Subsurface polarimetric migration imaging for full polarimetric ground-penetrating radar

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SUMMARY
Polarization is a property of electromagnetic wave that generally refers to the locus of the electric field vector, which can be used to characterize surface properties by polarimetric radar. However, its use has been less common in the ground-penetrating radar (GPR) community. Full polarimetric GPR data include scattering matrices, by which the polarization properties can be extracted, at each survey point. Different components of the measured scattering matrix are sensitive to different types of subsurface objects, which offers a potential improvement in the detection ability of GPR. This paper develops a polarimetric migration imaging method. By merging the Pauli polarimetric decomposition technique with the Krichhoff migration equation, we develop a polarimetric migration algorithm, which can extract three migrated coefficients that are sensitive to different types of objects. Then fusing the three migrated coefficients, we can obtain subsurface colour-coded reconstructed object images, which can be employed to interpret both the geometrical information and the scattering mechanism of the subsurface objects. A 3-D full polarimetric GPR data set was acquired in a laboratory experiment and was used to test the method. In the laboratory experiment, four objects—a scatterer, a ball, a plate and a dihedral target—were buried in homogeneous dry sand under a flat ground surface. By merging the reconstructed image with polarization properties, we enhanced the subsurface image and improved the classification ability of GPR.

Key words: Image processing; Inverse theory; Electrical properties; Electromagnetic theory; Ground penetrating radar; Microstructures.

1 INTRODUCTION
Ground-penetrating radar (GPR) is a widely used geophysical technique that exploits the wave character of electromagnetic (EM) fields to investigate the shallow subsurface (Daniels 2004; Jol 2009; Tronicke & Hamann 2014), including earthquake-affected area (Roberts et al. 2010; Beaupretre et al. 2012) and fault systems (Salvi et al. 2003; Wallace et al. 2010; Dujardin et al. 2014; Ercoli et al. 2014). A few wave characteristics, such as wavelength, frequency, amplitude, velocity and arrival time, are typically measured and used. Polarization is also a property of EM waves. However, polarimetric attributes have been used less commonly in the GPR community.

Traditional radar uses the polarization properties of reflected EM waves to characterize surface properties. During recent decades, polarimetric technology has been one of the most important advances in microwave remote sensing (Cloude 2010; Jin & Xu 2013). The analysis of the polarization properties of reflected EM waves has been successfully used in polarimetric synthetic aperture radar (SAR) applications to characterize objects (Lee & Pottier 2009; Cloude 2010; Jin & Xu 2013).

More recently, a few published works have demonstrated that the polarization properties of GPR can help to better analyse signals from subsurface objects. A calibration technique for a polarimetric GPR system was developed (Feng et al. 2012). Polarimetric borehole radar and polarimetric GPR were also developed and used to detect subsurface fractures (Miwa et al. 1999; Tsolfias et al. 2004; Tsolfias & Hoch 2006; Zhao & Sato 2006; Sassen & Everett 2009). Polarimetric analysis was also used to distinguish buried land-mine and unexploded ordnance (UXO; Chen et al. 2001; Morrow & Van Genderen 2001; O’Neill 2001; Sadjadi et al. 2005), and to detect buried utility pipes (Boniger & Tronicke 2012).

Polarimetric decomposition, which can extract polarization characteristics, is a type of polarimetric analysis that has been commonly used in SAR terrain and land-use classification (Lee & Pottier 2009). Pauli decomposition based on the coherent Sinclair scattering matrix is a simple decomposition method. Three Pauli decomposition components correspond to three scattering mechanisms, which
enables classification of objects into three groups (Cloude & Pottier 1996; Martinez et al. 2005; Lee & Pottier 2009). The Pauli decomposition technique previously has been applied to borehole radar for fracture classification (Zhao & Sato 2006).

The migration technique is well developed in seismic data processing. Migration essentially reconstructs a reflector surface from the recorded data (Yilmaz 2001; Daniels 2004). The similarity between GPR and seismic reflection has led to the application of the same techniques used in seismic data processing for GPR data processing (Sena et al. 2006; Forte et al. 2014). Many types of migration algorithm have been applied successfully to a range of different applications of GPR (Hermance 2001; Di & Wang 2004; Bradford 2006; Streich et al. 2007), for example, reverse time migration (Fisher et al. 1992; Leuschen & Plumb 2001; Zhou et al. 2005), F-K migration (Hayakawa & Kawanaka 1998) and Kirchhoff migration (Moran et al. 2000; Feng & Sato 2004; Feng et al. 2011).

Migration can relocate reflected or diffracted signal energy back to its proper origin in space, and thereby reconstruct a subsurface image, which in turn can be used to better interpret the geometrical information of the subsurface objects.

The purpose of this paper is to develop a polarimetric migration imaging method, which combines advantages of the polarimetric decomposition and migration techniques, applied to full polarimetric GPR data. The polarimetric migration algorithm merges the Pauli decomposition with the Kirchhoff migration to extract migrated coefficients. By fusing the migrated coefficients, we can obtain subsurface colour-coded reconstructed object images, which can be employed to better interpret both the geometrical information and the scattering mechanism of the subsurface objects. A 3-D full polarimetric GPR data set was acquired in a laboratory experiment and was used to test the technique. By merging the subsurface reconstructed image with polarization properties, we enhance the subsurface image and improve the classification ability of GPR.

2 METHODOLOGY

The polarimetric migration imaging technique, which includes preprocessing, migration and imaging, is developed based on the assumption that full polarimetric GPR data are available.

2.1 Full polarimetric GPR data

A plane EM wave is said to be linearly polarized. The transverse electric field wave is accompanied by a magnetic field wave. Here,
polarization refers to the locus of the electric field vector in the plane perpendicular to the direction of propagation. The two most common polarizations are horizontal linear or H, and vertical linear or V. In a vertically polarized wave, the electric field vector lies in a vertical direction. In a horizontally polarized wave, the electric field vector lies in a horizontal direction. Any polarization can be synthesized by using H and V components. For this reason, full polarimetric GPR systems that transmit and receive both of these linear polarizations are commonly used. With the GPR, there can be four combinations of transmitter antenna and receiver antenna polarizations: HH—for horizontal transmitter antenna and horizontal receiver antenna, VV—for vertical transmitter antenna and vertical receiver antenna, HV—for horizontal transmitter antenna and vertical receiver antenna and VH—for vertical transmitter antenna and horizontal receiver antenna. A full polarimetric GPR system can have all four of these polarimetric transmitter/receiver antenna combinations. However, the coordinate systems of the transmitter antenna and receiver antenna of the full polarimetric GPR are identical, such that the role of the transmitter antenna and the receiver antenna can be interchanged. Hence, only one of the HV or VH antenna combinations is required. Consequently, a full polarimetric GPR acquires three types of polarimetric data at each measurement point to form the measured scattering matrix,

$$\begin{bmatrix} S_{HH}(x, y, w) & S_{VH}(x, y, w) \\ S_{VH}(x, y, w) & S_{VV}(x, y, w) \end{bmatrix}, \quad (1)$$

where, $S_{HH}$, $S_{VV}$ and $S_{VH}$ are the data acquired respectively by the HH, VV and VH antenna combinations in the frequency domain.

Figure 3. Experiment setting. Object 1 is a metallic scatterer. Object 2 is a metallic plate. Object 3 is a metallic ball. Object 4 is a metallic dihedral.
2.2 Pre-processing for full polarimetric GPR data

The pre-processing procedures include removing antenna coupling in the frequency domain, bandpass filtering, inverse fast Fourier transform (IFFT; Feng & Sato 2004; Feng et al. 2009) and subtracting averaged signal in time domain.

Because of small offset of transmitter/receiver antenna combination, the direct antenna coupling is much stronger than reflected signals. Therefore, it is necessary to remove antenna coupling. First, the three antenna coupling data, HH coupling, $C_{HH}(w)$, VV coupling, $C_{VV}(w)$ and VH coupling, $C_{VH}(w)$, are determined by pointing three types of transmitter/receiver antenna combinations, which have fixed transmitter/receiver offset, into the air to form the measured coupling matrix,

$$[C(w)] = \begin{bmatrix}
C_{HH}(w) & C_{VH}(w) \\
C_{VH}(w) & C_{VV}(w)
\end{bmatrix}. \quad (2)$$

Then, the antenna coupling will be removed from the measured scattering matrix in the frequency domain by

$$[S_{RC}(x, y, w)] = [S(x, y, w)] - [C(w)]. \quad (3)$$

A digital bandpass filter is used to suppress noise in frequency domain. Then after the inverse Fourier transform, the scattering matrix elements are transformed into the time domain to get $[s_{RC}(x, y, t)]$. Because the strongest reflection is from the air/soil interface, the procedure of subtracting averages can decrease the reflection amplitude of the ground surface and enhance the scattering signature of subsurface objects. First, we average the scattering matrix over the measurement points to obtain averaged scattering matrix, including averaged HH signal, averaged VV signal and averaged VH signal. Then, the averaged scattering matrix elements will be removed from the scattering matrix elements in the time domain by

$$[s(x, y, t)] = [s_{RC}(x, y, t)] - \frac{1}{N} \sum x \sum y [s_{RC}(x, y, t)]. \quad (4)$$

**Figure 4.** Vertical profile of VV polarimetric data. Object 1 marks the signals from the scatterer. Object 2 marks the signals from the plate. Object 3 marks the signals from the ball. Object 4 marks the signals from the dihedral.
where, $N$ is the number of measurement points. If there have slight terrain variation, it can be operated to subtract averaged scattering matrix elements in a space range window.

### 2.3 Polarimetric migration

For reconstructing the subsurface image, we choose pre-stack Kirchhoff migration (Schneider 1978; Yilmaz 2001; Feng & Sato 2004), which can be expressed as follows:

$$
[s_{\text{out}}(x_{\text{out}}, y_{\text{out}}, z)] = \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} [s(x, y, t)] \, dx \, dy.
$$

Where, $v_{\text{rms}}$ is the root-mean-square (RMS) velocity at the scatter point $(x_{\text{out}}, y_{\text{out}}, z)$, and $r$ is the distance between the measurement point and the scatter point. $\theta$ describes the angle between the direction of propagation and the vertical axis $z$. $[s_{\text{out}}(x_{\text{out}}, y_{\text{out}}, z)]$ is the migrated scattering matrix, and $[s(x, y, t)]$ is the pre-processed scattering matrix in the time domain, which is given by the following $2 \times 2$ matrix,

$$
[y(x, y, t)] = \begin{bmatrix} s_{hh}(x, y, t_{hh}) & s_{vh}(x, y, t_{ch}) \\ s_{vh}(x, y, t_{ch}) & s_{vv}(x, y, t_{vv}) \end{bmatrix}.
$$

Here, $t_{hh}$, $t_{vv}$ and $t_{ch}$ are the propagation times of EM wave in the three types of transmitter/receiver antenna combinations, HH, VV and VH, respectively. When the transmitter/receiver antenna combinations are configured in the common middle point (CMP) architecture, shown in Fig. 1, the propagation time is given by

$$
t_{hh} = \left[ \frac{z^2 + (x + d_{hh} - x_{out}) + (y - y_{out})}{v_{\text{rms}}^2} \right]^{1/2} + \left[ \frac{z^2 + (x - d_{hh} - x_{out}) + (y - y_{out})}{v_{\text{rms}}^2} \right]^{1/2}
$$

$$
t_{vv} = \left[ \frac{z^2 + (x + d_{vv} - x_{out}) + (y - y_{out})}{v_{\text{rms}}^2} \right]^{1/2}
$$

**Figure 5.** Vertical profile of HH polarimetric data. Object ① marks the signals from the scatterer. Object ② marks the signals from the plate. Object ③ marks the signals from the ball. Object ④ marks the signals from the dihedral.
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\[
\begin{align*}
+ \left( \frac{z^2 + (x - d_{hh} - x_{out}) + (y - y_{out})}{v_{rms}^2} \right)^{1/2}, \\
n_{hh} = \left( \frac{z^2 + (x + d_{hh} - x_{out}) + (y - y_{out})}{v_{rms}^2} \right)^{1/2}, \\
+ \left( \frac{z^2 + (x - d_{hh} - x_{out}) + (y - y_{out})}{v_{rms}^2} \right)^{1/2},
\end{align*}
\]

(7)

where, \(d_{hh}, d_{vv}\) and \(d_{vh}\) are the half offset between transmitter antenna and receiver antenna in the HH, VV and VH combinations, respectively. If we assume that the offset between transmitter antenna and receiver antenna in the three types of antenna combinations is the same, \(d_{hh} = d_{vh} = d_{vv} = d\), so that \(n_{hh} = n_{vh} = n_{vv} = t\),

\[
t = \left( \frac{z^2 + (x + d - x_{out}) + (y - y_{out})}{v_{rms}^2} \right)^{1/2}
+ \left( \frac{z^2 + (x - d - x_{out}) + (y - y_{out})}{v_{rms}^2} \right)^{1/2},
\]

(8)

In this case, the pre-processed scattering matrix is given by

\[
[s (x, y, t)] = \begin{bmatrix}
s_{hh} (x, y, t) \\
s_{vh} (x, y, t) \\
s_{vv} (x, y, t)
\end{bmatrix}.
\]

(9)

The scattering matrix can also be expressed as follows (Martinez et al. 2005; Lee & Pottier 2009):

\[
[s] = \alpha [s]_a + \beta [s]_b + \gamma [s]_c,
\]

(10)

where \([s]_a, [s]_b, [s]_c\) is the Pauli basis, given by the following matrices (Martinez et al. 2005; Lee & Pottier 2009):

\[
[s]_a = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
[s]_b = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},
\]

\[
[s]_c = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.
\]

(11)

In general, the matrix \([s]_a\) corresponds to the single- or odd-bounce scattering, for example, the scattering of a sphere, a plate or a trihedral. The matrix \([s]_b\) represents the scattering mechanism of

Figure 6. Vertical profile of VH polarimetric data. Object○ marks the signals from the scatterer. Object□ marks the signals from the plate. Object△ marks the signals from the ball. Object□ marks the signals from the dihedr.
a dihedral. In general, it is referred to the double- or even-bounce scattering, because the polarization of the returned wave is mirrored respect to the one of the incident wave. The matrix $[s]$, indicates a scattering mechanism characterized by volume scattering, for example forest canopy, which is able to return the orthogonal polarization (Martinez et al. 2005; Lee & Pottier 2009). Here, $\alpha$, $\beta$ and $\gamma$ are the coefficients, which represent the contribution of $[s]_a$, $[s]_b$ and $[s]_c$ to the scattering matrix $[s]$, respectively. They are given by

$$\begin{align*}
\alpha &= \frac{s_{hh} + s_{vv}}{\sqrt{2}} \\
\beta &= \frac{s_{hh} - s_{vv}}{\sqrt{2}} \\
\gamma &= \sqrt{2}s_{hv}.
\end{align*}$$

Consequently, the migration formula can be expressed as follows

$$[s_{\text{out}}(x_{\text{out}}, y_{\text{out}}, z)] = \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \left( \alpha [s]_a + \beta [s]_b + \gamma [s]_c \right) dx dy.$$  \hspace{1cm} (14)

$$[s_{\text{out}}] = A [s]_a + B [s]_b + C [s]_c.$$ \hspace{1cm} (15)

where, $A$, $B$ and $C$ are the migrated $\alpha$, $\beta$ and $\gamma$, respectively. They are given by,

$$\begin{align*}
A &= \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \alpha dx dy \\
B &= \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \beta dx dy \\
C &= \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \gamma dx dy
\end{align*}$$

Consequently, the migration formula can be expressed as follows

In this case, the total power is invariant. The Frobenius norm (Span) of the scattering matrix $[s]$ is given by,

$$\text{SPAN} = |s_{hh}|^2 + |s_{vv}|^2 + 2|s_{hv}|^2 = |\alpha|^2 + |\beta|^2 + |\gamma|^2.$$ \hspace{1cm} (13)

Consequently, the migration formula can be expressed as follows

$$[s_{\text{out}}(x_{\text{out}}, y_{\text{out}}, z)] = \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \left( \alpha [s]_a + \beta [s]_b + \gamma [s]_c \right) dx dy.$$  \hspace{1cm} (14)

$$[s_{\text{out}}] = A [s]_a + B [s]_b + C [s]_c.$$ \hspace{1cm} (15)

where, $A$, $B$ and $C$ are the migrated $\alpha$, $\beta$ and $\gamma$, respectively. They are given by,

$$\begin{align*}
A &= \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \alpha dx dy \\
B &= \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \beta dx dy \\
C &= \frac{1}{2\pi} \int \int \cos \theta \frac{\partial}{\partial t} \gamma dx dy
\end{align*}$$

Consequently, the migration formula can be expressed as follows

In this case, the total power is invariant. The Frobenius norm (Span) of the scattering matrix $[s]$ is given by,

$$\text{SPAN} = |s_{hh}|^2 + |s_{vv}|^2 + 2|s_{hv}|^2 = |\alpha|^2 + |\beta|^2 + |\gamma|^2.$$ \hspace{1cm} (13)
2.4 Full polarimetric data imaging

The migrated $\alpha$, $\beta$ and $\gamma$ can be combined into single data set. The polarization properties of subsurface objects can be represented by the combination of the intensities $|\text{migrated } \alpha|^2$, $|\text{migrated } \beta|^2$ and $|\text{migrated } \gamma|^2$ into a single RGB datum in which every of the previous intensities, $|\text{migrated } \alpha|^2$, $|\text{migrated } \beta|^2$ and $|\text{migrated } \gamma|^2$, are coded as a colour channel. The RGB data, which correspond to physical scattering mechanisms, can be employed to interpret the physical information from a qualitative point of view. In general, the matrix $[s]_a$ corresponds to the single- or odd-bounce scattering, for example, the scattering of a sphere, a plate or a trihedral. Thus, the coefficient $\alpha$ represents the contribution of $[s]_a$. In particular, $|\text{migrated } \alpha|^2$ determines the power scattered by targets characterized by the single- or odd-bounce. The matrix $[s]_b$ corresponds to the double- or even-bounce scattering, for example, the scattering of a dihedral. Thus, the coefficient $\beta$ represents the contribution of $[s]_b$. In particular, $|\text{migrated } \beta|^2$ determines the power scattered by targets characterized by the double- or even-bounce scattering. The matrix $[s]_c$ corresponds to the volume scattering, for example, forest canopy. Thus, the coefficient $\gamma$ represents the contribution of $[s]_c$. In particular, $|\text{migrated } \gamma|^2$ determines the power scattered by targets characterized by the volume (Martinez et al. 2005). Consequently, the merged data can show both the image and the polarization properties of subsurface objects from which we can obtain the geometrical information and the physical scattering mechanisms of subsurface objects of interest.

3 APPLICATION TO EXPERIMENT DATA

3.1 Full polarimetric GPR and experiment setting

We built a full polarimetric GPR system for use in the laboratory, which consisted of a vector network analyser, a Cartesian coordinate robot, polarimetric transmitter/receiver antenna combinations,
and a control computer, all shown in Fig. 2. Due to the capability of network analyser, the system is a stepped frequency GPR, which can generate stable signals in the frequency domain. The network analyser is an Agilent E5071C whose frequency band is 9 KHz–8.5 GHz. The Cartesian coordinate robot can move the antenna combinations along 3-D axes within 0.5 mm error. The network analyser is an Agilent E5071C whose frequency band is 9 KHz–8.5 GHz. The Cartesian coordinate robot can move the antenna combinations along 3-D axes within 0.5 mm error. The polarimetric transmitter/receiver antenna HH, VV and VH combinations use Vivaldi antenna (Guillanton et al. 1998; Sato et al. 2004). The HH and the VV combination are referred to as co-polarized because the polarizations of the transmitter antenna and the receiver antenna are the same. The VH combination is referred to as cross-polarized. Because the three polarimetric transmitter/receiver antenna combinations share a common midpoint, they are in the CMP architecture. The offset between transmitter antenna and receiver antenna is 8 cm in all three combinations. We acquire common offset CMP polarimetric data sets, HH, VV and VH data sets, in the frequency domain at each measurement point.

Fig. 3 shows the experimental objects and acquisition setup. Four objects were chosen. Object 1 is a metallic scatterer with many branches; object 2 is a horizontal metallic plate; object 3 is a metallic ball; object 4 is a metallic dihedral. All four objects were buried in the homogeneous dry sand, with the ground surface almost flat. Objects 1, 2, 3 and 4 were buried at a depth of about 25, 23, 17 and 32 cm, respectively. To acquire full polarimetric data, the transmitter/receiver antenna combinations were moved at an elevation of 13 cm, and 2-D scans were executed. In the 2-D survey, there are 80 lines. The distance between survey lines is 2 cm. In each survey line, there are 72 measurement points. The distance between measurement points is 2 cm. The frequency range used in the experiment is from 2.5 to 4.5 GHz. After we acquired three 3-D full polarimetric data sets, the HH, VV and VH data were assembled to form the measured scattering matrices in the frequency domain.

3.2 Pre-processing

We use the aforementioned procedures to process the measured scattering matrices. After antenna coupling is removed and the ground
Figure 10. Subsurface colour-coded RGB image of migrated $\alpha$, $\beta$ and $\gamma$. Red: $|\text{migrated } \beta|^2$; green: $|\text{migrated } \gamma|^2$; blue: $|\text{migrated } \alpha|^2$. Object ① marks the signals from the scatterer. Object ② marks the signals from the plate. Object ③ marks the signals from the ball. Object ④ marks the signals from the dihedral.
surface reflection is suppressed, scattering signals from subsurface objects are enhanced in all sections. Figs 4–6 each show two vertical sections of pre-processed VV, HH and HV data sets at $X = 0.34$ m and $X = 1.06$ m, respectively. The labels $\mathbb{1}$, $\mathbb{2}$, $\mathbb{3}$ and $\mathbb{4}$ mark the signals scattered from the scatterer, plate, ball and dihedral, respectively. In these figures, the reflection due to object $\mathbb{1}$ is discontinuous and disordered. The reflection due to object $\mathbb{2}$ has the geometrical feature of distinct hyperbolic curve. The reflections due to objects $\mathbb{3}$ and $\mathbb{4}$ have similar geometrical features in that the reflections are almost flat.

### 3.3 Polarimetric migration processing

Next, the algorithm of polarimetric migration is applied to the pre-processed scattering matrices. Figs 7–9 each show two vertical sections of the migrated $\alpha$, $\beta$ and $\gamma$ at $X = 0.34$ m and $X = 1.06$ m, respectively. Fig. 7 shows two vertical cross-line sections of migrated $\alpha$. Fig. 8 shows two vertical cross-line sections of migrated $\beta$. Fig. 9 shows two vertical cross-line sections of migrated $\gamma$. The reflections due to objects $\mathbb{1}$, $\mathbb{2}$ and $\mathbb{4}$ have the similar geometrical features as in those Figs 4–6, which display sections of pre-processed data sets. The reflections due to object $\mathbb{3}$ has a curve that is different with the pre-processed data sets, because in this case the diffractions are collapsed by the migration processing.

### 3.4 Subsurface colour-coded reconstructed object image

After polarimetric migration processing, we obtain the migrated images of $\alpha$, $\beta$ and $\gamma$. Each of the intensities was coded as a colour to form an RGB datum. The employed codification is

$$|\text{migrated } \beta|^2 \rightarrow \text{Red}$$
$$|\text{migrated } \gamma|^2 \rightarrow \text{Green}$$
$$|\text{migrated } \alpha|^2 \rightarrow \text{Blue}.$$

Then, we combined the RGB data, which are shown in Fig. 10. Fig. 10(a) shows the intersectional image, which includes four vertical slices at $X = 0.34$ m, $X = 1.06$ m, $Y = 0.4$ m and $Y = 1.26$ m.
Figure 12. Pre-processed signals of the four objects—the scatterer, the plate, the ball and the dihedral—acquired by the VV, HH and VH antenna combinations, respectively. Blue dotted line: VV signal; red dashed line: HH signal; green solid line: VH signal.

Fig. 10(b) shows 3-D isocontours, which are produced based on the RGB colour values.

In Fig. 10, object ⃗{\gamma} is shown in green colour mainly, which means that |migrated \gamma| is larger than |migrated \alpha| and |migrated \beta| mostly, because \gamma represents the contribution of matrix \[s\] that indicates a scattering mechanism characterized by volume scattering, and object \(\odot\) is a type of volume scattering object. Objects \(\oplus\) and \(\otimes\) are shown in blue colour mainly, which means that |migrated \alpha| is the largest value mostly, because \alpha represents the contribution of matrix \[s\], that corresponds to the scattering
(a) Migrated alpha, beta and gamma of the scatterer at X=0.4m and Y=0.4m.

(b) Migrated alpha, beta and gamma of the plate at X=1.06m and Y=0.4m.

(c) Migrated alpha, beta and gamma of the ball at X=0.4m and Y=1.26m.

(d) Migrated alpha, beta and gamma of the dihedral at X=1.06m and Y=1.26m.

**Figure 13.** Signals of the migrated $\alpha$, $\beta$ and $\gamma$ of the four objects—the scatterer, the plate, the ball and the dihedral, respectively. Blue dotted line: migrated $\alpha$; red dashed line: migrated $\beta$; green solid line: migrated $\gamma$.

matrix of a sphere, a plate or a trihedral, and object $\odot$ is a plate and object $\oplus$ is a ball. Object $\odot$ is shown in red colour mainly, which means that $|\text{migrated } \beta|$ is the largest value mostly, because $\beta$ represents the contribution of matrix $[s]$, that represents the scattering mechanism of a dihedral. Therefore, the resulting colour of the RGB image can be interpreted in terms of scattering mechanism of objects.

Fig. 11 shows two vertical sections of the colour-coded image at $X = 0.34$ m and $X = 1.06$ m, respectively. The figure can show both the colour properties and the geometrical features of objects. In the figure, object $\odot$ has discontinuous and disordered geometrical feature and green colour mainly. Object $\oplus$ has flat reflection and blue colour mainly. Object $\odot$ has a low curvature reflection and blue colour mainly. Object $\oplus$ has flat reflection and red colour mainly.
Consequently, combining the colour properties and the geometrical features of objects, we have improved the classification ability of GPR.

3.5 Discussion

Comparing Figs 4–6, we can find that the signals from objects are different between co-polarized data, VV and HH data, and cross-polarized data, VH data. Object ① gets clearer scattering signals in the cross-polarized section than in the co-polarized section. On the contrary, object ② has the clearer signals in the co-polarized section. Therefore, combing the full polarimetric data together, we may obtain better subsurface object images than single-polarization data.

Mean fusion is a useful method to combine different data sets into a single fused data set. But direct mean fusion is not suitable for the present case. Fig. 12 shows the pre-processed signals acquired above the four objects, scatterer, plate, ball and dihedral, by VV, HH and VH antenna combinations, respectively. In the figure, the amplitude of VV signal is larger than other signals. Therefore, the method of direct mean fusion of VV, HH and VH data may mask the advantage of the full polarimetric data that co-polarized data and cross-polarized data are sensitive to different types of objects. Because the signal acquired by VH antenna combination has always the strongest amplitude, the VH data always contribute more than the HH and VH data in the fused data.

Fig. 13 shows the signals of the migrated α, β and γ of the four objects: scatterer, plate, ball and dihedral, respectively. In the figure, we can find that migrated α, β and γ are sensitive to different types of objects, respectively. Plate and ball have the maximum amplitude of migrated α, and dihedral has the maximum amplitude of migrated β. Scatterer has a complex situation that all of the migrated α, β and γ have chances to become the maximum value. Because there possibly have single-bounce scattering and double-bounce scattering signals from some parts of the scatterer, and have volume scattering signals from the most other parts of the object, object ② has some red and blue colours, and has green colour mainly shown in Fig. 11(a). Therefore, it is possible to exhibit the advantage of the full polarimetric data by the data fusion method based on the migrated α, β and γ.

4 Conclusion

At each measurement point, a full polarimetric GPR consists of a 2 × 2 scattering matrix, which is composed of three polarimetric data components, VV, HH and VH. VV and HH data are co-polarized data components, and VH datum is a cross-polarized data component. Different components are sensitive to different types of objects, which offers a potential advantage that can improve the detection ability of the full polarimetric GPR.

The method of polarimetric migration imaging is developed to exploit the advantage of the full polarimetric measured matrices. By merging the Pauli decomposition technique with the Kirchhoff migration method, we can extract three migrated coefficients, migrated α, β and γ, which indicate different scattering mechanisms. We fused the three migrated coefficients into a single RGB data, in which each of the coefficients intensities is coded as a colour channel. The colours of the RGB data are employed to interpret the scattering mechanism of object. Finally, we can achieve the subsurface colour-coded reconstructed object image, which can show both the geometrical information and the physical scattering mechanisms of subsurface object that can enhance the subsurface image and improve the classification ability of GPR.

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