Incorporating Drift in Long-hole Stope Optimization Using Network Flow Algorithm

Xiaoyu Bai, Department of Civil, Geological and Mining Engineering, École Polytechnique de Montréal, C.P. 6079 Succ.Centre-ville, Montréal, (QC), H3C 3A7 Canada. Corresponding author, e-mail: xiaoyu.bai@polymtl.ca.

Denis Marcotte, Department of Civil, Geological and Mining Engineering, École Polytechnique de Montréal, C.P. 6079 Succ. Centre-ville, Montréal, (QC), H3C 3A7 Canada.

Richard Simon, Department of Civil, Geological and Mining Engineering, École Polytechnique de Montréal, C.P. 6079 Succ. Centre-ville, Montréal, (QC), H3C 3A7 Canada.

ABSTRACT

We present an algorithm aimed at optimizing stope design accounting for drift development. The approach is typically suitable for the sublevel stope mining method with a vertical parallel drilling pattern. The algorithm consists of two parts. In the first part, a graph theory based stope optimizer is adapted to integrate the drift. In the stope optimizer, the raise initiates a stope and provides the parameters of drift level and draw point level. An ore model in cylindrical coordinates is defined around the raise. The geometric constraints on hanging wall and footwall slopes are translated as vertical precedence relations between blocks. The stope width is controlled by horizontal precedence relations. Similarly, the dependency of blocks in stope to blocks in drift is expressed in the graph. By solving the maximum flow problem with efficient push-relabel algorithm, the conditional optimal stope corresponding to the current raise location and height is obtained. The second part finds the best raise location and height. The genetic algorithm is used with the objective function defined as the (conditional) stope profit associated to a given raise location and height, computed in the first part. The performance of the algorithm is evaluated with a synthetic deposit and a real deposit. Comparison is made with the stope optimizer without a drift. The proposed method is shown to provide higher stope profit, with less dilution and less costs for the drift.

Keywords: Underground mining stope optimization, drift, maximum flow algorithm, parallel drilling

INTRODUCTION

Stope design is an important aspect of underground mining design as it influences considerably the economic benefit and the operation safety of a mining project. For long-hole stoping method, the design of opening includes the positions and boundaries of stopes, the location of raises, and of the drifts needed to perform drilling. Usually, the stope boundary is defined first to maximize the total profit of contained volumes subjected to global and local geotechnical requirements. The optimized stope determines the drift levels and accessing raises. This workflow has a significant drawback due to neglecting the economical dependency of the stope boundary to the associated drift. For example, a cluster of ore that is profitable in a stope may
not be economic after counting cost of drift development to drill to the cluster. This can result in the loss of optimality of stope, especially when the cost of drift is relatively large. As an example, with parallel drilling pattern, the drift development can reach as much as 30% of stoping cost (Oraee and Bangian, 2007). This calls for the integration of drift design into the stope boundary optimization.

For the state-of-art stope optimization techniques, Ataee-Pour (2005) and Alford et al. (2007) provided reviews on most of existing stope optimizers. The several techniques, including dynamic programming method (Riddle, 1977), and branch and bound technique (Ovanic and Young, 1995), were developed to optimize a stope in 1D or 2D. These methods are mathematically rigorous and can yield optimal stope, however, the simplification of mining and geotechnical requirements could hardly lead to realistic stopes in 3D. A series of 3D approaches were proposed, such as mathematical morphology tools (Der aisme et al, 1984; (Serra, 1982), floating stope technique (Alford, 1996), maximum value neighborhood method (Ataee-Pour, 2000), and octree division approach (Cheimanoff et al, 1989). These heuristic methods did not incorporate comprehensively the geotechnical constraints. Manchuk and Deutsch (2008) provided an algorithm based on simulated annealing, with the geometric constraints directly integrated. Nevertheless, the computation time and the convergence to a global optimum could be a problem. Above all, none of these methods incorporate directly the drift in the optimization of stope boundary.

More recently, a new stope optimizer was developed by Bai et al (2013a) and an improved approach was presented by Bai et al (2013b). These methods are based on powerful graph theory inspired from successful open pit optimizers. The key to the approaches is to recognize that the vertical raise in underground mining plays a similar role of the ground surface in open pit. This calls for an ore block model in cylindrical coordinates around the raise. In the cylindrical ore model, the blocks are linked toward the raise constituting a graph, in such a way that the geotechnical constrains are implemented. Solving this graph, with efficient maximum flow algorithm, an optimal stope can be obtained. This is the core of the proposed approach: a stope generator that creates an optimal stope for a given raise location and elevation. The raise parameters, location and elevation, are externally optimized based on the stope generator to ensure the global optimality of stope. An improved algorithm introduced by Bai et al (2013b) generalizes the approach to multiple raises. Each raise defines an optimal sub-stope with its own optimized distance of influence. The union of all sub-stopes provides a globally close to optimal stope with more adaptability to orebody shapes. The appealing points of this heuristic method are: 1) the mining constrains are comprehensively implemented in 3D, and 2) the optimization parameters have a clear engineering meaning.

In this paper, the authors attempt to develop further the approach by including the drift directly in the stope optimization. It is assumed that long hole stoping method with parallel drilling pattern is adopted.

**METHODOLOGY**

**Economical function of long hole stoping**

The stope design calls for a geological model quantifying the ore grade in discrete blocks. The
estimation and simulation techniques to obtain such models are well documented in geostatistics literature (David, 1988; Journel and Huijbregts, 1978). The ore grades are assumed to be known in this study, therefore, the effect of grade uncertainty on stope design is neglected. Converting a geological model to an economical model requires the knowledge of economic outcome of relevant mining components. Lane (1988) proposed an economic function of an ore grade model:

\[ p_i = d_i v_i (g_i r f - c_i) \]  \hspace{1cm} (1)

Where \( p_i \) denotes the profit of mining block \( i \); \( g_i \) is the average ore grade of block \( i \); \( f \) is the unit metal price; \( r \) is the recovery rate; \( c \) is the unit cost of processing and mining; \( v_i \) and \( d_i \) are the block volume and density.

In the sublevel stoping method, the drift is developed using different drilling and blasting techniques from those used for stoping. Therefore, the unit cost to mine a block in drift or stope is different, sometimes by a large margin. We should write:

\[ p_i = d_i v_i (g_i r f - c_i) \]  \hspace{1cm} (2)

where \( c_i = C_{\text{drift}} \) when block \( i \) is in the drift; and \( c_i = C_{\text{stope}} \) when the block is in the stope. The \( C_{\text{drift}} \) and \( C_{\text{stope}} \) consist of not only the cost of drilling and blasting, but also the cost of transportation and ore treatment. Cost of stoping \( C_{\text{stope}} \) is assumed to be constant, hence the long-hole drilling is assumed going from top to bottom levels. Moreover, the development costs for accesses and main haulage levels, are considered similar for each possible stope in the zone.

**Stope optimization algorithm**

**Graph theory in stope optimization**

In graph theory techniques, the mining optimization problem is commonly modeled by a weighted graph \( G = (V, A) \). The ore blocks are delineated as vertices \( V \), and the mining constrains are represented by arcs \( A \), the connection between vertices. The profit \( p_i \) from mining a block \( i \) is denoted as the weight of the vertex. The stope or pit optimization finds a closed set of vertices \( V' \in V \) that maximizes \( \sum_{i \in V'} p_i \). This maximum closure problem can be formulated as:

Maximize \( \sum_{i=1}^{N} p_i x_i \)

Subject to \( x_j - x_i \leq 0, \quad \forall i \in V, j \in \Gamma_i \)  \hspace{1cm} (3)

\( x_i = 0 \) or \( 1, \quad \forall i \in V \)
Where $I_i$ is the subset of immediate successor nodes to node $i$, representing the set of blocks to be mined to get access to block $i$. $N$ denotes the number of blocks. As $N$ is usually large, solving this integer program can be time consuming. Lerchs-Grossman algorithm (Lerchs and Grossmann, 1965) is an effective and widely applied algorithm to solve the problem. However, maximum flow techniques proved to be more efficient (Goldberg and Tarjan, 1988; King et al, 1992; Picard, 1976).

**Implementation of stope geometric constraints in network**

Bai et al (2013a) showed the similarities of the underground optimization with the open pit method. They used a cylindrical coordinated system around each raise. The hanging wall and footwall slope limits are defined by the precedence links in the vertical direction as shown in Fig. 1 b). They also defined two design parameters, the radius of influence of the raise ($R$) and the minimum width ($y_R$) needed to remove the farthest block from the raise. The two parameters $R$ and $y_R$ control the links in the horizontal plane (see Fig. 2 a). The stope height is simply confined by the length of the raise. The blocks above the top of the raise or under its bottom are not part of the network, hence are not included in the stope.

![Figure 1](image_url) – Block model under cylindrical coordinates a); typical in vertical section b); additional arcs needed to integrate the drift in stope optimization c).
Figure 2 – Horizontal plane showing blocks and links defined in the cylindrical system a) and corresponding blocks and links in the Cartesian system b). Shaded blocks represent blocks to be removed to get access to block A. Trace of the envelopes defined by the lateral links in the cylindrical system c) as they appear in the Cartesian system d).

**Adding the drift in the network**

It is assumed that vertical parallel down-drilling is applied from the upper level to the bottom of the stope. The drift is made of the upper level blocks above the stope that need to be removed to allow drilling of the blocks in the stope. This is coded in the network by linking each block in the stope to the corresponding block in the upper level (see Figure 1c).

**Algorithm**

The optimization algorithm consists of two main parts. The first part, the stope optimizer, generates an optimal stope and associated drift for a specified raise location and height, with chosen design parameters \( R \) and \( y_a \). It includes the following steps:

1. With given raise parameters, recognize the drift blocks and stope blocks, and establish the economic block model in cylindrical coordinates with Equation 2;
2. Construct the graph with vertical arcs for wall slope constraints, horizontal arcs for width constraints, and arcs of drift dependency;
3. Build flow network by adding the source and sink nodes to the graph;
4. Solve the maximum flow problem. The generated stope is conditionally optimal to the raise location and height and accounting for the drift.

The second part seeks the best raise location and height. It is done by global optimization on the parameters, using as objective function the stope value found with the stope optimizer. Genetic algorithm (GA) is employed for this purpose. The raise is parameterized as \((x, y, z^t, z^b)\), where \(x\) and \(y\) indicate the location of raise, and \(z^t\) and \(z^b\) denote respectively the top level and bottom level of raise. Under GA framework, a vector of raise parameters is represented by an individual or a chromosome with the genes denoting the parameters. The profit of a stope represents the fitness of an individual to the environment. The process of searching the best parameters imitates the natural evolution of the individuals: new individuals are reproduced by mating existing individuals, and the least fitted individuals are eliminated. The mutated reproduction is allowed to generate a child dissimilar with its parents. The evolution iterations terminate when a chosen stopping criteria is met. The choice of parameters of GA is discussed by Bai et al (2013b).

**Comparative method**

Two methods are compared. The first one optimizes the stope not accounting for the drift. The cost of drift is simply added afterwards. The second method optimizes simultaneously the stope and drift. For both methods, we use the single raise stope optimizer (Bai et al, 2013a), and the global optimization of raise parameters is achieved with GA as is discussed by Bai et al (2013b). The drift is obtained by taking the blocks in the drift sublevel within the polygon enclosing the x-y coordinates of all stope blocks.

**RESULTS**

**Data and parameters in the algorithm**

To test the proposed approach, two deposit models are used, a synthetic one and a real one. The ore grade statistics and economic parameters are listed in Table 1. A single raise is used to create the stopes. The design parameter \(R\) is arbitrarily selected, respecting that all of the blocks in the study zone can be mined from the raise. The other design parameter \(y_R\) is chosen as \(R/3\). The geotechnical requirements of stope design are illustrated in Table 1.

For the GA method optimizing the raise parameters, the initial population size is 40. Three parents are used to create a new individual. For each iteration, 20 new individuals are created and the same numbers of least fitted individuals are eliminated to keep stable the size of the population. During reproduction, 10% genes are mutated. The optimization stops when the number of iterations reaches 100, or when the best fitted individual among the population does not improve over 10 successive iterations. The optimized raise parameters are listed in Table 2.
Table 1 – Ore grade model, discretization, and economic, geometric and design parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ore block model parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean grade (%)</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Cost of stoping, transportation and treatment ($/t)</td>
<td>30</td>
<td>56</td>
</tr>
<tr>
<td>Cost of developing drift (including the cost of transportation and treatment) ($/t)</td>
<td>34.2</td>
<td>60.2</td>
</tr>
<tr>
<td>Metal price ($/t)</td>
<td>8.4</td>
<td>12</td>
</tr>
<tr>
<td>Metal recovery rate</td>
<td>72%</td>
<td>80%</td>
</tr>
<tr>
<td>Rock density (t/m³)</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

| Geometric parameters                           |        |        |
| Minimum hanging wall angle (deg)               | 45     | 45     |
| Minimum footwall angle (deg)                   | 63     | 63     |
| Maximum height (m)                             | 70     | 130    |
| Minimum height (m)                             | 40     | 50     |
| Height of drift (m)                            | 4      | 4      |

| Design parameters                              |        |        |
| Stope width parameter y_R (m)                  | 23     | 35     |
| Maximum reference distance to raise R (m)      | 8      | 12     |

| Discretization of ore block model in cylindrical coordinate |        |        |
| dz (m)                                             | 0.5    | 1      |
| dr (m)                                             | 0.5    | 1      |
| dθ (deg)                                           | 0.89   | 0.57   |

Table 2 – Optimized raise parameters

<table>
<thead>
<tr>
<th>Optimized raise parameters</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with drift</td>
<td>without drift</td>
</tr>
<tr>
<td>Location X (m)</td>
<td>18.6</td>
<td>15.3</td>
</tr>
<tr>
<td>Location Y (m)</td>
<td>19.3</td>
<td>20.4</td>
</tr>
<tr>
<td>Bottom level (m)</td>
<td>-152.8</td>
<td>-153.1</td>
</tr>
<tr>
<td>Top level (m)</td>
<td>-111.7</td>
<td>-106.2</td>
</tr>
</tbody>
</table>

Test results

The first case is a vertical deposit (Fig. 3 a) to c). The orebody is approximately 30 m long by 30 m wide by 50 m high. The resulting stope and drift are shown in Fig. 3 d) to i). With the
proposed method (i.e. with drift included in the optimization), the profit of stope is 627 k$, which is 15.5% higher than the profit from the traditional method, 543 k$. The stope contains less waste (-42.5 k$ vs -163.9 k$), and shows a lower dilution rate (7.4% vs 14.7%). A greater quantity of ore is excluded from the stope, but the missed ore is mostly located on the edge where the ore thickness is smaller (see Fig. 3 h). Hence, it is not profitable to extend the drift to access this ore. With this approach, because the drift is smaller, the drift cost is much lower (-30.5k$ vs -189.2k$).

In the second scenario, an ore body of a metal deposit in Canada is used (name and location of deposit undisclosed for confidentiality reasons). A part of the deposit of size 50 m × 30 m × 80 m (Fig. 4 a, b) and c)) is selected for long-hole stope design. The design parameters are shown in Table 1, and optimized raise parameters are shown in Table 2. The optimized stope generates 11.9% more profit than the traditional method (195 k$ vs 174 k$). The stope includes less waste (-38.6 k$ vs -61.2 k$), obtains lower dilution rates (10.8% vs 12.1%), and cost less for drift development (-77.3k$ vs -109.3k$).

<table>
<thead>
<tr>
<th>Economic indicators</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit of stope (k$)</td>
<td>627</td>
<td>195</td>
</tr>
<tr>
<td>Profit of missed ore (k$)</td>
<td>56.1</td>
<td>13</td>
</tr>
<tr>
<td>Values of waste in stope (k$)</td>
<td>-42.5</td>
<td>-38.6</td>
</tr>
<tr>
<td>Dilution Volume rate1</td>
<td>7.4%</td>
<td>10.8%</td>
</tr>
<tr>
<td>Net benefit from drift(k$)</td>
<td>-30.5</td>
<td>-77.3</td>
</tr>
</tbody>
</table>

1Dilution volume rate = Volume of waste in stope / Volume of stope

DISCUSSION

We developed an algorithm that can jointly optimize stope and drift, specifically for long-hole stoping method with parallel drilling pattern. The proposed algorithm is based on the stope optimizer using the network flow method proposed by Bai et al (2013a, 2013b). The algorithm is shown to produce stopes more economical than with the classical approach where drift costs are added only after the stope is determined. The improved benefit comes from the smaller drift dimensions, and the abandon of the ore at boundaries, where the ore value does not pay for the additional drift costs to access them. The search of raise parameters based on GA is a heuristic method, but it is proved stable as it converged quickly to similar values upon a series of runs from different initial solutions. The comutation times of the two approaches, with and without drift, are similar.

Because a single raise is used in the stope design, the distance of influence $R$ is taken large enough to allow the raise to reach all the ore blocks in the study zone. The effect of $y_R$ is not studied here. The parameter $y_R$ should be large enough to avoid generating too narrow stope, which may hinder the flow of ore. A value around $y_R = R/3$ produced visually sensible shapes in our tests.
Figure 3 – Case 1, simulated ore model and created stopes: a) 3D-view of the orebody, b) yz vertical section of the orebody at x=20, c) xy horizontal section at z=-130; d) optimized stope and drift with traditional method view in 3D, e) yz vertical section at x=20, f) xy horizontal section at z=-130; g) optimized stope and drift with proposed method view in 3D, h) yz vertical section at x=20, i) xy horizontal section at z=-130; For the 3D view of stopes in d) and g), stopes are marked light meshes, and drifts are marked in dark squares. For stope slices in e), f), h) and i), stope (shaded area), drifts (square); waste in stope (+), and ore out of stope (x). Raises are marked in black lines with dots. Design parameters are given in Table 1.
Figure 4 – Case 2, test with real ore deposit: a) 3D-view of the orebody, b) xz vertical section of the orebody at x=3168, c) xy horizontal section at z=-144; d) optimized stope and drift with traditional method view in 3D, e) xz vertical section at x=3168, f) xy horizontal section at z=-130; g) optimized stope and drift with proposed method view in 3D, h) xz vertical section at x=20, i) xy horizontal section at z=-144; For the 3D view of stopes in d) and g), stopes are marked light meshes, and drifts are marked in dark squares. For stope slices in e), f), h) and i), stope (shaded area), drifts (square); waste in stope (+), and ore out of stope (x). Raises are marked in black lines with dots. Design parameters are given in Table 1.

Although tested here with a single raise, the approach can be extended easily to multiple raises (Bai et al., 2013b). To incorporate the drift in this algorithm, the graph under each raise need to be modified to incorporate the drift arcs as presented here. Also, the top level and bottom levels of the raises should be made equal, so that the drifts from different sub-stopes locate at the same elevation. The improvement is expected to bring more flexibility to follow various ore body shapes.
In the proposed method, parallel drilling of long hole stoping is assumed to be vertical. The proposed method may not provide good results when the orebody is inclined, which requires inclined parallel drilling. Also, for other drilling patterns like ring drilling, further developments are required.

In the proposed model, the cost of raise to access stope is neglected. The influence of cost of raise to the decision of the stope to be mined can be also integrated in the model, by recognizing the blocks of raise, and assigning specific cost as is done for integrating drift.

CONCLUSIONS

The proposed approach, integrating the drift in the optimization of long-hole stope, is shown to provide stope more profitable compared to the method not involving drift in optimization. The algorithm inherits the merit of previously developed stope optimizer based on network flow method, providing optimal 3D stope with geotechnical constraints incorporated.

ACKNOWLEDGEMENT

Research grants are provided by Chinese Scholarship Council and NSERC.

REFERENCES


