Energy Saving for Automatic Train Control in Moving Block Signaling System

Qing Gu, Xiao-Yun Lu and Tao Tang

Abstract—With rapid development of the railway traffic, the moving block signaling system (MBS) method has become more and more important for increasing the track capacity by allowing trains to run in a shorter time-headway while maintaining the required safety margins. In this framework, the tracking target point of the following train is moving forward with its leading train. This paper focuses on the energy saving tracking control of two successive trains in MBS. Nonlinear programming method is used to optimize the energy saving speed trajectory of the following train. The real-time location of the leading train could be integrated into the optimization process. Due to simplicity, it can be used for online implementation. Finally, a smoothing algorithm is proposed to improve the speed trajectory for ride comfort. The feasibility and effectiveness are verified through simulation. The results show that the new method is efficient on energy saving with guaranteed ride comfort even when disturbances present.

Key words—moving block, energy saving, ATP (Automatic Train Protection), ATO (Automatic Train Operation), tracking, train speed trajectory, ride comfort, trajectory smoothing

I. INTRODUCTION

With safe, comfortable, punctual, and efficient features, mass rapid transit system can bring tremendous convenience to daily life and is an effective solution to modern traffic congestion problem. As railway networks could consume large amounts of energy and contribute significantly in emission, to ensure the punctuality and ride comfort of trains while achieving energy-saving represents an interesting and challenging problem.

In Fixed Block Signaling System, Energy-saving on railways has been studied for many years and got many effective results. The earliest and most noticeable work is from the Scheduling and Control Group (SCG) in North Australia. Researchers in SCG conducted theoretic research and project on train energy-saving operation. Milroy, Lee and Tyler proposed energy-saving operation model in [1]. They showed that the energy-saving control could be divided into four phases: traction, speed holding, coasting and braking as illustrated in Fig. 1. Based on that, many people investigated energy saving strategies, e.g. Chang and Wong applied different algorithms to find coasting points before the train’s departure in fixed running distance [2, 3]. Since MBS has more implementations in modern days, research on driving strategies in MBS will be the trend in the future [4].

However, there has not been much research in train’s tracking control strategy in Moving Block Signaling (MBS). MBS was proposed in [5] to reduce headway among successive trains. In MBS, two successive trains are separated by a safety margin (a pre-determined distance) and the required braking distance of the following train. During the running process, Automatic Train Protection (ATP) system calculates the braking curve of the following train to guarantee the safety. At the same time, the follower could obtain the position and speed of the leader. Therefore the speed trajectory of the follower could be optimized with respect to some objective function.

This paper designs a new energy saving driving strategies for trains’ tracking control. It could be implemented by Automatic Train Operation (ATO) system or as the reference speed for driving assistance. A trajectory smoothing algorithm is proposed to improve the ride comfort. The simulation results show that, compared with the traditional approaches, the new method is significantly more efficient on energy-saving without sacrificing the operation time when the leading train running without interference or only sacrificing little (due to smoothing) operation time when the leading train has interferences.
**Nomenclature:** The following notations are used throughout the paper.

- \( t \) is the time parameter;
- \( S_{\text{leading}}(t) \) is the position of the leading train’s head;
- \( S_{\text{following}}(t) \) is the position of the following train’s head;
- \( L_i(t) \) is the instantaneous distance of two successive trains;
- \( L_s \) is the length of the train;
- \( L_{\text{safety}} \) is the length of the safety margin;
- \( V_{\text{following}}(t) \) is the instantaneous speed of the following train;
- \( a_1, a_2, a_3 \) are service acceleration and deceleration of the train;
- \( b' \) is the deceleration in approximate coasting phase;
- \( \beta \) is the position of the following train’s head;
- \( \gamma \) is the position of the destination station;
- \( z_{\text{leading}} \) is the position of the leading train’s head;
- \( z_{\text{safe}} \) is the position of the destination station.

\[ S_{\text{leading}}(t) - S_{\text{following}}(t) \]

The distance between two successive trains must be larger than the safety margin at any moment even if the leading train comes to a sudden halt, so we have

\[ L_i(t) \geq L_{\text{safety}} + L_s + V_{\text{following}}(t)^2 / 2b', \]

Based on equation (1) and equation (2), the relation between the leading train and following train should satisfy:

\[ S_{\text{leading}}(t) - S_{\text{following}}(t) \leq L_{\text{safety}} + L_s + V_{\text{following}}(t)^2 / 2b', \]

which implies that the instantaneous speed and position of the following train should satisfy

\[ V_{\text{following}}(t) \leq \sqrt{2 \times b \times (S_{\text{leading}}(t) - S_{\text{following}}(t) - L_{\text{safety}})}, \]

\[ S_{\text{following}}(t) \leq S_{\text{leading}}(t) - L_{\text{safety}} - L_s - V_{\text{following}}(t)^2 / 2b'. \]

**II. TRACKING INTERVAL MODEL IN MBS**

Under MBS, the tracking target point of the following train moves forward continuously as the leading train travels. The instantaneous distance \( L_i(t) \) of two successive trains is expressed as:

\[ L_i(t) = S_{\text{leading}}(t) - S_{\text{following}}(t) \]  

The distance between two successive trains must be larger than the safety margin at any moment even if the leading train comes to a sudden halt, so we have

\[ L_i(t) \geq L_{\text{safety}} + L_s + V_{\text{following}}(t)^2 / 2b', \]

Based on equation (1) and equation (2), the relation between the leading train and following train should satisfy:

\[ S_{\text{leading}}(t) \geq L_{\text{safety}} + L_s + S_{\text{following}}(t) + V_{\text{following}}(t)^2 / 2b', \]

which implies that the instantaneous speed and position of the following train should satisfy

\[ V_{\text{following}}(t) \leq \sqrt{2 \times b \times (S_{\text{leading}}(t) - S_{\text{following}}(t) - L_{\text{safety}} - L_s)}, \]

\[ S_{\text{following}}(t) \leq S_{\text{leading}}(t) - L_{\text{safety}} - L_s - V_{\text{following}}(t)^2 / 2b'. \]

**III. ANALYSIS AND MODELING OF NEW ENERGY SAVING TRACKING STRATEGY**

In MBS, the following train could get the leading train’s speed and position. Two problems should be considered in speed trajectory design for energy optimization. The first is how to modify the running strategy for the real scenario because, in some circumstance, it is unnecessary to go through all the four phases listed above. The second is how to establish a simple model for easy online implementation.

For the first problem, the energy saving strategy could be achieved by arranging the sequence of the phases appropriately. It is known that some running phases may not happen in some circumstances. On the other hand, if the leading train has a long stop, the following trains should stop to wait until the leading train starts. To ensure the new model to fit all running scenarios, the new strategy should reflect all the cases. Since the leading train’s position will be involved in the optimization of the following train’s speed trajectory, we can build two models based on different phases of the leading train (running or stopped).

For the second problem, a quadratic programming method is used to obtain the optimal speed trajectory due to the availability of several efficient numerical methods. Since the coasting phase has a variable deceleration, it is hard to represent the running distance with respect to the deceleration (control variable) and speed (state variable). In order to simplify the model, we replace the coasting phase with a slowdown phase and call it the approximate coasting phase in this paper. For energy saving, the deceleration should close to but not less than the mean deceleration of the coasting phase, because the extra deceleration would lead to significant friction forces which cause tear and wear to the braking systems. On the other hand, if the deceleration is too small, the traction system needs to provide more traction force to support the train, which will consume more energy. It is therefore necessary to choose an appropriate value for a better trade-off. After analyzing the practical data of coasting phase from Dalian Fast Track [6], it can be observed that the speed in coasting phase declines very slowly. Therefore, the speed-time profile is close to a straight line and the slope of the line is the deceleration in the coasting phase. We have used the Least-Square procedure to fit the speed-time curve with the results shown in Table 1.

From Table 1, it is clear that the higher the coasting starting speed, the greater the deceleration. In order to supply a small traction force to keep the train moving in a constant deceleration, -0.01 m/s² seems appropriate.

**TABLE 1**

<table>
<thead>
<tr>
<th>Speed range (km/h)</th>
<th>Slope (m/s²)</th>
<th>Average Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>37-31</td>
<td>-0.0147</td>
<td>0.0227</td>
</tr>
<tr>
<td>41-40</td>
<td>-0.0125</td>
<td>0.0361</td>
</tr>
<tr>
<td>43-42</td>
<td>-0.0147</td>
<td>0.0364</td>
</tr>
<tr>
<td>54-48</td>
<td>-0.0213</td>
<td>0.0236</td>
</tr>
<tr>
<td>60-59</td>
<td>-0.0237</td>
<td>0.0296</td>
</tr>
<tr>
<td>62-61</td>
<td>-0.0210</td>
<td>0.0301</td>
</tr>
</tbody>
</table>

For the two successive trains running on the track, assuming...
two situations of the leader (running and stopped), we design corresponding driving strategies for the following train.

A. Problem Formulation for Moving Leader Train

When the leading train is running, the new operation strategy is consisted of four running phases: traction, speed holding, approximate coasting and braking. The initial speed of each running phase is $v_i$, $(i=1, 2, 3, 4).$ The running distances and time of each phase are $\Delta S_i$, $\Delta T_i$, $(i=1, 2, 3, 4)$ respectively. It is easily known that $v_2=v_3$. In this paper, we use kinetic energy to measure the energy consumption of trains. The kinetic energy difference between $v_1$ and $v_2$, as well as $\Delta T_1$ should be minimized because the traction and speed holding phases are highly energy consuming. Since the approximate coasting phase is relatively low energy consuming, it is hoped that $\Delta T_1$ can be longer instead of $\Delta T_2$, which could reduce the time periods for traction and speed holding phases.

![Fig.2. Operation strategy when the leading train is running](image)

There are two situations for time and distance constraints. Before the leading train re-starts from the station, the time constraint is $\Delta T_{tacking}$. And the distance constraint is the current distance between these two successive trains minus the distance of safety margin ($L_{safe}$). However, after the leading train clears the station, the distance constraint is the current distance between the following train and the station. Therefore, the time constraint should be changed accordingly. The time constraint depends whether the leading train is delayed. If it is not delayed, the time constraint changes to $\Delta T_{tacking}+T_{np}$, where $T_{np}$ is the running time of the following train at strategy update. On the contrary, if it is delayed, more passengers may wait at the station. Therefore, there is no need to apply the new strategy to the following train since it has to arrive at the destination as soon as possible. Based on the analysis above, the model is as follows:

$$\begin{align*}
\min f &= \frac{1}{2} m (v_i^2 - v_{i-1}^2) + \alpha (\Delta T_i - \Delta T_{i-1}) \\
\text{s.t. } & \Delta T_i \geq 0 \quad i=1,\ldots,4 \\
& \Delta S_i = \Delta T_i \frac{v_i^2 - v_{i-1}^2}{a} + \Delta S_{tacking} \frac{v_i - v_{i-1}}{b} - \frac{v_i^2}{2b} = -\Delta T_{tacking} - \Delta T_{np} \\
& \sum_{i=1}^{4} \Delta S_i = \sum_{i=1}^{4} \Delta T_i \frac{v_i^2 - v_{i-1}^2}{2a} + \sum_{i=1}^{4} \Delta S_{tacking} \frac{v_i - v_{i-1}}{2b} = \sum_{i=1}^{4} \Delta T_{tacking} \frac{v_i^2 - v_{i-1}^2}{2a} + \sum_{i=1}^{4} \Delta S_{tacking} \frac{v_i - v_{i-1}}{2b} = m \frac{v_i^2}{2} - m \frac{v_{i-1}^2}{2} + m \frac{v_i^2}{2} - m \frac{v_0^2}{2} = m \frac{v_f^2}{2} - m \frac{v_0^2}{2}
\end{align*}$$

where:
- $\alpha$ is penalty factors, $\alpha > 0$;
- $v_i$ is the starting velocities of each running phase, $i \in Z$, $i=1,2,3,4$;
- The decision parameters here are $v_2$, $\Delta T_2$.

B. Problem Formulation for Stopped Leader Train

When the leader train stops, the distance between it and the following train is getting shorter and shorter. However, the tracking time between them is unchanged. Therefore, the following train does not need traction phase any more. At the same time, if the leading train has a long stop, the following train may need stop to wait until the leading train’s re-starting. Therefore, we remove the traction phase from the strategy in the last case and add stopping phase to it as shown in Fig. 3.

![Fig.3. Operation strategy when the leading train stops](image)

The tracking interval between two successive trains is divided into 4 phases: speed holding, approximate coasting, braking and waiting. The running time and distance of each phase are $\Delta T_i$ and $\Delta S_i$, $(i=1,\ldots,4)$, respectively. The initial speed of each running phase is $v_i$, $(i=1,2,3,4)$. It is known that $v_1=v_2$.

The time and distance constraints are the same as those in (7). The new driving strategy is formulated as follows:

$$\begin{align*}
\min f &= (a \frac{v_i^2}{2} + a \frac{v_1^2}{2}) \Delta T_i + \alpha (\Delta T_i - \Delta T_{i-1}) \\
\text{s.t. } & v_i \geq v_1 \geq 0 \\
& \Delta T_i \geq 0 \quad i=1,\ldots,4 \\
& \sum_{i=1}^{4} \Delta T_i = \sum_{i=1}^{4} \frac{v_i - v_{i-1}}{b} + \Delta T_{tacking} - \Delta T_{np} \\
& \sum_{i=1}^{4} \Delta S_i = \sum_{i=1}^{4} \frac{v_i^2 - v_{i-1}^2}{2a} + \sum_{i=1}^{4} \frac{v_i - v_{i-1}}{b} = \sum_{i=1}^{4} \frac{v_i^2}{2a} - \sum_{i=1}^{4} \frac{v_{i-1}^2}{2a} + \sum_{i=1}^{4} \frac{v_i}{2b} - \sum_{i=1}^{4} \frac{v_{i-1}}{2b} = m \frac{v_f^2}{2} - m \frac{v_0^2}{2} - m \frac{v_i^2}{2} + m \frac{v_{i-1}^2}{2}
\end{align*}$$

where:
- $v_i$ is the starting velocities of each running phase, $i \in Z$, $i=1,2,3$;
- The decision parameters here are $v_i$, $\Delta T_i$.

IV. SIMULATION AND DISCUSSION

In this section, a simulation is used to test and verify the new strategy. We use the actual data to do the simulation. The two successive stations are Tongji South Road Station and Jianghai Station in Mass Transit Beijing Yi Zhuang Line. The distance between them is 2545m. The length of train ($L_i$) is 140m, safety
margin($L_{safe}$) is 50m, service tracking headway ($\Delta T_{tracking}$) is 100s, dwell time ($T_{dwell}$) is 20s, running time in timetable is 177s, target speed ($v$) is 16 m/s, service acceleration rate ($a$) is 1 m/s$^2$, and service barking deceleration rate ($b$) is 1 m/s$^2$ with $b=-0.01$ m/s$^2$. The simulation update rate 1s. We use interior-point method to solve the model in Matlab. By applying the traditional strategy, the speed curve is shown in Fig. 4 and Fig. 5.

![Fig.4. Ordinary operation trajectory in the inner station (v-s)](image)

![Fig.5. Ordinary operation trajectory in the inner station (v-t)](image)

Under the normal circumstance, successive trains depart every 100s following the speed trajectory shown in Fig 4 and Fig 5. With traditional control strategy, no matter how the leading train behaves, the following train speed follows the curve shown in Fig. 4 and Fig. 5 until the constraint (4) is violated. In such a case, the following train will brake to stop. In other words, with this strategy, if the leading train stops due to some interruptions, the following train may still accelerate or keep at a constant speed followed by breaking if the (4) is satisfied. This causes the waste of energy.

In the new strategy, the speed curve is calculated online taking into account the leading train’s position. Therefore, if the leading train speed curve does not match with those depicted in Fig. 4 and Fig 5, the following train will avoid unnecessary traction to reduce energy consumption.

Two simulation scenarios have been presented below to show the effectiveness on energy saving of the following train by applying the new strategy even when the leading train runs abnormally. In the following figures, the red dotted line is the running trajectory of the leading train and the blue solid line is the trajectory of the following train.

A. Leading Train with Short Stop at Inner Station

When the leading train has a short stop at the position of 1696 meters for 35 seconds, Fig. 6 and Fig. 7 show the speed curve created by traditional control strategy.

From Fig. 6, it can be observed that the following train has no coasting phase. This is because when the leading train stops, the following train is in the speed holding phase and even after the leading train’s re-starting, it is still in the same phase. However, since the leading train is delayed, more passengers may wait at the station, which requires that the following train arrive at the station as soon as possible. Therefore, the coasting phase is replaced by speed holding phase. The running time of the following train is 175s, 2s earlier than what was scheduled. The total energy consumption is 152.29 kwh.

![Fig.6. Following train’s trajectory when the leading train has a short stop under traditional strategy (v-s)](image)

![Fig.7. Following train’s trajectory when the leading train has a short stop under traditional strategy (v-t)](image)

Fig. 8 and Fig. 9 show the speed curve created with the new strategy with update rate 1s. When the leading train stops, the following train is acknowledged immediately, which is incorporated in speed curve design of the next step. It can be observed that the train only coasts at beginning. After the leading train re-starts, the following train accelerates immediately. After the leading train stops at the destination, the following train reduces to a lower speed and keeps it until the leading train clears the destination. The total run time of following train is 185s which is 10s later than scheduled. The total energy consumption is reduced to 28.02 kwh. Compared with the traditional strategy, energy consumption reduced to 18.4% of the original with 5% time delay, which is worthwhile.

![Fig.8. Following train’s trajectory when the leading train has a short stop](image)
under new strategy (v-s)

Fig.9. Following train’s trajectory when the leading train has a short stop under new strategy (v-t)

B. Leading Train with Long Stop at Inner Station

When the leading train has a long time stop at the position of 1696 meters for 120 seconds, Fig. 10 and Fig. 11 show the speed curve created by traditional strategy.

Fig.10. Following train’s trajectory when the leading train has a long stop with traditional control strategy (v-s)

Fig.11. Following train’s trajectory when the leading train has a long stop under traditional strategy (v-t)

It is noted that the following train stops at 1506 m for 35 seconds, due to the leading train’s long time stop. Moreover, since the leading train was delayed, what can be achieved for the two trains is run under the constraint (4) to arrive at the next station as soon as possible. Since the following train accelerates immediately after the leading train re-starts, the following train still has a stopping phase, followed by acceleration and then deceleration before it arrive at the station because of the dwell time of the leading train at the station. The total running time of the following train is 258.6 s with energy consumption 115.1kwh.

Fig.12. Following train’s trajectory when the leading train has a long stop under new strategy (v-s)

Fig.13. Following train’s trajectory when the leading train has a long stop under new strategy (v-t)

Figure 12 and 13 show the speed curve generated with the new strategy. It can be observed that there is only one stopping phase for the following train instead of two (vs. Fig. 10 and Fig. 11). Therefore, one acceleration phase has been avoided. In addition, the speed holding phase (with energy consumption) in the front part is replaced by an approximate casting phase (lower energy consumption) because the leading train’s long stop. Both imply energy savings. The running time of the following train is 289s versus 30.4s with traditional strategy. However, the total energy consumption is reduced to 28.2 kwh. Compared with the traditional strategy, energy consumption reduced to 24.5% of the original with 11.8% time delay.

Table 2 shows the comparison between the traditional and new strategies in different scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Short time stop</th>
<th>Long time stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Consumption (kwh)</td>
<td>152.29</td>
<td>28.02</td>
</tr>
<tr>
<td>Running Time (s)</td>
<td>175</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>258.6</td>
<td>289</td>
</tr>
</tbody>
</table>

V. FURTHER IMPROVEMENT FOR RIDE COMFORT

Based on the discussion above, it is known that the new strategy is rather effective on energy saving. However, the ride comfort may be deteriorated due to non-smooth phase changes. To improve this, a smooth algorithm is proposed below.

In calculating the online speed curve as above, we know the trend of the curve on both sides of the phase changing point. Therefore, some smoothing techniques can apply around the phase changing point for smooth transitions. Smooth transitions for speed trajectory design with piecewise sinusoidal function were proposed in [7, 8]. However, since the transition time and distance are too short and fixed in train control system, in this
In this paper, we propose a technique for properly selecting the parameters of the quadratic functions to achieve smooth transition.

![Smoothing transition](image)

**Fig. 14. Smoothing transition**

In Figure 14, the red line shows the smooth transition between two different phases. We design the following smoothing equation:

$$v_{\text{sub}}(t) = A(t - B)^2 + C$$  \hspace{1cm} (9)

and the derivative function is

$$\dot{v}_{\text{sub}}(t) = a_{\text{sub}}(t) = 2A(t - B)$$  \hspace{1cm} (10)

To ensure the smooth transition at both sides, the following boundary conditions are imposed:

$$v_{\text{sub}}(t_s) = v(t_s), \quad v_{\text{sub}}(t_e) = v(t_e)$$  \hspace{1cm} (11)

$$a_{\text{sub}}(t_s) = a(t_s), \quad a_{\text{sub}}(t_e) = a(t_e)$$  \hspace{1cm} (12)

where:

- $v_{\text{sub}}(t)$ is the speed in the smooth speed curve of the transition at time $t$;
- $v(t)$ is the speed in the un-smoothed speed curve at time $t$;
- $t_s$ is the starting time of the transition;
- $t_e$ is the ending time of the transition.

$A$, $B$, $C$ are parameters to be determined with the boundary conditions;

$$a_{\text{sub}}(t_s)$$ is the acceleration in smooth curve of the transition at time $t_s$;

$$a(t)$$ is the acceleration in the un-smoothed speed curve at time $t$.

After applying the smoothing technique to the speed curves in Figure 12 and Figure 13, the results are shown in Figure 15 and Figure 16. It can be observed that the smooth causes a little delay. This is because the acceleration during the transition is smaller than the one used before. Therefore, the running distance in the same time period is shorter. This means that the following train has to take longer time to reach the next station. The total running time becomes 304s and energy consumption 28.4 kwh with significant ride comfort improvement.

**VI. CONCLUSION**

This paper discusses energy saving tracking control for successive trains in Moving Block Signalling system (MBS). A new strategy is proposed for speed trajectory design and smoothing technique is used to improve the ride comfort. Since the model is simple to solve, the new strategy could be implemented online for Automatic Train Operation system (ATO). Simulation shows the feasibility and effectiveness of the new strategy on energy saving. Compared with traditional strategy, the new strategy has the same effect on energy saving when the leading train has no interruption. More importantly, the new strategy can significantly reduce energy consumption with the cost a little bit time delay when the leading train has a running interruption. At the same time, ride comfort can be guaranteed with a little bit more cost on run-time and energy consumption as expected.

**REFERENCES**


