CHAPTER 9

A GAME THEORETICAL APPROACH FOR MODELING MERGING AND YIELDING BEHAVIOR AT FREEWAY ON-RAMP SECTION

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ABSTRACT

Traffic conflicts between merging and through vehicles are typical phenomena near freeway on-ramp sections, yet few microscopic models describing the interaction of these vehicles in the merging process have been proposed. In this paper, vehicle interactions during merging process are modeled under an enhanced game-theoretic framework. Freeway on-coming through vehicle and on-ramp merging vehicle are considered as competing players that seek to maximize their respective rewards during the merging process. As the freeway vehicle aims to maintain their initial car-following state and minimize speed variations, the on-ramp merging vehicle strives to join mainline traffic in the minimal time possible subject to safety constraints. Considering non-cooperative nature of the game, drivers at the merging section would eventually adopt strategies that form Nash equilibrium. To assess the model parameters, we propose a bi-level estimation methodology with the upper level as a least square problem and the lower level a linear complementarity problem searching for the equilibria. Applicability of the proposed model is examined and validated using trajectory data collected from field. Testing results indicate that this framework can effectively capture vehicle interactions at freeway merging sections while achieving a relatively high accuracy of predicting vehicles’ actions.

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INTRODUCTION

Traffic conflicts between merging and through vehicles are typical phenomena near freeway on-ramp sections. Such conflicts often slow down the freeway vehicles and trigger shockwaves that propagate and dissipate over time and space, or result in localized congestion that could evolve into long-lasting bottlenecks throughout entire peak periods. A clear understanding of the merging process renders indispensable technical foundations for designing freeway on-ramps and developing sophisticated traffic management strategies such as ramp control. In previous studies, freeway merging process was often described in a “one-way” fashion, focusing on the influence of through traffic on merging vehicles in terms of the latter’s gap acceptance behavior (Hidas, 2002; Kita, 1993; Kosonen, 1999; Owen and Zhang, 1998; Yang and Koutsopoulos, 1996). However, in real life it is frequently observed that a through vehicle performs courtesy yield or accelerates when seeing a merging vehicle on the ramp, while an on-ramp vehicle accelerates or decelerates looking for appropriate gaps in response to the movements of through traffic. Clearly, there exists mutual influence between the merging and through vehicles, i.e., they are not independent of but affecting each other’s decision in an “interactive” manner. Such pattern has been noted as typical in freeway merging area and considered a dominant factor affecting traffic characteristics near on-ramp sections (Kita et al., 1999).

A few research efforts have been devoted to model freeway merging process taking into account vehicular interactions. Troutbeck (1999) analyzed give-way behavior of mainline vehicles before approaching on-ramps. Rysgaard and Nielsen (1998) performed a study of motorway merging-giveaway behavior in Europe. In addition to these studies, freeway merging process with vehicle interactions has also been studied from macroscopic perspective using aggregate variables (Cassidy et al., 1990; Vermijs, 1991). Notably, Kita et al. (1999, 2002) were one of the first to model vehicles merging interactions as a “game”, where each involving vehicle determines its final action by considering each other’s alternatives. Specifically, Kita’s model considers collision risk as an incentive factor to build players’ payoff functions. That is, in order to minimize collision risks, freeway through vehicle can opt to give way to ramp merging vehicle, while ramp merging vehicle will choose to merge to freeway mainline or stay on the ramp merging section. However, in Kita’s model vehicle speeds are assumed constant during the merging process, which is not true in reality. In addition, as pointed out by Troutbeck (1999), freeway vehicle usually perform giveaway behavior even before the appearance of merging vehicle, so the alleged interaction may not exist.

In this paper, merging and yielding behavior at freeway on-ramp sections are modeled under an improved game-theoretic framework. Vehicle speeds are no longer assumed constant as in Kita’s study, while minimum safety gaps are explicitly considered in players’ payoff functions. Comparing to previous modeling efforts, more realistic behavioral rules are proposed in this study to describe typical behavior at merging sections. To be sure, it is assumed that during the merging process freeway on-coming vehicles would try to maintain...
their initial car-following state and minimize speed variations, while on-ramp merging vehicles would strive to join mainline traffic in the minimum time possible subject to safety constraints. These behavioral rules are incorporated into respective payoff functions of conflicting vehicles, and the resultant pair of their actions is formulated as an equilibrium solution. Finally, an estimation methodology based on bi-level programming technique is proposed for assessing model parameters. Applicability of the proposed model is examined and validated using trajectory data collected from field. Testing results indicate that this framework can effectively capture vehicle interactions at freeway merging sections while achieving a relatively high accuracy of predicting vehicles’ actions.

GAME THEORETICAL MERGING MODEL

Game Definition

Consider a typical merging situation shown in Figure 1, where vehicles involved include a merging vehicle, a lag vehicle (i.e., oncoming through vehicle) and possibly a lead vehicle. As illustrated in this figure, merging vehicle is that vehicle in acceleration lane trying to join the freeway; lag vehicle is the vehicle in the target lane just behind the merging vehicle, and lead vehicle is the one immediately in front of the lag vehicle in the target lane. The process is modeled as an independent game by taking merging and lag vehicles as players. This means, immediately upon seeing each other, both the merging vehicle and lag vehicle have to decide a set of moves to maximize their respective rewards in the game. The decisions are based on their instantaneous states including speed and acceleration rates as well as their predictions on the interaction situation. The action strategies for each player (vehicle) are assumed as follows:

1. The merging vehicle can select either to merge into the mainline traffic immediately or to wait until the next available gap;
2. The lag vehicle’s options are whether to keep its current car-following state or decelerate to yield in order to facilitate a smooth merge.

It is important to clarify that albeit the lead vehicle is not directly considered as a player, its influence is implicitly accounted for by incorporating it into the payoff functions of the lag vehicle.
For the game defined above, equilibrium is achieved when no player can unilaterally increase his expected payoff by changing his probability of selecting a particular strategy. This essentially gives Nash equilibrium:

\[
E_1(p^*, q^*) \geq E_1(p, q^*) \\
E_2(p^*, q^*) \geq E_2(p^*, q)
\]

where \(E_1\) and \(E_2\) is the expected payoff at equilibrium, and \(p^*\) and \(q^*\) represent the equilibrium strategy sets for merging and lag vehicles respectively. In case of multiple equilibriums, a superior solution is considered to be the one that gives the highest payoffs for both players.

**Payoff Functions Formulation**

In the literature, payoff functions are often formulated assuming that minimizing collision risks is the behavioral goal for each individual player (Kita 1993; Kita and Fukuyama 1999). This assumption may result in trivial equilibrium solutions as collision risks are affecting both players thus each player would have similar magnitude of effects on game equilibrium (Kita 1993). In this study, it is assumed that freeway through vehicle’s objective is to minimize speed variations, i.e., try to employ a lowest possible acceleration rate during the merging process. By contrast, merging vehicle’s objective is to minimize the time spent in acceleration lane subject to safety constraints. These rules are mapped to mathematical functions describing each player’s payoffs. Prior to detailing these functions, it should be noted that lag vehicle’s payoffs are in the unit of acceleration (\(ft/s^2\)), and merging vehicle’s, in the unit of time (second). Also the typical merging scenario is assumed as follows: (1) prior to approaching the merging section, the lag and lead vehicles are interacting with each other as in a normal car-following situation, and (2) lag and merging vehicle immediately constructs their respective payoff matrix once the merging vehicle appears on the acceleration lane and the distance between lag and merging vehicle is less than 200 feet. Vehicle beyond this distance are assumed out of the interaction range (Toledo, 2003). The time at which they construct payoff matrix and make decisions will be referred to as decision time henceforth.

**Payoffs for the lag vehicle**

At the decision time, lag vehicle needs to decide whether to perform a courtesy yield, or maintain its current car following state as dictated by its lead vehicle. First, consider the case if the lag vehicle chooses to maintain its current car-following state with instantaneous acceleration rate \(a_i\). If its opponent on the acceleration lane chooses to wait for the next available gap, then the lag vehicle can indeed maintain its current state as it desires. In this case, the lag vehicle’s payoff is \(a_i\), i.e., it can keep the acceleration rate dictated by the lead
vehicle. Note $a_i$ is directly observable at the decision time. However, if the merging vehicle decides to merge right away, the lag vehicle may have to apply an unexpected braking to avoid potential collision in response to the sudden cutting-in of the merging vehicle. This sharp and unfavorable deceleration rate is not directly observable at the decision time thus has to be projected based on the instantaneous states of both lag and merging vehicles. The projected time is the time at which the lag vehicle anticipates the merging vehicle enters the freeway. The initial states at the decision time are denoted by the following:

- $v_m$: Instantaneous speed of the merging vehicle at decision time;
- $v_i$: Instantaneous speed of the lag vehicle at decision time;
- $a_m$: Instantaneous acceleration of the merging vehicle at decision time;
- $a_i$: Instantaneous acceleration of the lag vehicle at decision time;
- $RD$: Remaining distance on the acceleration lane for the merging vehicle at decision time;
- $X$: Initial gap distance between lag and merging vehicles at decision time.

The projected states, from the lag vehicle’s perspective are as follows:

- $v'_m$: Instantaneous speed of the merging vehicle at projected time;
- $v'_i$: Instantaneous speed of the lag vehicle at projected time;
- $t'_m$: The time duration that the lag vehicle anticipates the merging vehicle would need to complete the remaining distance ($RD$) on the acceleration lane;
- $X'$: Gap distance between lag and merging vehicle when the latter joins the freeway.

Given the initial states at the decision time, projected states can be computed as:

\begin{align}
    v'_m &= \sqrt{(v_m)^2 + 2a_mRD} \\
    t'_m &= \frac{v'_m - v_m}{a_m} \\
    v'_i &= v_i + a_i t'_m \\
    X' &= X - \frac{(v'_i)^2 - (v_i)^2}{2a_i} + RD
\end{align}

With these projected state variables, the lag vehicle is able to estimate the braking rate needed to avoid a potential collision when the merging vehicle suddenly cuts in. Using $X'$ to approximate the braking distance the payoff of the lag vehicle $a_s$ can be estimated as:

\begin{align}
    a_s &= \begin{cases} 
        \beta_1 \frac{2(X' - v'_it_h)}{t_h^2} + \beta_2, & \text{if } X' > 0 \\
        a_i, & \text{if } X' \leq 0
    \end{cases}
\end{align}

where $t_h$ is the braking time anticipated by the lag vehicle; $\beta_1$ and $\beta_2$ are free coefficients to be estimated from data. $X' \leq 0$ indicates the lag vehicle should have surpassed the merging vehicle when the latter joins the freeway. In this case, there is no need for the lag vehicle to brake and it just keeps its initial car-following state $a_i$. Also it should be stressed that $a_i$ is essentially a quantity assumed to be “perceived” as necessary by the lag vehicle at the
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decision time; it reflects the lag vehicle’s prediction about possible interactions if lag vehicle maintains its car-following state while merging vehicle selects to merge anyway. Moreover, even though various approaches are available for obtaining \( a_y \) analytically, the relationship assumed in Equation (6) has been found to yield best accuracy with \( X’, v’, t_b \), which are directly obtainable from the collected trajectory data.

The other option for the lag vehicle is to conduct an early courtesy yield, giving a clear sign of invitation for the on-ramp vehicle to merge. This yielding action produces a gentle decrease in speed by applying a comfortable deceleration rate, therefore the payoff \( a_y \) is determined using the following equation:

\[
a_y = \beta_3 \max \left[ \frac{v_m - v_i}{t_m - 1.0}, -10 \right] \text{ ft/s}^2
\]  

(7)

where \( \beta_3 \) is a parameter to be calibrated from observation data, and 1.0 is the assumed safety time margin. Equation (7) gives a braking rate that ensures the lag vehicle to achieve relatively low speed some time before the merging vehicle joins the freeway. If the merging vehicle’s speed \( v_m \) is lower than the lag vehicle’s speed \( v_i \), the lag vehicle will brake at a rate bounded by \(-10 \text{ ft/sec}^2\), which is the limit of comfortable deceleration rate suggested in Traffic Engineering Handbook. Note when the lag vehicle takes yielding action, its payoff is regardless of merging vehicle’s action.

The payoff matrix of the lag vehicle is summarized in Table 1.

<table>
<thead>
<tr>
<th>Players</th>
<th>Lag Vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merging Vehicle</td>
<td>Actions</td>
</tr>
<tr>
<td>Merge</td>
<td>( a_y )</td>
</tr>
<tr>
<td>Wait</td>
<td>( a_y )</td>
</tr>
</tbody>
</table>

Table 1. Payoff matrix of freeway lag vehicle

Payoffs for the merging vehicle

The merging vehicle driver creates its payoff matrix as soon as he/she enters the acceleration ramp and recognizes freeway conditions. The payoff functions proposed here are the times required to join freeway traffic. These times are calculated based on initial conditions of both vehicles as well as anticipated actions to be carried by the freeway vehicle. As being pointed out earlier, these times are associated with the acceleration/deceleration rates the merging vehicle anticipates the lag vehicle to adopt, therefore reflecting strong interactions between both decision makers.
First consider the situation where the merging vehicle decides to merge instead of waiting for the next available gap. If the freeway lag vehicle selects to yield, then the merging vehicle can smoothly join the freeway traffic using a comfortable acceleration rate \( a_{\text{comfort}} \) as there would be no conflict at the merging point. On the other hand, if the lag vehicle chooses not to yield, the merging vehicle would need to adopt a more aggressive acceleration rate \( a_{\text{max}} \) in order to arrive at the merging point earlier than the lag vehicle to avoid potential collision risks. The specific payoffs are expressed as follows:

\[
\begin{align*}
  t_{m-y} &= \beta_4 - \frac{v_m^2}{a_{\text{comfort}}} + \frac{2a_{\text{comfort}} RD}{a_{\text{comfort}}} + \beta_5 \\
  t_{m-ny} &= \beta_6 - \frac{v_m^2}{a_{\text{max}}} + \frac{2a_{\text{max}} RD}{a_{\text{max}}} + \beta_7
\end{align*}
\]

where

- \( t_{m-y} \): The payoff that the merging vehicle needs to join freeway traffic if lag vehicle selects to yield;
- \( t_{m-ny} \): The payoff that the merging vehicle needs to join freeway traffic if lag vehicle selects not to yield;
- \( \beta_4, \beta_5, \beta_6, \beta_7 \): Free coefficients to be calibrated from observation data;
- \( v_m \): Merging vehicle’s initial speed when entering the acceleration lane;
- \( RD \): Remaining distance on the acceleration lane;
- \( a_{\text{comfort}} \): Comfortable acceleration rate merging vehicle adopts if lag vehicle selects to yield;
- \( a_{\text{max}} \): Maximum acceleration rate merging vehicle adopts if lag vehicle selects not to yield;

Alternatively, merging vehicle can also select to wait for the next available gap rather than competing with the freeway lag vehicle. In this case, if the lag vehicle still performs a courtesy yield, signaling a clear invitation for the merging vehicle to take the move first, the latter doesn’t really have to wait till next available gap, rather, it will wait for a while till recognizing the yielding gesture, then accelerate and merge immediately with a comfortable acceleration rate \( a_{\text{comfort}} \). However, if the lag vehicle selects not to yield but keeps its initial acceleration dictated by the car-following situation, the merging vehicle needs to wait till the lag vehicle overpasses it, and takes the next immediate gap with a more aggressive acceleration rate \( a_{\text{max}} \). The payoffs for this situation are expressed as:

\[
\begin{align*}
  t_{w-y} &= \beta_8 t_0 + \beta_9 - \frac{v_m^2}{a_{\text{comfort}}} + \frac{2a_{\text{comfort}} (RD - v_m t_0)}{a_{\text{comfort}}} + \beta_{10} \\
  t_{w-ny} &= \beta_{11} t_0 + \beta_{12} - \frac{v_m^2}{a_{\text{max}}} + \frac{2a_{\text{max}} (RD - v_m t_0)}{a_{\text{max}}} + \beta_{13}
\end{align*}
\]
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\[ t'_0 = \frac{(v_m - v_f) + \sqrt{(v_m - v_f)^2 + 2a_lX}}{a_f} + 1.0 \]  

(12)

where

- \( t_{w-y} \): The payoff that the merging vehicle spends on acceleration lane before joining freeway traffic if the lag vehicle selects to yield;
- \( t_{w-ny} \): The payoff that the merging vehicle spends on acceleration lane before joining freeway traffic if the lag vehicle selects not to yield;
- \( \beta_1, \beta_2, \beta_3, \beta_4 \): Free coefficients to be calibrated from observation data;
- \( v_m \): Merging vehicle’s initial speed when entering the acceleration lane;
- \( t_0 \): Waiting time merging vehicle has to wait before recognizing lag vehicle’s yielding gesture;
- \( t'_0 \): Waiting time merging vehicle has to wait till lag vehicle overpasses it; is safety margin of time headway;
- \( X \): Initial lag distance;
- \( a_{i_1} \): Lag vehicle’s initial acceleration rate;
- \( RD \): Remaining distance on the acceleration lane for the merging vehicle;
- \( a_{comfort} \): Comfortable acceleration rate merging vehicle adopts if lag vehicle selects to yield;
- \( a_{max} \): Maximum acceleration rate merging vehicle adopts if lag vehicle selects not to yield;

The payoff matrix of the merging vehicle is summarized in Table 2.

**Table 2. Payoff matrix of on-ramp merging vehicle**

<table>
<thead>
<tr>
<th>Players</th>
<th>Lag Vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actions</td>
</tr>
<tr>
<td>Merging Vehicle</td>
<td>Merge</td>
</tr>
<tr>
<td></td>
<td>Wait</td>
</tr>
</tbody>
</table>

Summarizing from the above, a payoff bi-matrix can be constructed in Table 3:

**Table 3. Merging-yielding game in normal form**

<table>
<thead>
<tr>
<th>Players</th>
<th>Lag Vehicle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actions</td>
</tr>
<tr>
<td>Merging vehicle</td>
<td>Probability</td>
</tr>
<tr>
<td></td>
<td>Merge</td>
</tr>
<tr>
<td></td>
<td>Wait</td>
</tr>
</tbody>
</table>
Parameter Estimation

Parameters estimation of the proposed model is achieved by solving a bi-level programming problem. The upper level is a non-linear programming problem minimizing system total deviation from actual observed actions:

$$\min_{(p,q)} \sum_{i=1}^{n} [(Q_i - \hat{Q}_i(p,q))^2 + (P_i - \hat{P}_i(p,q))^2]$$

where $i$ is the index of observations, $Q_i$ is the observed choice of through vehicle (1 yield, 0 otherwise), $P_i$ represents the observed choice of merging vehicle (1 merge, 0 otherwise), while $\hat{Q}_i$ is the model predicted choice of through vehicle (1 yield, 0 otherwise), $\hat{P}_i$ is the model predicted choice of merging vehicle (1 merge, 0 otherwise). Both $\hat{Q}_i$ and $\hat{P}_i$ are functions of yielding and merging probabilities $p$ and $q$, which are the optimizers for the upper level programming problem. The optimal value of $p$ and $q$ should minimize the square difference between observed choices and model predicted choices.

The lower level program seeks solution for Nash equilibrium. The bi-matrix game may have several equilibrium solutions in pure strategies, as well in mixed strategies. The non-uniqueness of Nash equilibrium of bi-matrix games is a serious theoretical and practical problem. For our modeling purpose, any local solution should suffice. Since the lower level problem is actually a two-player game, Stengel (1999) has showed that there exists an equivalent linear complementarity formulation as follows:

$$0 \leq (e - M \cdot S) \perp S \geq 0$$

Here $S = [S_1, S_2] \in R^4$ is an auxiliary variable with $S_1, S_2 \in R^2$. Also, $e$ is a vector of all 1’s with a proper dimension, and $M = \begin{bmatrix} 0 & A \\ T^T & 0 \end{bmatrix}$ with $A$ and $T$ being the freeway vehicle and merging vehicle payoff matrix respectively.

Therefore, given the payoff matrices $A$ and $T$, $S$ can be obtained by solving (14). The probability for choosing each strategy can then be computed as:

$$\begin{align*}
P & = \frac{S_2}{e^T S_2} \\
Q & = \frac{S_1}{e^T S_1}
\end{align*}$$

Provided equations (14) and (15), the bi-level program (13) can be formulated as a mathematical program with complementarity constraints (MPCC). MPCC has been recently extensively studied, and Ferris et al. (2002) implemented a solver of nonlinear program with equilibrium constraints (NLPEC) as sub-system of GAMS (General Algebraic Modeling System). NLPEC can automatically convert MPCC into an equivalent single level NLP using a number of reformulation techniques. We adopt NLPEC in this study for the parameter estimation. Figure 2 illustrates the schematic workflow for this bi-level programming process.
It should be noted that the parameter estimation method above is consistent with the method of "probability of equilibrium selection" originally developed by Kita and Fukuyama (2002), as there may exist multiple equilibria. Our method can jointly estimate the payoff and the probability of equilibrium selection so that multiple equilibria can be accommodated. Table 3 lists how the probabilities of different equilibrium strategies are defined and Equation (13) demonstrates the objective function of the estimation model that incorporates these probabilities. The method does not require any selection criteria form their resultant actions (i.e., no need for identifying the correspondence between realized equilibrium and the values of the explanatory variables).

Figure 2. Schematic workflow for bi-level programming
Model Validation

Model validation is the process of quantifying predicting capability of the calibrated model using validation data set. The following metrics are employed in model validation:

Root Mean Square Error (RMSE)

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - y_i)^2}
\]  

(16)

Where  
- \(x_i\) is the model predicted value indexed by \(i\);
- \(y_i\) is the actual observation indexed by \(i\);
- \(n\) is the number of total observations.

Correlation Coefficient

\[
R = \frac{1}{n-1} \sum_{i=1}^{n} \frac{(x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \sigma_y}
\]  

(17)

where  
- \(\bar{x}\) is the mean of model predicted values;
- \(\bar{y}\) is the mean of the actual observed values;
- \(\sigma_x\) is the standard deviation of model predicted values;
- \(\sigma_y\) is the standard deviation of the actual observed values.

Mean Absolute Error (MAE)

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - x_i|
\]  

(18)

CASE STUDY

The field observation data were used to calibrate and validate the proposed model. These data were obtained from Freeway Data Collection for Studying Vehicle Interactions (DCSVI) project conducted by FHWA in 1983. The data collection site was an on-ramp section of I-405 at Roscoe Boulevard, Van Nuys, California. This site includes a 4-lane freeway section that is 1728 feet in length with a metered entrance ramp. The length of the ramp acceleration lane after meter signal light is about 400 feet. A full-frame 33 mm motion picture camera was mounted on a fixed-wing, short-take-off-and-landing aircraft. The site was then filmed at one frame per second with the aircraft flying clockwise at altitudes ranging between 2,500 and 4,500 feet. Individual vehicle trajectories were then extracted from the film at 1-second resolution. The extracted data set contains 200,000 data records (vehicle-seconds), including detailed information about vehicle speed, acceleration, front and lag distances and other variables. Figure 3 demonstrates the geometry of the test site.
The extracted data set was carefully examined to identify merging cases consistent with the assumptions prescribed earlier. With each identified case, trajectories of involved vehicles were traced back from their actual choices (i.e., yield/not yield, wait/merge) while examining their respective speed and acceleration profiles. In the end this screening process identified a total of 86 merging cases that meet model assumptions. For each merging case, the lead, lag, and merging vehicle’s trajectories are meticulously investigated and analyzed. Figure 4 illustrates an example of speed profiles for merging and lag vehicles in four typical merging scenarios. In this figure, the time when the merging vehicle joins freeway mainline is marked as “merge point” by the vertical line, and the first point in merging vehicle’s speed profile represents the decision point for all the players. This way, merging situation can be effectively deduced and reconstructed using speed profiles between the merge and decision point. For instance, in the scenario where lag vehicle choose to yield and merging vehicle chooses to wait (see Figure 4(d)), it can be clearly seen from the figure that lag vehicle slowly decelerates signalling his yielding intention while the merging vehicle has a relatively long accelerating time frame suggesting the latter selects not to merge immediately but rather to wait. Likewise, when the lag vehicle chooses not to yield and merging vehicle chooses to merge immediately (see Figure 4(b)), corresponding speed profiles depict a steep increase for merging vehicle, while a drop in lag vehicle’s speed can be seen right before the merge point. This indicates that a sudden cutting in of the merging vehicle causes the lag vehicle to decelerate unexpectedly.
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From the total 86 cases, the extracted trajectories from 63 cases were used in calibrating model parameters via bi-level programming, while the rest of 23 cases were used for validation. Table 4 summarizes the calibrated parameters and Table 5 summarizes the validation results.

**Table 4. Calibrated Model Parameters**

<table>
<thead>
<tr>
<th>Freeway Lag Vehicle</th>
<th>$\beta_1$</th>
<th>1.0658166</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>-0.01398846</td>
</tr>
<tr>
<td></td>
<td>$\beta_3$</td>
<td>6.06633624</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>On-Ramp Merging Vehicle</th>
<th>$\beta_4$</th>
<th>0.00432315</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_5$</td>
<td>1.10E+02</td>
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<td></td>
<td>$\beta_6$</td>
<td>-0.00589191</td>
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<td></td>
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<td></td>
<td>$\beta_{10}$</td>
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<td>$\beta_{11}$</td>
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<td></td>
<td>$\beta_{12}$</td>
<td>-0.00717059</td>
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<tr>
<td></td>
<td>$\beta_{13}$</td>
<td>2.06E+03</td>
</tr>
</tbody>
</table>

**Table 5. Validation results**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Validation Cases</td>
<td>23</td>
</tr>
<tr>
<td>No. of Pure Strategy Equilibrium</td>
<td>12</td>
</tr>
<tr>
<td>No. of Mixed Strategy Equilibrium</td>
<td>11</td>
</tr>
<tr>
<td>Mean Average Error</td>
<td>0.087</td>
</tr>
<tr>
<td>Root Mean Square Error</td>
<td>0.280</td>
</tr>
<tr>
<td>Correlation Coefficient</td>
<td>0.915</td>
</tr>
</tbody>
</table>
As shown in Table 5, a total of 23 merging cases were used in model validation. About one half of these cases resulted in mixed strategy equilibrium while the other half pure strategy equilibrium. The Mean Average Error is 0.087, which essentially equals to the false alarm rate, in other words, the model successfully predicted vehicles actions with 91.3% of all the cases. The Root Mean Square Error of model predictions is 0.289 while Correlation Coefficient between real choices and model predicted choices is 0.915. This indicates that the proposed model has a good capability to replicate and predicate vehicle actions at merging sections.

**CONCLUDING REMARKS**

This paper describes a game-theoretical framework that can model driver’s behavior during the complex merging maneuver. In the game, freeway on-coming vehicles aims at maintaining their initial car-following state and minimize speed variations, while on-ramp merging vehicles strive to join mainline traffic in the minimum time possible subject to safety constraints. These behavioral rules are incorporated into respective payoff functions of conflicting vehicles, and the selected actions become the outcome of a game with each player trying to maximize his own rewards. An estimation methodology based on bi-level programming technique is proposed for assessing model parameters. Vehicle trajectory data from the field is used for the estimation and validation of the proposed model.

This study is an attempt to better understand merging behavior from game perspective. The proposed model can be implemented in simulation package to improve current modeling technique. Currently only two players, each with two alternatives are considered in the game, yet the framework could be further expanded “horizontally” to include more players, or “vertically” to consider multiple choices and sequential moves for each player. Albeit a game with more than 4 players may involve tremendous amount of computation thus practically not feasible, however a 3-player game with multiple options for each player merits further exploration. For example, the lag vehicle can have more options including accelerating to surpass the merging vehicle or conducting a lane change to avoid potential conflicts. This should also include collecting new high-resolution trajectory data to aid detailed analysis and model calibration/validation. Such work is the subject of a research project currently underway by the authors.

**REFERENCE**


Owen, L. E., and Zhang, Y. "A multi-regime microscopic traffic simulation approach." the 5th International Conference on Applications of Advanced Technologies in Transportation Engineering, Newport Beach, California, 199-206.


