Stochastic Finite-Time Stabilization for a Class of Nonlinear Markovian Jump Stochastic Systems With Impulsive Effects

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This paper is dedicated to the study of stochastic finite-time stability (SFTS) and control synthesis for a class of nonlinear Markovian jump stochastic systems with impulsive effects. By introducing a time-varying stochastic Lyapunov function with discontinuities at impulse times, an improved criterion for SFTS is derived in terms of linear matrix inequalities (LMIs). Based on the new SFTS criterion, four kinds of finite-time hybrid/continuous-time state feedback controllers are constructed by using the solutions to certain sets of LMIs. The effectiveness of the proposed method is validated through one numerical example. [DOI: 10.1115/1.4028874]

1 Introduction

During the last decades, Markovian jump systems (MJSs) have received considerable attention from the control and mathematics communities, since they are very appropriate to model dynamical systems whose structures are subject to random variation, such as random component failures, sudden environment disturbance, and changes of the interconnections between subsystems. In MJSs, the random jump modes are described by finite-state Markov processes. When the mode is fixed, the evolution of the state is governed by a differential (difference) equation. The problem of stability analysis and controller synthesis of MJSs has been investigated in a number of papers (see, for instance, [1-6], and the references therein). When the jumping process is not accessible, the mode-independent stabilization and estimation problem has been studied in Refs. [7,8].

On the other hand, the impulsive phenomena exist in many real systems in which the states experience sudden changes at certain moments. From the viewpoint of control theory, the impulses can be classified into two types, according to the effect it causes on the stability property: destabilizing impulses and stabilizing impulses. The former can suppress the stability, while the latter may enhance the stability. It is worth mentioning that additional appropriate impulsive feedbacks in the continuous control can improve convergence rate and system performance. So it is of importance to study the effects of impulses on system stability. Recently, some important results have appeared on stability and stabilization of MJSs with impulsive effects. In Ref. [9], Wu and Sun established some criteria on p-moment stability of nonlinear MJSs with impulsive effects based on Lyapunov functions. The authors in Ref. [10] investigated the stabilization problem of discrete-time linear MJSs using time delay and impulsive controllers. A hybrid control scheme for continuous-time linear MJSs was proposed in Ref. [11]. The works of Refs. [9,10] deals with the asymptotic behavior of a MJS over an infinite time-interval. But in some cases, it is useful to consider the bounds of system trajectories over a finite time-interval. For this purpose, we can use the concept of finite-time stability (FTS) to characterize the desired bound properties of system responses [12]. The problems of FTS and finite-time stabilization for state-dependent impulsive systems were investigated in Refs. [13,14]. Very recently, stochastic finite-time stability (SFTS) for a class of nonlinear MJSs with impulsive effects was developed in Ref. [15], and a criterion for SFTS in terms of LMIs was derived therein. We note that the stability analysis in Refs. [11,15] was conducted by using time-invariant stochastic Lyapunov functions. However, impulsive systems are a class of hybrid systems whose states are discontinuous at impulse instants. Applying time-invariant stochastic Lyapunov function method may neglect the discontinuous dynamical characteristics of impulsive MJSs and the resulting stability criteria are usually conservative. In fact, the stability criteria derived in Refs. [11,15] require both continuous and discrete dynamics to be stable. Such stability criteria are not suitable for synthesis of impulsive/hybrid controllers. Therefore, how to construct an appropriate Lyapunov function such that the dynamical characteristics of the studied impulsive MJS can be fully utilized is crucial for deriving a solution to various control problems. This has motivated the present research.

In this paper, a time-varying stochastic Lyapunov function with discontinuities at the impulse times is introduced for dealing with SFTS of impulsive MJSs. The merit of this Lyapunov function lies in that it can efficiently capture the discontinuous dynamical characteristics of impulsive MJSs and thus leads to a less conservative stability criterion. Based on the new SFTS criterion, four types of finite-time state feedback controllers are constructed by using the solutions to certain sets of LMIs. We note the synthesis problem is not considered in Refs. [11,15].

2 Problem Formulation

Given a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}$, let $w(t) = (w_1(t), \ldots, w_m(t)) \in \mathbb{R}^m$ be an $m$-dimensional Brownian motion defined on the probability space. $E[\cdot]$ denotes the expectation operator with respect to the given probability measure $P$. Let $\{r_i, t \geq 0\}$ be a right-continuous Markov process defined on the probability space which takes values in the finite set $\mathcal{M} = \{1, \ldots, N\}$ with generator $\Pi = (\pi_{ij}), (i, j) \in \mathcal{M}$, given by

$$P\{r_{i+1} = j | r_i = i\} = \begin{cases} \pi_{0j} + o(\delta), & i \neq j, \\ 1 + \pi_{ij} + o(\delta), & i = j \end{cases}$$

where $\delta > 0$, $\lim_{\delta \to 0} \frac{o(\delta)}{\delta} = 0$, and $\pi_{ij} \geq 0$ for $i \neq j$, $\pi_{ii} \leq 0$ with

$$\sum_{j=1, j \neq i}^{\infty} \pi_{ij} = -\pi_{ii} \quad \text{for each} \quad i \in \mathcal{M}.$$ 

Consider the following class of uncertain Markovian jump stochastic systems with impulsive effects:

$$\begin{align*}
\dot{x}(t) &= [A(t, r_t) x(t) + f(t, x(t), r_t) + B_1(t, r_t) u_1(t)] dt \\
&+ \sum_{k=1}^{\infty} W_k(t) r(t) dw_k(t), \quad t \neq t_k, \\
\Delta x(t_k) &= E_k x(t_k^-) + B_{2k} u_2(t_k^-), \quad t = t_k, \\
x(0) &= x_0, \quad r_0 = r_0, \quad k \in \mathbb{N} \ni \{1, 2, \ldots\}
\end{align*}$$

(1)

The following class of uncertain Markovian jump stochastic systems with impulsive effects:
where \( x(t) \in \mathbb{R}^n \) is the state, \( u_i(t) \in \mathbb{R}^{p_i} \) is the continuous control input, \( w_i(t) \in \mathbb{R}^{r_i} \) is the impulsive control input, \( W_i(t) \in \mathbb{R}^{m_i \times r_i} \), \( l = 1, 2, \ldots, m \), are known constant matrices, \( \Delta x(t) = x(t^+) - x(t^-) \) is the impulse at instant \( t_k \), where \( \lim_{t \to t_k^-} x(t_k - h) = x(t_k)^+ \),

\[
\lim_{t \to t_k^+} x(t_k + h) = x(t_k^-), \quad \text{and} \quad \{t_k \} \in S(\sigma_1, \sigma_2) \triangleq \{t_k : t_0 = 0, \sigma_1 \leq t_k - t_{k-1} \leq \sigma_2, k \in \mathbb{N}\}.
\]

Moreover, we assume that \( x(t_k) = x(t_k^-), k \in \mathbb{N} \).

For each \( i \in \mathcal{M} \), \( A(t, i) = A(i) + \Delta A(t, i) \), \( B(t, i) = B(i) + \Delta B(t, i) \), where \( A(i) \in \mathbb{R}^{n \times r_i} \) and \( B(i) \in \mathbb{R}^{n \times p_i} \) are known matrices, \( \Delta A(t, i) \) and \( \Delta B(t, i) \) are unknown matrices denoting the norm bounded parameter uncertainties in the system. For each \( k \in \mathbb{N} \), \( E_k = E + \Delta E_k \), \( B_{2k} = B_1 + \Delta B_{2k} \), where \( E \in \mathbb{R}^{r} \) and \( B_1, B_2 \in \mathbb{R}^{n \times p_i} \) are known constant matrices, and \( \Delta E_k \) and \( \Delta B_{2k} \) are uncertain matrices. The admissible parameter uncertainties are assumed to be of the form

\[
[\Delta A(t, i) \ \Delta B(t, i)] = D_k(i) F(t, i) [N_1(i) \ N_0(i)],
\]

where \( D_k(i), N_1(i), N_0(i), D_0, N_2 \), and \( N_{02} \) are known real constant matrices with appropriate dimensions, \( F(t, i) \), with unknown Lebesgue measurable matrix functions satisfying \( F(t, i)F(t, i)^T \leq I \), and \( F(t, i) \) are unknown matrices satisfying \( F(t, i)F(t, i)^T \leq I \). Throughout the paper, we will assume that the functions \( f(t, x, i) \), with appropriate dimension such that \( \|f(t, x, i)\| \leq \|G_i(t, x)\| \), \( \forall t \in [t_0, t_0 + T] \).

**Definition 1.** Given three positive scalars \( c_1, c_2, T \), with \( c_1 < c_2 \), positive definite matrices \( R(i), i \in \mathcal{M} \), and positive definite matrix-valued functions \( \Gamma(t, i) \), defined over \([0, T] \times \mathbb{M} \), system (1) is said to be robustly uniformly SFTS (RUSFTS) over \( S \) with respect to \( c_1, c_2, T, R(i), \Gamma(t, i) \), if for any \( t \in [0, T] \), \( E_i \{x(t, i) \mid (x(t_0, i), x(t_k, i), x(t_{k-1}, i)) \in c_1, \forall k \leq T \} \leq c_2 \), for all admissible uncertainties satisfying (2).

The following lemma will be used to prove our main results.

**Lemma 1.** For matrices \( A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{n \times m}, \Xi \in \mathbb{R}^{n \times n}, \Xi_0 \in \mathbb{R}^{n \times N}, \Xi_1 = \Xi_0', X_0, X_i, H_i \in \mathbb{R}^{n \times n} \), \( i = 1, 2, \ldots, p \), if they satisfy the following inequalities for all \( t \in \{1, \ldots, n\} \):

\[
\Xi + B X_0 A + (B X_0 A)^T ((X_i - X_0)^T + B H_i) - H_i H_i^T < 0
\]

then it holds that

\[
\Xi + B X_0 A + (B X_0 A)^T < 0, \quad i = 1, \ldots, p
\]

**Proof.** Multiplying Eq. (3) to the left by \( [I_b \ B] \) and to the right by its transpose yields Eq. (4).

3 Robust Stochastic Finite-Time Stability

In this section, we will establish a new criterion for SFTS of system (1) with \( u_i(t) = 0, i = 1, 2 \). For this purpose, we introduce two piecewise linear functions \( \rho_{11}, \rho_0 : [0, \infty) \rightarrow \mathbb{R}^+ \)

\[
\rho_{11}(t) = \frac{t - t_{k-1}}{t_k - t_{k-1}}, \quad \rho_0(t) = \frac{1}{t_k - t_{k-1}}, \quad t \in \{t_k, t_{k-1}\}, \quad k \in \mathbb{N}
\]

It is easy to see that there exists a function \( \rho_2(t) : [0, \infty) \rightarrow \mathbb{R}^+ \) such that

\[
\rho_0(t) = (1/\sigma_1) \rho_2(t) \quad \text{and} \quad (1/\sigma_2) \rho_2(t)
\]

Set \( \rho_2(t) = 1 - \rho_0(t), i = 1, 2 \).

**Theorem 1.** Consider the impulsive system (1) with \( u_i(t) = 0, i = 1, 2 \). Given three positive scalars \( c_1, c_2, T \), with \( c_1 < c_2 \), positive definite matrices \( R(i), i \in \mathcal{M} \), and positive definite matrix valued functions \( \Gamma(t, i), \) defined over \([0, T] \times \mathbb{M} \).

\[
\Xi_0(i) = \left[ \begin{array}{ccc} \Omega_0(i) & L_{2M}(i) & W^T(i)(I_M \otimes P_{1h}(i)) & L_{2M}(i) \\ \Phi_{1M}(i) & 0 & 0 \\ * & * & -I_M \otimes P_{1h}(i) & 0 \\ * & * & * & \Phi_{2M}(i) \end{array} \right] < 0
\]

where

\[
\Xi_0(i) = \left[ \begin{array}{cccc} -\mu P_1(i) & (I + E)^T P_2(i) & 0 & \varepsilon_1(i) N_2^T(i) \\ -P_2(i) & P_2(i) D_2 & 0 & -\varepsilon_2(i) I \\ * & * & * & 0 \\ * & * & * & * \end{array} \right] \leq 0
\]

\[
P_2(i) \leq \delta R(i)
\]

\[
(c_1/c_2) Z_i^T(i, \Gamma(i)) \leq \phi(i)(\rho_{11}(t)P_{1h}(i) + \rho_{12}(t)P_{2h}(i))
\]

Using the above notations, it can be obtained from condition (6) that

\[
\sum_{i=1}^{2} \rho_2(t) \rho_{1h}(t) \Xi_0(i) \leq \left[ \begin{array}{ccc} \Omega(i, t) & L_1(t, i) & W^T(i)(I_h \otimes P(t, i)) \\ \Phi_1(i, t) & 0 & 0 \\ * & * & -I_h \otimes P(t, i) \end{array} \right] < 0
\]

\[
\Xi_0(i) = \left[ \begin{array}{cccc} -\mu P_1(i) & 0 & 0 & \varepsilon_1(i) N_2^T(i) \\ 0 & P_2(i) D_2 & 0 & -\varepsilon_2(i) I \\ * & * & * & 0 \\ * & * & * & * \end{array} \right] \leq 0
\]

\[
-c_1(c_2/c_1) Z_i^T(i, \Gamma(i)) \leq \phi(i)(\rho_{11}(t)P_{1h}(i) + \rho_{12}(t)P_{2h}(i))
\]

\[
\lim_{t \to t_k^+} x(t_k + h) = x(t_k^-)
\]

\[
\lim_{t \to t_k^-} x(t_k - h) = x(t_k^+), \quad \text{and} \quad \{t_k \} \in S(\sigma_1, \sigma_2) \triangleq \{t_k : t_0 = 0, \sigma_1 \leq t_k - t_{k-1} \leq \sigma_2, k \in \mathbb{N}\}.
\]
Define a discontinuous stochastic Lyapunov function candidate
\[
V(t,x,i) = \phi(i)x^T(t)P(t,i)x(t)
\]
for \(t \in (t_0, t_{k+1})\), at the point \((t,x(t),r_i)\), the weak infinitesimal generator \(\mathcal{L}V\) along the trajectories of system (1) is given by
\[
\mathcal{L}V(t,x(i),i) = \gamma V(t,x(i),i) + \phi(t)x^T(t)\left(-\gamma + (\ln \mu)\rho_0(i)\right)P(t,i)
+ \rho_0(t)(P_1(i) - P_2(i)) + P(t,i)A(t,i) + A^T(t,i)P(t,i)
+ \sum_{j=1}^{N}W^j(t,i)W_j + \sum_{j=1}^{N}n_0P(t,j)
\]
\[
+ 2\chi(t)P(t,i)f(t,x(i),i).
\]

Applying Lemma 1 of Ref. [2] and (A1), we obtain
\[
P(t,i)\Delta A(t,i) + \Delta A^T(t,i)P(t,i)
\]
\[
\leq \varepsilon_1^2(t)P(t,i)D_1(t,i)D_1^T(t,i)P(t,i) + \varepsilon_1(t)N_1^T(t,i)N_1(t,i),
\]
\[
0 \leq \chi(t)\phi(t)^2 x^T(t)G_2(t,i) \chi(t)
\]
\[
- \chi(t)\phi(t)f(t,x(t),i),
\]
\[
t \in [t_0, t_0 + T]
\]

Applying the above two inequalities into Eq. (12) yields
\[
\mathcal{L}V(t,x(i),i) \leq \gamma V(t,x(i),i) + \eta^T(t,i)\Xi_1(i)\eta(t,i),
\]
\[
t \in (t_k, t_{k+1}) \cap [t_0, t_0 + T]
\]

Then, by Eq. (11), we obtain
\[
\mathcal{L}V(t,x(i),r_i) < \gamma V(t,x(i),r_i),
\]
\[
t \in (t_k, t_{k+1}) \cap [t_0, t_0 + T]
\]

In view of Lemma 3 of Ref. [15], it follows that:
\[
\mathbb{E}\{V(t,x(r_i),r_i)\} < e^{\gamma(t-s)}\mathbb{E}\{V(t_k,x(t_k),r_k)\},
\]
\[
t \in (t_k, t_{k+1}) \cap [t_0, t_0 + T]
\]

On the other hand, applying Schur complement to Eq. (7) gives
\[
-\mu_2(t) + \varepsilon_2(t)N_2^T N_2 + (I + \mu)\hat{P}^{-1}(i)(I + \mu) < 0,
\]
\[
i \in \mathcal{M}
\]

where \(\hat{P}(i) = P_2^{-1}(i) - \varepsilon_1^2(i)D_1D_1^T\).

Combining Eqs. (13) and (15) together yields
\[
\mathbb{E}\{V(t,x(r_i),r_i)\} < e^{\gamma(t-s)}\mathbb{E}\{V(t_{k_0},x(t_{k_0}),r_{k_0})\}
\]
\[
\leq e^{-\gamma T}x^{T}_{k_0}P_{2}X_{k_0}x_{k_0},
\]
\[
t \in [t_0, t_0 + T]
\]

For \(t \in [t_0, t_0 + T]\), consider Eqs. (8) and (9), we have
\[
x^T_{k_0}P_{2}(r_{k_0})x_{k_0} \leq \beta x^{T}(t)\Gamma(t,r_{i})x(t)
\]
\[
\leq V(t,x(i),r_{i})
\]

Substituting the above two inequalities into Eq. (16) yields
\[
\mathbb{E}\{x^T(t)\Gamma(t,r_{i})x(t)\} < \frac{c_1}{c_2}x^T_{k_0}R(r_{k_0})x_{k_0},
\]
\[
t \in [t_0, t_0 + T]
\]

In light of the above inequality, we can conclude that for any \(t \in [0, T]\)
\[
\mathbb{E}\{x^T(t)\Gamma(t,r_{i})x(t)\} < c_2
\]

This proves that system (1) with \(u_d(t) = 0\), \(i = 1, 2\), is RUSFTS over \(S\) with respect to \(\{c_1, c_2, T, R(i), \Gamma(t,i)\}\).

When \(\Gamma(t) = \Gamma_1(t) = \rho_1(i)R_1(i) + \rho_2(i)R_2(i) + R_0(i) > 0\), \(h = 1, 2, i \in \mathcal{M}\), from Theorem 1, we easily derive the following corollary.

**Corollary 1.** Consider the impulsive system (1) with \(u_d(t) = 0\), \(i = 1, 2\). Given a parameter set
\[
S_p = \{c_1, c_2, T, R(i), \Gamma_1(i)\}
\]

where \(c_1, c_2, T\) are positive scalars, and \(c_1 < c_2\). If for prescribed scalars \(\gamma > \mu > 0\), there exist matrices \(P_2(i) > 0\), positive scalars \(z_{\alpha}(i), z_{\beta}(i), e_{\alpha}(i), e_{\beta}(i), h, l = 1, 2, i \in \mathcal{M}\), and \(\delta\), such that the LMs (6)–(8) and the following LMs (18) are satisfied for all \(h, l \in \{1, 2\}\) and \(i \in \mathcal{M}\):
\[
\max\{\frac{1}{h}, \frac{\mu_1}{1 - \mu_1}\}\varepsilon_{\alpha}(i)\delta_{\beta}(i) \leq \varepsilon_{\beta}(i)\delta_{\alpha}(i) \leq \varepsilon_{\alpha}(i)\delta_{\beta}(i)\]
\[
\text{then the system is RUSFTS over } S(\alpha_1, \alpha_2) \text{ with respect to set } S_p.
\]

4 Robust Stochastic Finite-Time Stabilization

This section will be devoted to designing a mode-dependent hybrid state feedback controller (MDHSC) of the following form:
\[
u_k(t) = K_1(r_i)x(t), \quad u_2(t_k) = K_2x(t_k), \quad k \in \mathbb{N}
\]

such that the closed-loop system obtained by the connection of (1) and (19) is RUSFTS over \(S(\alpha_1, \alpha_2)\) with respect to the parameter set \(S_p\) defined in Eq. (17).

First, we present the following lemma, which is useful for the design of the hybrid state feedback controller (19).

**Lemma 2.** Consider the closed-loop system consisting of impulsive system (1) and the MDHSC (19). Given a parameter set \(S_p\) defined in Eq. (17). If for prescribed scalars \(\gamma > \mu > 0\), there exist \(n \times n\) matrices \(\alpha_2(i) > 0, H_1(i), H_2(i), \alpha_1(i), \beta_1(i), \beta_2(i), h, l = 1, 2, i \in \mathcal{M}\), and \(\delta\), such that (20)–(21), and the following LMs are satisfied for all \(h, l \in \{1, 2\}\) and \(i \in \mathcal{M}\):
\[
\Xi_{\alpha}(i) + B_1(i)X_{\alpha}(i)I_1 + I_1^T X_{\alpha}(i)B_1^T(i) \Psi_{\alpha}(i) - H_1(i) - H_1^T(i) > 0
\]
\[
\Xi_{\beta}(i) + B_2(i)X_{\beta}(i)I_2 + I_2^T X_{\beta}(i)B_2^T(i) \Psi_{\beta}(i) - H_2(i) - H_2^T(i) > 0
\]
\[
\delta R^{-1}(i) \leq X(i)
\]
\[
X(i) \leq \min\{\mu_1, \frac{c_1}{c_2}\} e^{-\gamma T}\delta R^{-1}(i)
\]
where $\mathbf{I}_1 = [I\ 0\ 0\ 0]$ and $\mathbf{I}_2 = [I\ 0\ 0\ 0]$.

$$\hat{\mathbf{\xi}}_{10}(i) = \begin{bmatrix} \mathbf{\Omega}_0(i) & X_0(i) & X_0(i)^{W^T} & X_0(i)\mathbf{N}_1^T(i) \\ * & -\mathbf{z}_0(i)I & 0 & 0 \\ * & * & -L_0 \otimes X_0(i) & 0 \\ * & * & * & -\hat{e}_0(i)I \end{bmatrix},$$

$$\hat{\mathbf{\xi}}_{20}(i) = \begin{bmatrix} \mathbf{\varepsilon}_1(i) & X_1(i) & X_1(i)^{W^T} & X_1(i)\mathbf{N}_2^T(i) \\ * & -X_2(i) + \hat{e}_2(i)D_1D_2^T & 0 & 0 \\ * & * & * & -\hat{e}_2(i)I \end{bmatrix}.$$ 

$\mathbf{\Omega}_0(i) = (-\gamma + \ln\mu/\sigma)X_0(i) + \mathbf{A}(i)X_0(i) + X_0(i)\mathbf{A}^T(i) + \mathbf{z}_0(i)I$

$+ \hat{e}_0(i)D_1D_1^T + \sum_{i=1}^{\mathbf{N}} \pi_{0\mathbf{x}_0}(i)X_0(i)^T \mathbf{x}_0(i)\mathbf{K}_1(i).$

$\mathbf{B}_1^T(i) = \begin{bmatrix} (\mathbf{x}_0(i) - \mathbf{K}_1(i)) \mathbf{^T} + \mathbf{B}_1(i)H_1(i), \mathbf{B}_2(i) = ((\mathbf{x}_1(i) - \mathbf{X}_1(i)\mathbf{L}_2(i)) + \mathbf{B}_2H_2(i) \end{bmatrix}$

$N_1(i) = N_0(i) + N_0(i)K_1(i),$

$B_1^T(i) = \begin{bmatrix} (\mathbf{B}_1(i)K_1(i))^T & 0 & 0 & (N_0(i)K_1(i))^T \end{bmatrix},$

$B_2^T(i) = \begin{bmatrix} 0 & (\mathbf{B}_2K_2(i))^T & (N_0(i)K_2(i))^T \end{bmatrix}.$

$\Psi_{10}(i) = ((\mathbf{x}_0(i) \mathbf{X}_0(i)) \mathbf{^T} + \mathbf{B}_1(i)H_1(i),$

$\Psi_{20}(i) = ((\mathbf{x}_1(i) - \mathbf{X}_1(i)\mathbf{L}_2(i)) + \mathbf{B}_2H_2(i)$

then the closed-loop system is RUSFTS over $S(\sigma_1, \sigma_2)$ with respect to $S_p$. 

**Proof.** For the purpose of illustration, we use the notation (6) and (7) to denote the ones corresponding to Eqs. (6) and (7), respectively, in which $\mathbf{A}(i)$ is replaced by $\mathbf{A}(i) + \mathbf{B}(i)\mathbf{K}_1(i)$ and $N_0(i)$ is replaced by $N_0(i) + N_0(i)\mathbf{K}_1(i).$ In light of Corollary 1, a sufficient condition for RUSFTS of the closed-loop system over $S(\sigma_1, \sigma_2)$ with respect to $S_p$ is that for prescribed positive scalars $\gamma$ and $\mu > 0$, there exist matrices $P(i) > 0$, positive scalars $\mathbf{z}_0(i), \mathbf{e}_0(i), \mathbf{e}_1(i), \mathbf{e}_2(i)$, $h, l = 1, 2$, $i \in \mathcal{M}$, and $\delta$, such that the set of matrix inequalities (8), (18), (6), and (7) are satisfied for all $h, l \in [1, 2]$ and $i \in \mathcal{M}$. 

By means of Lemma 5, the matrix inequalities (20) and (21) imply the matrix inequalities

$$\hat{\mathbf{\xi}}_{10}(i) + \mathbf{B}_1(i)X_0(i)\mathbf{I}_1 + \mathbf{I}_1^T X_0(i) \mathbf{B}_1^T(i) < 0 \quad (24)$$

and

$$\hat{\mathbf{\xi}}_{20}(i) + \mathbf{B}_2X_1(i)\mathbf{I}_1 + \mathbf{I}_2^T X_1(i) \mathbf{B}_1^T(i) < 0 \quad (25)$$

respectively, where $h, l = 1, 2$, $i \in \mathcal{M}$. Let $P_0(i) = X_0^{-1}(i), \mathbf{z}_0(i) = \mathbf{e}_0(i), \mathbf{e}_1(i) = \mathbf{e}_2(i), \mathbf{e}_2(i) = \mathbf{e}_2(i), h, l = 1, 2, i \in \mathcal{M}. \delta = \delta^{-1}$. Premultiplying and postmultiplying the both sides of Eq. (24) by $\mathbf{diag}(P_0(i), \mathbf{z}_0(i), \mathbf{I}_0 \otimes \mathbf{P}_0(i), \hat{e}_0(i), \hat{e}_1(i))$, and using Schur complement, it is easy to find that LMI (24) are equivalent to (6). 

Similarly, one can prove that Eqs. (22), (23), and (7) are equivalent to Eqs. (8), (18), and (7), respectively. Thus, the conditions of Corollary 1 are satisfied. This completes the proof. 

Using Lemma 5, we can obtain a sufficient condition for the design of an MDHFS which makes the resulting closed-loop system RUSFTS.

**Theorem 2.** Consider impulsive system (1). Given a parameter set $S_p$ defined in Eq. (17). If for prescribed scalars $\gamma, \mu > 0$, and $\sigma_1(i) > 0, i \in \mathcal{M}$, there exist $n \times n$ matrices $X_0(i) > 0, X_0(i), i \in \mathcal{M}$, $X_{p0}, p_0 \times n$ matrices $K_1(i), i \in \mathcal{M}$, a $p_0 \times n$ matrix $K_2$, and positive scalars $\mathbf{z}_0(i), \mathbf{e}_0(i), \mathbf{e}_1(i), \mathbf{e}_2(i), h, l = 1, 2, i \in \mathcal{M}$, and $\delta$, such that (22)–(23) and the following LMI are satisfied for all $h, l \in [1, 2]$ and $i \in \mathcal{M}$:

$$\begin{bmatrix} \mathbf{z}_0(i) & \mathbf{\theta}_{11}(i) & \mathbf{I}_1 \end{bmatrix}^T + \begin{bmatrix} \mathbf{\theta}_{20}(i) & \mathbf{\theta}_{21}(i) & \mathbf{I}_2 \end{bmatrix}^T < 0 \quad (26)$$

and

$$\begin{bmatrix} \mathbf{\varepsilon}_0(i) & \mathbf{\theta}_{11}(i) & \mathbf{X}_2(i) \end{bmatrix}^T + \begin{bmatrix} \mathbf{\theta}_{20}(i) & \mathbf{\theta}_{21}(i) & \mathbf{X}_2(i) \end{bmatrix}^T < 0 \quad (27)$$

then there exists an MDHFS (19) with $K_1(i) = \mathbf{K}_1(i)X_0^{-1}(i)$ and $K_2 = K_2X_0^{-1}, i \in \mathcal{M}$, such that the resulting closed-loop system is RUSFTS over $S(\sigma_1, \sigma_2)$ with respect to $S_p$. 

**Proof.** Note that conditions (26)–(28) imply that the matrices $X_0(i), i \in \mathcal{M}$, and $X_{10}$ are nonsingular. Applying the change of variables such that $K_1(i) = \mathbf{K}_1(i)X_0^{-1}(i)$, $K_2 = K_2X_0^{-1}, H_1(i) = \mathbf{K}_1(i)X_0(i)$, and $H_2(i) = \mathbf{K}_1(i)X_{10}(i)$, and using Schur complement, Eqs. (27) and (28) are changed to Eq. (20) with $h = 2$ and (21), respectively. Then, the inequalities $-X_1(i)X_0^{-1}(i)X_0(i) < \hat{e}_0(i)X_1(i) + \hat{e}_0(i)X_{10}(i)$, we obtain (20) with $h = 1$. 

Thus, Theorem 2 follows from Lemma 5.

In the case of $K_2 = 0$, the hybrid state feedback controller (19) reduces to the following mode-dependent continuous-time state feedback controller (MDCTSFC):

$$u_1(i) = K_1(i)x(i), u_2(x_2) = 0, k \in \mathbb{N} \quad (29)$$

Applying the similar technique as used in the proof of Theorem 2, we can obtain the following result.

**Theorem 3.** Consider impulsive system (1). Given a parameter set $S_p$ defined in Eq. (17). If for prescribed scalars $\gamma, \mu > 0, \kappa_1(i) > 0, \sigma_1(i) > 0, i \in \mathcal{M}$, there exist $n \times n$ matrices

with $\mathbf{I}_3 = [I\ 0\ 0\ 0]$.
exists an MIHSFC (31) with $K_i$ of Theorem 2 and so are omitted.

then there exists an MDCTSFC (29) with $K_i(t) = K_i(t)X_0^{(1)}(i)$, $i \in \mathbb{M}$, such that the resulting closed-loop system is RUSFTS over $\mathcal{S}(\sigma_1, \sigma_2)$ with respect to $S_p$.

When the jumping modes are inaccessible, it is of importance to design a mode-independent controller to stabilize the studied system. The mode-independent hybrid state feedback controller (MIHSFC) and the mode-independent continuous-time state feedback controller (MICTSFC) can be represented as the following forms:

$$u_1(t) = K_1x(t), \quad u_2(t_k) = K_2x(t_k^-), \quad k \in \mathbb{N}$$  \hspace{1cm} (31)$$

respectively. The following theorems provide sufficient conditions for the existence of mode-independent control laws for impulsive system (1).

**Theorem 4. Consider impulsive system (1). Given a parameter set $S_p$ defined in Eq. (17). If for prescribed scalars $\gamma, \mu > 0, \nu(i) > 0, \kappa(i) > 0, l = 1, 2, i \in \mathbb{M},$ there exist $n \times n$ matrices $X_0, X_0, a_i \in \mathbb{M}, X_0, a_i \in \mathbb{R}^{n \times n}$ $K_1, a_p \in \mathbb{R}^{n \times n}$ $K_2, a_p \in \mathbb{R}^{n \times n}$ $K_3, a_p \in \mathbb{R}^{n \times n}$ $K_4,$ and positive scalars $\mu_0, \nu(i), \kappa(i), \gamma(i), h, l = 1, 2, i \in \mathbb{M},$ and $\delta$, such that Eqs. (22)–(23), (26)–(28), and the following LMIs are satisfied for all $h, l \in [1, 2]$ and $i \in \mathbb{M}$:

$$\begin{bmatrix} X_0^T(i) & \Theta_1(i)I_1 \\ \ast & \Theta_2(i) \end{bmatrix} < 0$$  \hspace{1cm} (33)$$

$$\begin{bmatrix} X_0^T(i) & \Theta_3(i)I_1 \\ \ast & \Theta_4(i) \end{bmatrix} < 0$$  \hspace{1cm} (34)$$

where $X_0^T(i), h, l = 1, 2, i \in \mathbb{M},$ are derived from $\mathbb{X}_0^T(i)$ in which $X_0$ is replaced by $X_0$ and $K_0$ is replaced by $K_0$, then there exists an MIHSFC (31) with $K_1 = K_1X_0^{(1)}$ and $K_2 = K_2X_0^{(2)}$, such that the closed-loop system obtained by the connection of (1) and (31) is RUSFTS over $\mathcal{S}(\sigma_1, \sigma_2)$ with respect to $S_p$. 

**Theorem 5. Consider impulsive system (1). Given a parameter set $S_p$ defined in (17). If for prescribed scalars $\gamma, \mu > 0, \nu(i) > 0, \kappa(i) > 0, a_i \in \mathbb{M}, X_0, a_i \in \mathbb{R}^{n \times n}$ $K_1, a_p \in \mathbb{R}^{n \times n}$ $K_2, a_p \in \mathbb{R}^{n \times n}$ $K_3, a_p \in \mathbb{R}^{n \times n}$ $K_4,$ and positive scalars $\mu_0, \nu(i), \kappa(i), \gamma(i), h, l = 1, 2, i \in \mathbb{M},$ and $\delta$, such that the LMIs (22)–(23) and (30)–(34) are satisfied for all $h, l \in [1, 2]$ and $i \in \mathbb{M},$ then there exists an MICTSFC (32) with $K_1 = K_1X_0^{(1)}$, such that the closed-loop system obtained by the connection of (1) and (32) is RUSFTS over $\mathcal{S}(\sigma_1, \sigma_2)$ with respect to $S_p$.

The proofs of Theorems 4 and 5 are quite similar to the proof of Theorem 2 and so are omitted.

5 **Numerical Example**

**Example 1. Consider a simple economic system based on Samuelson’s multiplier–accelerator model [16], which can be modeled by Eq. (1) with three modes: $1$ = “Norm,” $2$ = “Boom,” and $3$ = “Slump.” The system data are**

<table>
<thead>
<tr>
<th>$A(1)$</th>
<th>$A(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} 0 &amp; 0 &amp; 0 \ -0.545 &amp; 0.626 &amp; 0 \ 0 &amp; -0.106 &amp; 0.087 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 &amp; 0 &amp; 0 \ -0.545 &amp; 0.626 &amp; 0 \ 0 &amp; -0.106 &amp; 0.087 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

$A(3) = \begin{bmatrix} 3.14 & 0.100 & -0.28 \\ -19.06 & -0.148 & 1.56 \\ 0 & 1 \end{bmatrix}$

$N_0 = 1.0D_1(t), N_0(t) = 0, E = D_2 = N_2 = N_0 = 0, B_1(t) = [0 \quad -0.283 \quad 0.333]^T, B_2(t) = [0 \quad 0.087]^T, B_3(t) = [-0.064 \quad 0.195 \quad -0.080]^T, B_4 = [1 \quad 1 \quad 0]^T$.

Type 1: MDHSFC. Applying Theorem 2 by solving the LMIs (22)–(23) and (26)–(28) with the choice of $\gamma = 0.2, \mu = 1, \nu_1 = 1.5, \nu_2 = 1.04, \nu_3 = 1.15, \kappa_1(1) = 0.22, \kappa_1(2) = 0.13, \kappa_2(1) = 0.05, \kappa_2(2) = 0.65, \kappa_2(3) = 0.87$, it has been found that the minimum value of $c_2$ such that the conditions of Theorem 2 are satisfied is $c_{min} = 0.01$. The corresponding gain matrices of the MDCTSFC (19) are given by

$K_1(1) = [9.2274 \quad -1.7216 \quad -24.5772], K_1(2) = [65.4892 \quad 34.1314 \quad -193.1498], K_1(3) = [143.0652 \quad -90.0237 \quad -63.0270], K_2 = [-0.4159 \quad -0.0965 \quad -0.0297]$.

Type 2: MDCTSFC. Applying Theorem 3 by solving the LMIs (22)–(23) and (26)–(28), and (30) with the choice of $\gamma = 0.69, \mu = 1, \nu_1 = 1.5, \nu_2 = 1.04, \nu_3 = 1.15, \kappa_1(1) = 0.24, \kappa_1(2) = 0.1, \kappa_1(3) = 0.05$, the derived minimum value of $c_0$ is $c_{min} = 15.45$. The corresponding gain matrices of the MDCTSFC (29) are given by $K_1 = [-2.6475 \quad 1.5024 \quad -10.3103], K_1(2) = [-74.2152 \quad 42.6369 \quad -177.9439], K_1(3) = [59.7281 \quad 0.422014.3281]$.

The above calculation results indicate that the hybrid state feedback control achieves better performance than the continuous state feedback control.

6 **Conclusions**

The problems of SFTS and stochastic finite-time stabilization for a class of nonlinear Markovian jump stochastic systems with impulsive effects have been investigated. A discontinuous stochastic Lyapunov function based method has been presented to develop a less conservative SFTS criterion. Based on the newly established SFTS criterion, four types of state feedback controllers that make the corresponding closed-loop system SFTS have been constructed in terms of the solutions to certain sets of LMIs. Finally, a numerical example has demonstrated the effectiveness of the proposed results.

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**References**


