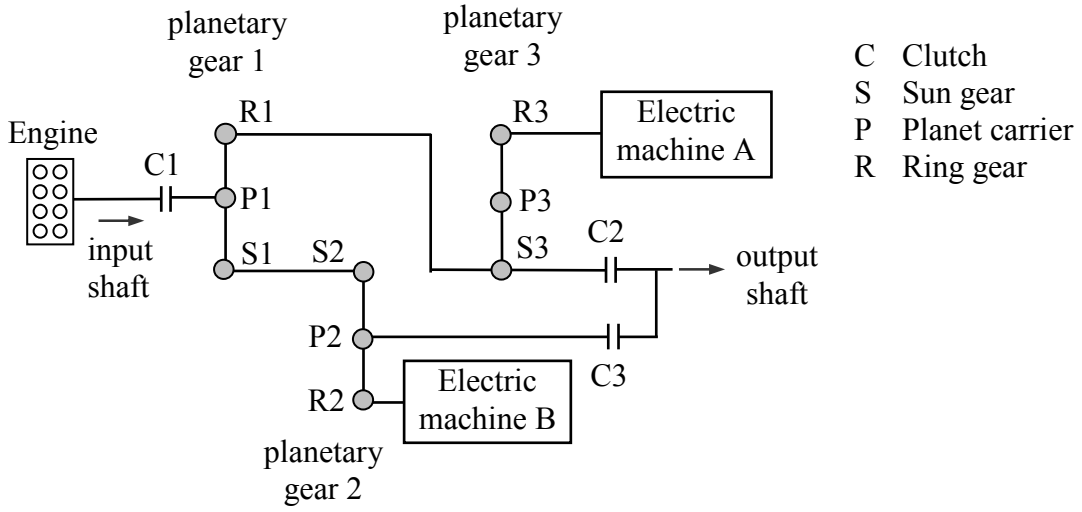
(c) Combining of two lever representations

Figure 5. Two connected gear sets and lever representations

3. Application of Lever Analogy in Hybrid Powertrain

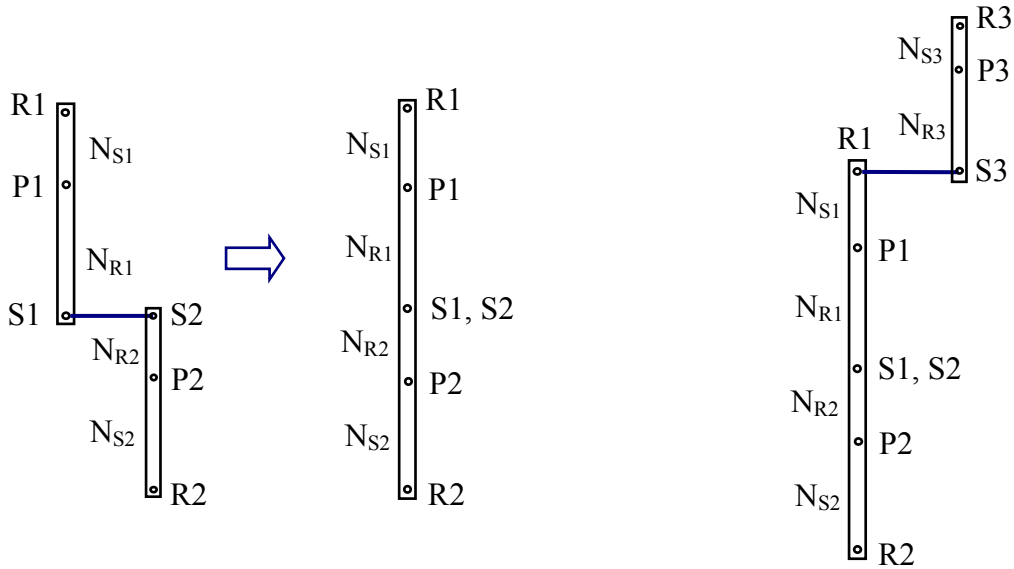
To bring the lever analogy into a more interest stage, an Electric Variable Transmission (EVT) used in hybrid powertrain is selected as an example. Figure 6 shows an EVT consisting of three planetary gear sets, three clutches, and three power sources (engine and two electric motors). The planetary gear system in the EVT plays an important role in distributing the power among the engine and two electric machines. In planetary gear set 1, the planet carrier shaft is connected to the engine output shaft through a clutch (C1). This EVT may perform in a hybrid or purely electrical manner depending on the on/off status of the C1. The ring gear shaft (in planetary gear set 1) and the sun gear shaft (in planetary gear set 3) are connected to the transmission output shaft through clutch 2 (C2). Similarly, the planet carrier shaft in planetary gear set 2 is connected to the transmission output shaft through clutch 3 (C3). The lever diagram is shown in Figure 7.

When C1 and C3 are engaged (with C2 is off), the EVT runs in the first mode. In this mode, the engine drives the output shaft through planetary gear sets 1 and 2. Electric machine B, serving as a motor, also gives additional power to the output shaft through planetary gear set 2 and C3. Simultaneously, through the ring gear in planetary gear set 3, part of the engine power drives electric machine A which serves as a generator. Unit A converts the mechanical energy into electrical energy which is stored in the battery and drives unit B. The speed diagram for the first mode is illustrated in Figure 8.



- C Clutch
- S Sun gear
- P Planet carrier
- R Ring gear

Figure 6. Schematics of two-mode EVT (from US Patent 5,558,588)¹⁰



(a) Combining planetary gears 1 and 2

(b) Combining planetary gears 1, 2 and 3

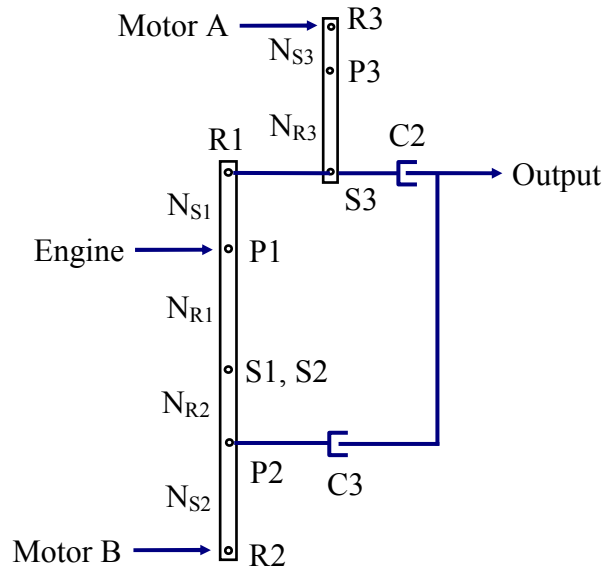


Figure 7. Lever diagram of EVT

In the second mode shown in Figure 9, the engine drives the output shaft through planetary gear sets 1 and 3. Electric machine A, serving as a motor, also gives additional power to the output shaft through planetary gear set 3 and C2. Simultaneously, through the ring gear in planetary gear set 2, part of the engine power drives electric machine B that serves as a generator, or the power of unit B will drive the output shaft as B serves as a motor.

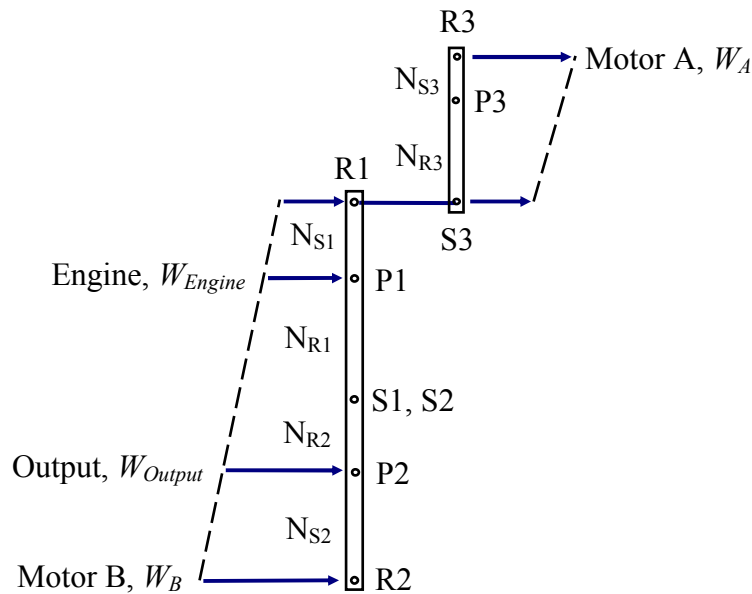


Figure 8. Speed diagram of first mode

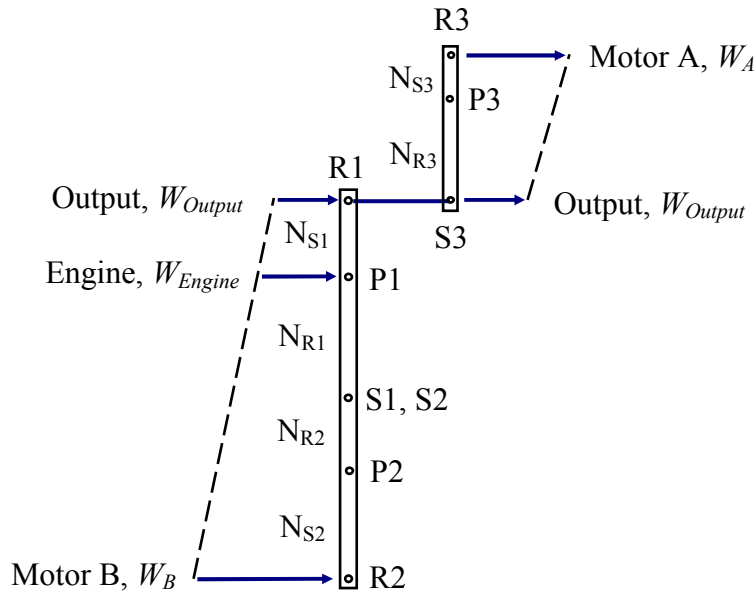


Figure 9. Speed diagram of second mode

Based on speed diagram, the output shaft speed in the first mode is derived as:

$$W_{output} = \frac{N_{R1} + N_{R2}}{N_{R1} + N_{R2} + N_{S2}} * W_B + \frac{N_{S2}}{N_{R1} + N_{R2} + N_{S2}} * W_{Engine} \quad (21)$$

The speed equations in the second mode is illustrated as:

$$W_{Engine} = \frac{N_{S1}}{N_{R1} + N_{R2} + N_{S2} + N_{S1}} * W_B + \frac{N_{R1} + N_{R2} + N_{S2}}{N_{R1} + N_{R2} + N_{S2} + N_{S1}} * W_{Output} \quad (22)$$

4. Summary

Using lever analogy diagrams, the increasing complex mechanical system, such as automotive transmission, is much easier to understand. The lever analogy allows easy analysis of mechanical transmissions of angular velocity and torque. It is our intent in this paper to describe the lever analogy method of analysis and to present a miniature ‘cookbook’ of levers for various planetary arrangements. The following steps should be followed:

- (1) Replace planetary gear sets with their equivalent levers
- (2) Rescale the levers such that their interconnections are horizontal
- (3) Combine levers if possible
- (4) Identify inputs, outputs, and reaction for each gear
- (5) Solve lever system for angular speeds and torques respectively

References

1. Sumi, Y., "A study on automatic detective system for planetary gear trains", Japan Society of Automotive Engineers (JSAE), paper number 9436620, 1994.
2. Benford, H. L. and Leising, M. B., "The lever analogy: a new tool in transmission analysis", Society of Automotive Engineers (SAE), paper number 810102, 1981.
3. Achtenova, G. and Svoboda, J., "Computer-aided calculation of planetary gear sets", SAE paper number 2003-01-0129, 2003.
4. Ross, C. S. and Route, W. D., "A method for selecting parallel-connected planetary gear train arrangements for automotive automatic transmissions", SAE paper number 911941, 1991.
5. Cleghorn, W. and Dechev, N., "Enhancements to an undergraduate mechanisms course", *Proceedings of 2003 American Society for Engineering Education (ASEE) Annual Conference*, 2003.
6. Echempati, R and Mazzei, A., "Teaching and learning experiences of an integrated mechanism and machine design course", *Proceedings of 2002 ASEE Annual Conference*, 2002.
7. Buchal, R., "Using animations and interactive simulations to learn how machines work", *Proceedings of 2002 ASEE Annual Conference*, 2002.
8. Vavrek, E., "Incorporating working model into an applied kinematics course", *Proceedings of 2002 ASEE Annual Conference*, 2002.
9. Wang, S. L., "Motion simulation of cycloidal gears, cams, and other mechanisms", *Proceedings of 2002 ASEE Annual Conference*, 2002.
10. Zhang, D., Chen, J., Hsieh, T., Rancourt J. and Schmidt, M. R., "Dynamic modelling and simulation of two-mode electric variable transmission", Proc. Instn Mech. Engrs, Part D, *Journal of Automobile Engineering*, 215(D), 1217-1223, 2001.