A new camera calibration method from vanishing points in a vision system

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The problem of finding intrinsic parameters of a camera is extremely important for practical applications. In this paper, we present an approach for automatic estimation of intrinsic parameters from images by using the vanishing points in orthogonal directions. Firstly, image lines are extracted and clustered into groups corresponding to three dominant vanishing points. Secondly, the intrinsic parameters of the camera are determined by using the vanishing points in each image. The rotation matrix of the projection matrix is computed from the vanishing points and image edges while the translation matrix is obtained with additional translation motion between the viewpoints. Our approach does not need any prior information about the cameras being used. Experiments with real images show that our new technique is robust and accurate.

Key words: camera calibration; projection matrix; rotation matrix; vanishing point

1. Introduction

Camera calibration is an important task in computer vision. The purpose of the camera calibration is to establish the projection from the 3D world co-ordinates to the 2D image co-ordinates. However, the traditional calibration algorithm requires known 3D co-ordinates of the feature points. Auto-calibration only requires the corresponding points of images, and thus provides more flexibility in practical applications.
In general, an auto-calibration algorithm results in a non-linear optimization problem using constraints from the intrinsic parameters of the camera. Thus, it requires proper initialization for the non-linear minimization. Traditional approaches to initialization assume unchanged intrinsic parameters while dealing with the situation where the intrinsic parameters of the camera may actually change.

Faugeras and Laveau (1994) proposed a self-calibration algorithm that uses the Kruppa equation. It enforces that the planes through two camera centres that are tangent to the absolute conic should also be tangent to both of its images. Hartley (1996) proposed another method based on the minimization of the difference between the internal camera parameters for the different views. Pollefeys et al. (1998) proposed a stratified approach that first recovers the affine geometry using the modulus constraint and then recovers the Euclidean geometry through the absolute conic. Heyden and Astrom (1996), Triggs (1997) and Pollefeys et al. (1996a) used explicit constraints that related the absolute conic to its images. These formulations are interesting since they can be extended to deal with the varying internal camera parameters.

Recently, self-calibration algorithms that can deal with the varying camera’s intrinsic parameters have been proposed. Heyden and Astrom (1997) proposed a self-calibration algorithm that uses explicit constraints from the assumption of the intrinsic parameters of the camera. They proved that self-calibration is possible for varying cameras with the assumptions that the aspect ratio is known and there is no skew. They solved the problem by using a bundle adjustment that requires simultaneous minimization on all of the reconstructed points and cameras. Moreover, the initialization problem was not properly presented. Bougnoux (1998) proposed a practical self-calibration algorithm that used the constraints derived from Heyden and Astrom (1997). He proposed the linear initialization step in the non-linear minimization. He used a bundle adjustment in the projective reconstruction step. Similarly, Pollefeys et al. (1996b) proposed a versatile self-calibration method that can deal with a number of types of constraints on the camera. They showed a specialized version for the case when the focal length varies, possibly also the principal point. In Zhang’s work (2004), data for calibration were collected from images of the calibration object, a 1D stick with three or more markers, rotating around a fixed point. Wu et al. (2005) proved that the planar motion of the 1D object can be converted to a rotational one, and solved the calibration problem using the constraints of conjugate points with respect to the absolute conic. Most recently, Hammarstedt et al. (2005) analysed the critical motion patterns of a 1D object for the calibration purposes and provided simplified closed-form solutions in Zhang’s configuration.

Vanishing points have been used to compute the principal points and the rotations (Beardsley and Murray, 1992; Caprile and Torre, 1990). Thus, the calibration problem can be solved by two steps. The first step focuses on the computation of the vanishing points and the related calibration parameters. The second step computes the remaining parameters in a system with reduced degrees of freedom. In Stevenson and Fleck (1995), the calibration equation is simplified by specifically controlled motions. However, there
are usually not many vanishing points in images and they are difficult to obtain. Thus, considerable research effort has been directed towards the computation of vanishing points from images (Shufelt, 1999). Most of the existing methods for computing vanishing points rely on line pair intersections obtained from parallel lines in the scene. Guillemaut et al. (2005) produced a method for decoupling translation and rotation for a collection of 3D scene points that are related to 2D image points by a projection. It does not require parallel lines, but points at infinity are computed from arbitrary straight lines in the scene, and a constraint on the location of the corresponding vanishing points is obtained. In Liu et al. (2004), according to the geometrical properties of the vanishing points and the square planar model, two constraints between vanishing points and the focal lengths can be found, from which a closed-form solution to focal lengths can be obtained. The interior orientation is automatically estimated from images with three vanishing points of orthogonal directions (Grammatikopoulos et al. (2007). In Xu et al. (2007), a robust and fast algorithm for skew distortion correction in 2D bar code images based on vanishing points is proposed.

Because of its simple geometrical structure, a 1D object is easy to be constructed. This is the main advantage of calibrating with a 1D object. In this paper, we proposed a simple approach to the construction of a 3D model by exploiting some constraints presented in the scenes to be modelled. In particular, the constraints that can be used are orthogonality or parallelism in the context of architectural environments. These constraints lead to a simple and geometrical method to calibrate the four intrinsic parameters of the camera from only two images from arbitrary positions. The process of camera calibration from vanishing points is shown as Figure 1.

2. Methodology

For a pin-hole camera, the perspective projection from Euclidean 3-space to an image can be conveniently represented in homogeneous co-ordinates by a $3 \times 4$ camera matrix, $P$

$$
\lambda_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}
$$

Figure 1 The process of camera calibration from vanishing points
The projection matrix has 11 degrees of freedom and can be decomposed into the orientation and position of the camera relative to the world co-ordinate system (a $3 \times 3$ rotation matrix $R$ and a $3 \times 1$ translation vector $T$):

$$ P = K[R \ T] $$ (2)

$K$ is called the camera intrinsic matrix,

$$ K = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} $$ (3)

where $f_u$ and $f_v$ are the scale factors in image $u$ and $v$ axes, $s$ is the parameter describing the skew of the two images axes, and $(u_0, v_0)$ are the co-ordinates of the principal point.

2.1 Using vanishing points

From Equation (1) and considering the points at infinity corresponding to the three orthogonal directions, we can derive simple constraints on the elements of the projection matrix:

$$ \begin{bmatrix} \lambda_1 u_1 & \lambda_2 u_2 & \lambda_3 u_3 \\ \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix} = P \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$ (4)

where $\lambda_i$ are the initially unknown scaling factors.

According to Equation (2), Equation (4) can be re-expressed in terms of the camera calibration matrix $K$ and camera orientation (rotation matrix $R$):

$$ \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = KR $$ (5)

2.2 Camera calibration and recovery of orientation

By exploiting the properties of the rotation matrix $R$, we can rearrange Equation (5) to recover constraints on the intrinsic parameters of the camera and the unknown scaling parameters $\lambda_i$:

$$ \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix}^T = KK^T $$ (6)
where

\[
KK^T = \begin{bmatrix}
  f_u^2 + u_0^2 & u_0v_0 & u_0 \\
  u_0v_0 & f_v^2 + v_0^2 & v_0 \\
  u_0 & v_0 & 1
\end{bmatrix}
\]  

(7)

Under the assumption of known zero skew, Equation (6) can be rewritten as six linear equations.

In our approach, the vanishing points corresponding to three mutually orthogonal directions can be used to determine the camera parameters:

1. The camera calibration matrix \( K \) under the assumption of zero skew;
2. The intrinsic parameters of the camera are determined by using the vanishing points in each image;
3. The rotation matrix of the projection matrix is computed from the vanishing points and image edges while the translation matrix is obtained with additional translation motion between the viewpoints.

3. Recovery of intrinsic parameters and projection matrix

3.1 Recovery of the principal point

Under the assumption of known zero skew, Equation (5) can be rewritten as:

\[
\lambda_1 u_1 \lambda_2 u_2 \lambda_3 u_3\begin{bmatrix}
\lambda_1 v_1 \\
\lambda_1 \\
\lambda_1
\end{bmatrix} = \begin{bmatrix}
 f_u & 0 & u_0 \\
 0 & f_v & v_0 \\
 0 & 0 & 1
\end{bmatrix} R
\]

thus

\[
R = \begin{bmatrix}
\lambda_1(u_1 - u_0)/f_u & \lambda_2(u_2 - u_0)/f_u & \lambda_3(u_3 - u_0)/f_u \\
\lambda_1(v_1 - v_0)/f_v & \lambda_2(v_1 - v_0)/f_v & \lambda_3(v_3 - v_0)/f_v \\
\lambda_1 & \lambda_2 & \lambda_3
\end{bmatrix}
\]  

(8)

The orthonormality of \( R \) can be used to provide the following equation:

\[
R^T R = I = \begin{bmatrix}
I_{12} \\
I_{23} \\
I_{31}
\end{bmatrix} = \begin{bmatrix}
R_{11}R_{12} + R_{21}R_{22} + R_{31}R_{32} \\
R_{12}R_{13} + R_{22}R_{23} + R_{32}R_{33} \\
R_{13}R_{11} + R_{23}R_{21} + R_{33}R_{31}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]  

(9)

Since \( \lambda_i \neq 0 \), \( f_u \neq 0 \) and \( f_v \neq 0 \), hence the following Equations are obtained from Equation (9):

\[
f_v^2(u_1 - u_0)(u_2 - u_0) + f_u^2(v_1 - v_0)(v_2 - v_0) + f_u^2f_v^2 = 0 \]  

(10)

\[
f_v^2(u_1 - u_0)(u_3 - u_0) + f_u^2(v_1 - v_0)(v_3 - v_0) + f_u^2f_v^2 = 0 \]  

(11)

\[
f_v^2(u_2 - u_0)(u_3 - u_0) + f_u^2(v_2 - v_0)(v_3 - v_0) + f_u^2f_v^2 = 0 \]  

(12)
Subtracting Equation (10) from Equation (11) gives:
\[ f_v^2(u_1 - u_0)(u_2 - u_3) + f_u^2(v_1 - v_0)(v_2 - v_3) = 0 \]  
(13)

Subtracting Equation (11) from Equation (12) gives:
\[ f_v^2(u_2 - u_0)(u_1 - u_3) + f_u^2(v_2 - v_0)(v_1 - v_3) = 0 \]  
(14)

Letting \( s = f_u^2/f_v^2 \), we have
\[
(u_1 - u_0)(u_2 - u_3) + s(v_1 - v_0)(v_2 - v_3) = 0
\]
(15)
\[
(u_2 - u_0)(u_1 - u_3) + s(v_2 - v_0)(v_1 - v_3) = 0
\]
(16)

As there are three unknown parameters in Equation (15) and Equation (16), we must find some more equations in order to obtain the solution.

3.2 Vanishing points constrains

Assuming a pair of orthogonal lines in the 3D space, the infinity points of them are \( P_{1\infty} \) and \( P_{2\infty} \), and the corresponding vanishing points are \( p_1 \) and \( p_2 \) on the image plane. According to the pinhole camera model, the relationship between the infinity points and their image projection is given by
\[
\begin{align*}
\alpha_1 p_1 &= K[R \ T]P_{1\infty} \\
\alpha_2 p_2 &= K[R \ T]P_{2\infty}
\end{align*}
\]
(17)

According to the main characteristics of the vanishing points, if the line between the optical centre \( O \) and the vanishing points is similar to the vector direction of the infinity point, the lines between the optical centre \( O \) and the infinity point \( P_{1\infty}, P_{2\infty} \) are orthogonal. The relationship between the lines could be described as:
\[
p_1^T K^{-T} K^{-1} p_2 = \alpha_1 P_{1\infty}^T [R \ T]^T \alpha_2 [R \ T] P_{2\infty} = \alpha_1 \alpha_2 P_{1\infty}^T P_{2\infty} = 0
\]
(18)

Let
\[
C = K^{-T} K^{-1} = \begin{bmatrix}
1/f_u^2 & 0 & -u_0/f_u^2 \\
0 & 1/f_v^2 & -v_0/f_v^2 \\
-u_0/f_u^2 & -v_0/f_v^2 & u_0^2/f_u^2 + v_0^2/f_v^2 + 1
\end{bmatrix}
\]

If two pairs of the orthogonal lines are observed, we have
\[
\begin{align*}
p_1^T K^{-T} K^{-1} p_2 &= 0 \\
p_3^T K^{-T} K^{-1} p_4 &= 0
\end{align*}
\]
(19)

Hence
\[
\begin{align*}
&u_1 u_2/f_u^2 - (u_1 + u_2) u_0/f_u^2 + v_1 v_2/f_v^2 - (v_1 + v_2) v_0/f_v^2 + u_0^2/f_u^2 + v_0^2/f_v^2 + 1 = 0 \quad (20) \\
&u_3 u_4/f_u^2 - (u_3 + u_4) u_0/f_u^2 + v_3 v_4/f_v^2 - (v_3 + v_4) v_0/f_v^2 + u_0^2/f_u^2 + v_0^2/f_v^2 + 1 = 0 \quad (21)
\end{align*}
\]
Subtracting Equation (20) from Equation (21) and letting $s = \frac{f_u^2}{f_v^2}$, then we have

$$(u_1u_2 - u_3u_4) - u_0(u_1 + u_2 - u_3 - u_4) + s(v_1v_2 - v_3v_4) - sv_0(v_1 + v_2 - v_3 - v_4) = 0$$

Combining Equations (15) and (16) with Equation (22), $s, u_0, v_0$ can be calculated.

### 3.3 Obtaining $\lambda_i^2$

In order to obtain a geometric interpretation of $\lambda_i^2$, row normality must be considered. According to the orthonormality of $R$, we have:

$$RR^T = I = \begin{bmatrix} I_{13} \\ I_{23} \\ I_{33} \end{bmatrix} = \begin{bmatrix} R_{11}R_{31} + R_{12}R_{32} + R_{13}R_{33} \\ R_{21}R_{31} + R_{22}R_{32} + R_{23}R_{33} \\ R_{31}R_{31} + R_{32}R_{32} + R_{33}R_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Hence:

$$\begin{bmatrix} \lambda_1^2(u_1 - u_0)/f_u + \lambda_2^2(u_2 - u_0)/f_u + \lambda_3^2(u_3 - u_0)/f_u \\ \lambda_1^2(v_1 - v_0)/f_v + \lambda_2^2(v_2 - v_0)/f_v + \lambda_3^2(v_3 - v_0)/f_v \\ \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

So from Equation (24) we can get the result of $\lambda_1^2$, $\lambda_2^2$, $\lambda_3^2$:

$$\begin{bmatrix} \lambda_1^2 \\ \lambda_2^2 \\ \lambda_3^2 \end{bmatrix} = \begin{bmatrix} (v_0 - v_3)(u_2 - u_3) - (u_0 - u_3)(v_2 - v_3) \\ (v_1 - v_3)(u_2 - u_3) - (u_1 - u_3)(v_2 - v_3) \\ (v_1 - v_3)(u_0 - u_3) - (u_1 - u_3)(v_0 - v_3) \\ (v_1 - v_3)(u_2 - u_3) - (u_1 - u_3)(v_2 - v_3) \\ 1 - \lambda_1^2 - \lambda_2^2 \end{bmatrix}$$

Since $u_0, v_0$ and $\lambda_i^2$ have been determined in the previous sections, the left side of Equation (6) is known. So, we have

$$f_u = \sqrt{\lambda_1^2 u_1^2 + \lambda_2^2 u_2^2 + \lambda_3^2 u_3^2 - u_0^2}$$

$$f_v = \sqrt{f_u^2/s}$$

As $u_0, v_0, f_u, f_v$ and $\lambda_i^2$ have been obtained, the rotation matrix $R$ can be determined according to Equation (8).

### 3.4 Recovery of the translation matrix $T$

The fourth column of the projection matrix depends on the position of the world co-ordinate system relative to the camera co-ordinate system. An arbitrary reference
point can be chosen as the origin. Its image co-ordinates fix the translation $T$, up to an arbitrary scale factor $\lambda_4$:

$$
\lambda_4 \begin{bmatrix}
u_4 \\
1
\end{bmatrix} = \begin{bmatrix}
p_{11} & p_{12} & p_{13} & p_{14} \\
p_{21} & p_{22} & p_{23} & p_{24} \\
p_{31} & p_{32} & p_{33} & p_{34}
\end{bmatrix} \begin{bmatrix}0 \\
0 \\
0 \\
1
\end{bmatrix} = KT
$$

(26)

Hence

$$
\begin{bmatrix}
\lambda_4 \\
\lambda_4 u_4 \\
\lambda_4 v_4
\end{bmatrix} = \begin{bmatrix}p_{34} \\
p_{14} \\
p_{24}
\end{bmatrix} = \begin{bmatrix}t_z \\
f_u t_x + u_0 \lambda_4 \\
f_v t_y + v_0 \lambda_4
\end{bmatrix} = \begin{bmatrix}Z \\
f_u t_x + u_0 \lambda_4 \\
f_v t_y + v_0 \lambda_4
\end{bmatrix}
$$

(27)

With a single viewpoint without metric information, $\lambda_4$ will be an arbitrary value. So, additional views with the image correspondence of a fifth point are required to fix this scale factor. This is equivalent to recovering pure unknown translation from the translational component of image motion under known rotation. So, only two points corresponding are required to recover the direction of translation.

Let $P$ be a visible point in the scene as in Figure 2, $X = (X, Y, Z)^T$ and $X' = (X', Y', Z')^T$ be its 3D co-ordinates with respect to the two viewpoints. $(x, y)^T$ and $(x', y')^T$ are the image co-ordinates of $P$ on the left and right images, respectively. The general motion equation of the camera is:

$$
X' = R(X - T)
$$

(28)

Figure 2 The imaging geometry (camera undergoes translational motion). O1 and O2 are the camera positions. T is the unknown camera translation vector.
where $T$ is an unknown translation vector $(T_x, T_y, T_z)^T$ and we can assume that it has unit length $\|T\|^2 = 1$. Therefore, there are only two unknowns for the translation vector. $R$ is a rotation matrix, which is known in this case.

A general relationship between the two sets of image co-ordinates – a relationship that expresses the condition that corresponding rays through the two centres of projection must intersect in the space – can be established as in Equation (10):

$$ x^TEx = 0 \quad (29) $$

where $x = (x, y, 1)^T$, $x' = (x', y', 1)^T$ are the matched image points in the two views. The essential matrix $E$ is defined as:

$$ E = RS \quad (30) $$

where $S$ is a skew-symmetric matrix

$$ S = \begin{pmatrix} 0 & -T_y & T_z \\ -T_z & 0 & T_x \\ T_y & -T_x & 0 \end{pmatrix} $$

When $R = I$, i.e., there is no rotation involved during the camera motion, we have $E = S$. Therefore, Equation (29) could be written as:

$$ \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -T_y \\ -T_z & 0 & T_x \\ T_y & -T_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \quad (31) $$

We can obtain a linear equation from Equation (31) via the elements of the translation vector $T$:

$$ (y' - y)T_x - (x' + x)T_y + (x'y - xy')T_z = 0 \quad (32) $$

For each pair of matched points, there will be one homogeneous equation showing the relationship among the variables $T_x, T_y, T_z$, and the corresponding points on the images. If a set of homogeneous equations like Equation (32) is found, the ratios of the three unknowns of $T$ can therefore be obtained.

Then, from $x' - u_0 = f_x X'/Z'$ and $y' - v_0 = f_y Y'/Z'$, it follows that:

$$ x' - u_0 = f_x X'/Z' = (X - T_z)/(Z - T_z) = (x - u_0 - T_x/Z)/(1 - T_z/Z) \quad (33) $$

Rearranging the above equation, $Z$ can therefore be derived as:

$$ Z = (T_x - (x' - u_0)T_z)/(x - x') \quad (34) $$
Substituting $Z$ into Equation (27), $t_x$, $t_y$ and $t_z$ can be calculated as follows:

$$
\begin{bmatrix}
t_x \\
t_y \\
t_z
\end{bmatrix} = \begin{bmatrix}
\lambda_4(u_4 - u_0)/f_u \\
\lambda_4(v_4 - v_0)/f_v \\
\lambda_4
\end{bmatrix} = \begin{bmatrix}
Z(u_4 - u_0)/f_u \\
Z(v_4 - v_0)/f_v \\
Z
\end{bmatrix}
$$

(35)

So, the translation matrix $T$ is recovered.

4. Experiments and results

4.1 Finding the vanishing points

The first step, which is to recover the projection matrices in the algorithm, requires finding the vanishing points of the parallel lines with known orientations. A vanishing point corresponds to the projection of the intersection of parallel lines at infinity.

4.2 Projection matrices

Having found the vanishing points, the second step is to recover the parameters $u_0$, $v_0$, $f_u$, $f_v$ and $\lambda_i$ scale factors. Then, the rotation matrix can be determined from Equation (8). Finally, when $\lambda_4$ is obtained, the translation matrix is obtained by Equations (27) and (35).

4.3 Experiments

In our experiments, the images are acquired using an Olympus FE-180/X745, 2048 x 1536 pixel camera (Figure 3), with a lens of focal length 6.3 mm. The primitive definition of the co-ordinates is defined as Figure 4. The process of finding vanishing points is shown as Figure 5. The calibration results are given in Table 1.
Figure 4 Primitive definition and localization of the co-ordinate

Figure 5 Finding vanishing points in the camera calibration

Table 1 Calibration results with synthetic data

<table>
<thead>
<tr>
<th>Parameter camera</th>
<th>Intrinsic parameters</th>
</tr>
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<tbody>
<tr>
<td>$f_u$</td>
<td>570.59</td>
</tr>
<tr>
<td>$f_v$</td>
<td>523.06</td>
</tr>
<tr>
<td>$f_u/f_v$</td>
<td>1.19</td>
</tr>
<tr>
<td>$(u_0, v_0)$</td>
<td>(922.79, 459.68)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Outer parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotation matrix</td>
</tr>
<tr>
<td>-0.980 0.164 0.109</td>
</tr>
<tr>
<td>-0.081 0.094 -0.992</td>
</tr>
<tr>
<td>0.182 0.982 0.054</td>
</tr>
</tbody>
</table>
In order to test the sensitivity of the obtained vanishing points to noise, we generate three images of model planes (shown in Figure 6) with the following three camera poses:

\[ r_1 = [0.8023, 0.5525, 0.2260]^T, \theta_1 = \pi/6, t_1 = [-40, 10, 420]^T; \]
\[ r_2 = [0.4468, 0.5525, 0.7036]^T, \theta_2 = \pi/5, t_2 = [-50, -10, 430]^T; \]
\[ r_3 = [0.7879, 0.4468, 0.4238]^T, \theta_3 = \pi/8, t_3 = [-50, -10, 430]^T. \]

Gaussian noise (unit: pixel) with standard deviation (noise level) is added to the projected image points. We varied the noise levels from 0 to 2.4 pixels with a setup of 0.2 pixels and performed 150 independent trials at each noise level. The test results shown are the average. During the tests, the eight corner points on each side are fitted into a line by a least-square algorithm. The vanishing points are computed from the intersection of several sets of parallel lines. The relative errors for \( f_u, f_v \) and absolute errors for \( u_0, v_0, s \) at each different noise level are shown in Figure 7. From this figure, we can see that the errors for the five camera intrinsic parameters increase linearly with the noise level and the proposed technique is accurate enough even in the presence of high levels of noise.

In order to validate further the accuracy and effectiveness of the proposed technique, we compare the calibration results with Zhang’s method (2004) under the same conditions, e.g. camera’s intrinsic and extrinsic parameters, noise level, image number and corner point number (totally 28 corners). Note that no non-linear optimization methods are used to refine the calibration results in both our technique and Zhang’s method. The relative errors for \( f_u, f_v, u_0 \) and \( v_0 \) are shown in Figure 8. The figure indicates that our proposed calibration technique is a little better than Zhang’s method.

In order to verify the validity of calibration results in Table 1, a standard stereovision technique was applied to reconstruct the calibration object. Two images of the calibration object were taken by the previous calibrated camera, which is shown in Figure 9. We manually picked out the corresponding points (marked by small circle).
from either of the two visible sides. The structure-from-motion algorithm was used to reconstruct the two visible sides of the calibration object. Two views from different viewpoints are shown in Figure 10. From this figure, we can see that reconstructed points on the same side of the calibration object are indeed coplanar. The angles between the two reconstructed sides are 90.4°, which accords well with the ground truth of 90°.

From the experiments, it can be seen that the proposed calibration method is easy to implement and has high accuracy.
Figure 8 The results Comparing with Zhang’s method
Figure 9  Two images of a calibration object

Figure 10  Two views of the reconstructed calibration object
5. Conclusions

A novel and simple method for recovering the projection matrix from 3D space to the CCD image plane, which exploits the use of vanishing points of three orthogonal directions, has been described. The simple but powerful constraints of parallelism and orthogonality in images can be used to recover projection matrices accurately with only a few corresponding points and lines. The technique presented has been successfully used for camera calibration.

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