Born inversion of surface-scattered SAR (synthetic aperture radar) wave field

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Received May 24, 1990

A new approach to synthetic aperture radar (SAR) digital image formation, based on inverse scattering theory, is derived as an alternative to the conventional method of imaging surface-scattered wave fields. The conventional image formation technique for high-resolution SAR data utilized azimuth compression using correlation in the range-Doppler domain. More recent approaches in SAR image formation algorithms exploit downward extrapolation of the wave field in the frequency–wavenumber ($f$–$k$) domain to perform not only the azimuth compression but also the range curvature correction at the same time, with improved quality of the final image. In this paper, imaging of the SAR wave field is formulated with the Born inversion approach, which includes a range-curvature-correction term that is valid at all ranges of image formation. This new inversion formula is established to exploit $f$–$k$ domain computation, from which the complex backscattering coefficient, defined by the ratio of the backscattered wave field to the incident wave field, can be accurately estimated from the observed backscattered wave field.

On propose, pour la formation d’images numériques ROS (radar à ouverture synthétique), une nouvelle méthode, basée sur la théorie de diffusion inverse, comme alternative à la méthode conventionnelle utilisée pour imagier les champs d’ondes diffusés par une surface. La technique conventionnelle de formation d’images pour les données ROS à haute résolution a utilisé la compression d’azimut et la correction dans le domaine portée-Doppler. Dans des approches plus récentes, les algorithmes de formation d’images ROS exploitent l’extrapolation vers le bas du champ d’onde dans le domaine $f$–$k$, pour effectuer non seulement la compression d’azimut mais aussi en même temps la correction de courbure de portée, ce qui améliore la qualité de l’image finale. Dans cet article, la formation d’image de champ d’onde ROS est formulée suivant l’approche d’inversion de Born, qui inclut un terme de correction de la courbure de portée et est valide pour toutes les portées de formation d’image. Cette nouvelle formule d’inversion est établie pour exploiter le calcul dans le domaine $f$–$k$, d’où le coefficient complexe de rétrodiffusion, défini par le rapport entre le champ d’onde diffusé vers l’arrière et le champ incident, peut être estimé de façon précise à partir du champ d’onde rétrodiffusé que l’on observe.

[Traduit par la rédaction]

1. Introduction

The Born series and the first-order inversion method, which were originally developed for quantum mechanics and optics, have been successfully applied in several fields of geophysics for reconstruction of true amplitude images. The conventional approaches of processing synthetic aperture radar (SAR) data have mostly been based on the matched-filter theory (1, 2). The azimuth compression in these approaches utilizes a one-dimensional or limited two-dimensional matched filter in the range-Doppler domain (the experimental configuration is explained in Fig. 1) and consequently the processing efficiency and performance depends very much on how the range migration correction is implemented.

Recently several different image-processing methods for the surface-scattered SAR wave field have been proposed (3–5), in which the azimuth compression is performed in the $f$–$k$ (frequency–wavenumber) domain. The Born series and first-order inversion methods (3, 4) are quite different in their basic approach. While the former was initially conceived and developed from the similarities between the reflection seismology and SAR configuration, the latter was developed from the SAR system’s theory. Interesting and common new features of these two algorithms lie in the fact that the range migration correction can be more precisely achieved through phase shifting in the $f$–$k$ domain. Some of the remaining problems include efficient use of Fast Fourier Transform (FFT) and effective $f$–$k$ domain interpolation techniques. In this research, attempts are made to tackle these problems and each formula is set up to resolve these inefficiencies more easily during numerical implementation. Some of the major differences between refs. 3 and 4 and this paper include;

(i) the carrying out in one step in this research of the 2-D image formation, including the $f$–$k$ domain interpolation
(ii) the difference in the treatment of the velocity variation between the origin and the scatterers by the seismic migration which is considered as a perturbation in refs. 3 and 4, while it is constant in this research, and
(iii) the inversion formula for the complex backscattering coefficient to be presented in this paper has never been discussed in any previous work.

The focus of this paper is the application of the Born inversion formula to accurately represent the complex backscattering coefficient. Some of the ambiguity in previous approaches (3, 4) has been the lack of explicit discussion of the estimation of the complex backscattering coefficient, which is very important for certain geophysical parameter estimations. In the following sections, a new SAR data inversion and image formation method is developed step by step from the classical Born inverse scattering theory (6, 7). Then the final inversion formulation for the complex backscattering coefficient, derived in a similar fashion as the seismic Born inversion method (5, 7, 8), is presented.

2. High-frequency backscattered wave field

The Helmholtz equation in frequency domain can be written

\[ \left( \nabla^2 + \frac{\omega^2}{c^2} \right) U(r, r_0, \omega) = -f(r - r_0) \]

where

\[ U(r, r_0, \omega) = \int_{-\infty}^{\infty} U(r, r_0, t) e^{-i\omega t} \, dt \]
represents the total electromagnetic (EM) wave field, \( f(r - r_s) \) is the source function, \( c \) is the velocity of light, and \( r_s \) is the source coordinate. The total wave field \( U(r, r_s; \omega) \) can be decomposed into the incident wave field \( U_i \) radiating from the source and the scattered wave field \( U_s \) due to scattering on the Earth's surface. Then \( U(r, r_s; \omega) \) can be written as

\[ U(r, r_s; \omega) = U_i(r, r_s; \omega) + U_s(r, r_s; \omega) \]

Applying Green's theorem and replacing the total wave field by \([2]\), one has

\[ \int \int \left[ U_s(r, r_s; \omega) \frac{\partial}{\partial n} G_0(r, r_s; \omega) - G_0(r, r_s; \omega) \frac{\partial}{\partial n} U_i(r, r_s; \omega) \right] \, d^2r \]

where \( B \) represents the surface boundary and \( \hat{n} \) is the unit vector normal to the boundary surface as shown in Fig. 1. The Green's function \( G_0(r, r_s; \omega) \) is the same as the source wave field \( U_i(r, r_s; \omega) \) and is given by

\[ G_0(r, r_s; \omega) = \frac{\exp \left\{ -i \omega \frac{|r - r_s|}{c} \right\}}{4\pi |r - r_s|} \]

In these derivations, we assume the zero-offset survey geometry where the receiver location is the same as the source location. Now if surface scattering is dominant and if one can assume that the backscattered wave field is linear with respect to the incident wave field in the vicinity of the surface boundary \( B \), we have

\[ U_s(r, r_s; \omega) = R(r) U_i(r, r_s; \omega), \quad r \text{ on } B \]

which corresponds to the Kirchhoff boundary condition in optics (7, 9).

The complex backscattering coefficient \( R(r) \) represents the ratio of the backscattered wave field to the incident wave field, and consequently implies phase changes as well as a decrease in amplitude of the backscattered wave field from the incident wave field. Equations [5] and [6] imply that other possible sources of the backscattered wave field, for instance the scattered waves themselves in volume-scattering cases, are negligible except the incident wave field.

Substituting [4]–[6] into [3], we have an approximation to the highest order of \( \omega \), given by

\[ U(r, r_s; \omega) = \frac{i \omega}{8\pi^2c} \int \int_B R_s(r) \frac{\exp \left\{ -i \omega r' \right\}}{r'^2} \, d^2r \]

where

\[ \theta(k_s; \omega) \approx \frac{2\omega}{8\pi^2c^2} \int \int_B R_s(r) e^{-ik_s \cdot (i\pi) H_0^2} \left( \frac{\sqrt{y^2 + z_1^2}}{4\frac{\omega^2}{c^2} - k_s^2} \right) \, dx \, dy \]
where $H_0^{(2)}(.)$ is the Hankel function of the second kind (10). Now, change of coordinates from $(y, z_i)$ to slant range coordinate $\eta = \sqrt{y^2 + z_i^2}$ gives

\[
\theta(k_x; \omega) \approx \frac{2\omega}{8\pi^2c^2} \int \int_B R'_\omega(x, \eta) \frac{i\pi \eta}{\sqrt{\eta^2 - \eta_i^2}} H_0^{(2)}(\eta k_\eta) e^{-i\omega x} \, dx \, d\eta
\]

where

\[
k_\eta = \text{sgn}(\omega) \sqrt{\frac{4\omega^2}{c^2} - k_x^2} \quad \text{for} \quad \frac{4\omega^2}{c^2} \gg k_x^2
\]

Since the argument of the Hankel function $(\eta k_\eta)$ is much larger than unity, the Hankel function can be expanded into an asymptotic expansion (11)

\[
H_0^{(2)}(\eta k_\eta) \approx \sqrt{\frac{2}{\pi \eta k_\eta}} \exp \left[ -i\eta k_\eta + i\left(\frac{\pi}{4}\right) \right]
\]

Then we have

\[
\theta(k_x; \omega) \approx \frac{i\omega \exp(\pi/4)}{4\pi c^2} \sqrt{\frac{2}{\pi k_\eta}} \int \int_B R'_\omega(x, \eta) \sqrt{\frac{\eta}{\eta^2 - \eta_i^2}} e^{-i(k_x x + k_\eta \eta)} \, dx \, d\eta
\]

where

\[
\theta(k_x; \omega) = \int_{-\infty}^{\infty} \theta(k_x; \omega) \exp(-ik_\eta \eta) \, dx_0
\]

and

\[
R'_\omega(x, \eta) = \sqrt{\frac{\eta}{\eta^2 - \eta_i^2}} R_\omega(x, \eta)
\]

Since the right-hand side of (12) is a two-dimensional Fourier transform formula, the inversion formula can easily be derived from (12) by two-dimensional inverse Fourier transform.

If we rewrite (12), we have

\[
\theta(k_x; \omega) \approx \frac{i\omega \exp(\pi/4)}{4\pi c^2} \sqrt{\frac{2}{\pi k_\eta}} R'_\omega(k_x, k_\eta)
\]

or equivalently

\[
R'_\omega(k_x, k_\eta) = \frac{4c^2 \sqrt{\pi} \omega \exp(\pi/4) \sqrt{k_\eta}}{\sqrt{2}} \theta(k_x; \omega)
\]

The complex backscattering coefficient $R'_\omega(x, \eta)$ can easily be obtained by taking the 2-D inverse Fourier transform of $R'_\omega(k_x, k_\eta)$ with respect to $k_x$ and $k_\eta$. Before taking the 2-D inverse Fourier transform, however, some special consideration should be given to the processing of the SAR data. There is a time gap of $(2\pi/\omega c)$ between the transmitting time and the first received echo, and this can be compensated for by multiplying $\exp\left[-i2\pi \left(\frac{\omega}{c}\right)\right]$ to $\theta(k_x; \omega)$. From the expression for $k_\eta$ above, the frequency $\omega$ is

\[
\omega = \frac{c}{2} \sqrt{k_x^2 + k_\eta^2}
\]

After multiplying $\exp\left[-i2\pi \left(\frac{\omega}{c}\right)\right]$ to $R'_\omega(k_x, k_\eta)$ and replacing $\omega$ with above expression, the 2-D inverse Fourier transform
Here we notice that the restored image $R'_a(x, \eta)$ does not provide any information for regions less than the minimum range $\eta_0$, and therefore the image is required to be shifted to $R'_a(x, \eta_0 + \eta)$ by multiplying $e^{i\eta_0 k}$ in the $(k_x, k_y)$ domain. Then the final inversion formula becomes

$$
R'_a(x, \eta + \eta_0) = -4\sqrt{2} \sqrt{\frac{\pi}{\lambda}} c e^{i(\pi/4)} \frac{1}{(2\pi)^2} \int \int \frac{\sqrt{k^2}}{\sqrt{k^2 + k^2}} e^{-i\eta_0 \sqrt{k^2 + k^2}} \left( k_x \frac{c}{2} \sqrt{k^2 + k^2} \right) e^{i(k_x x + k_y \eta)} \, dk_x \, dk_y
$$

where $\eta_0$ is the minimum slant range distance. Thus the complex backscattering coefficient of the surface can be estimated.

### 4. Discussion

The SAR data inversion method presented in this paper focuses on the formulation of an explicit inversion formula in the $f-k$ domain. Even though the forward formula is derived from the basic Helmholtz equation, the same, [7], is obtained as in the range-compressed SAR imaging approach except for the spreading factor $1/r^2$. While the spreading factor is excluded from the azimuth impulse response function in most common SAR image processing theories, the inverse-square term is exploited in this method to find an explicit Fourier transform formula of the azimuth impulse response function as mentioned before.

The inverse square of the distance term [7] is inherited from the theory of wave-field propagation, and corresponds to the spreading factor in the radar equation (12). A similar approximation to [7] for the backscattered wave field can also be established through the physical optics approach (1). For a monochromatic wave, the field may be written as

$$
U(r, t) = \text{Re} \left[ U(r) e^{i\omega t} \right]
$$

Harger (1) introduced the Kirchoff–Huygens diffraction integral to the SAR theory, after separation of variables. He then solved the space-dependent term $U(r)$, given by

$$
U(r) = \frac{ik}{4\pi} \int_A \frac{U_0(r) \exp \left\{ -ik |r - r'| \right\}}{|r - r'|} \, dA
$$

where $U_0$ is the spatial distribution of wave field over the aperture $A$, and $k = \omega/c$. The negative sign in the exponent is used instead of the positive sign in the original equation (equation [3.1] of ref. 1), because the sign depends on the sign of the exponent of the Green’s function. Harger (1) then followed the Fraunhofer approximation to obtain an analytical expression for the received signal. One can directly replace $U_0(r')$ by $G_d(r; \omega) \Psi(r, \ldots)$ where $\Psi(r, \ldots)$ denotes the reflectivity density of Harger (1) instead of the Fraunhofer approximation, since the backscattered wave field near the surface can be represented by a linear approximation equation (3.4) of ref. 1. Then, the resulting spatially dependent term $U(r)$ can be written as

$$
U(r) = \frac{i\omega}{(4\pi)^2 c} \int_A \frac{\Psi(r; \ldots) \exp \left\{ -i2\omega c |r - r'| \right\}}{|r - r'|^2} \, dA
$$

Comparing [17] with [7], one can easily see that the spatial dependency of the physical optics approach can be similarly derived using the first Born approximation as demonstrated in this paper. The backscattering coefficient $R'_a(r)$ corresponds to the reflectivity density $\Psi(r, \ldots)$ in ref. 1.

### 5. Conclusion

This paper shows that the SAR image formation can be achieved by the inverse scattering approach of Born (6) and that the complex backscattering coefficient can be accurately estimated. The new approach to the SAR image formation method proposed is similar to that of two previous works, refs. 3 and 4, but is analytically more compact and is expected to be more efficient in numerical implementation (13). Another advantage is the first-order range curvature correction term that is valid at all ranges of image formation (13). However, the most important result of this work is the new explicit inversion formula for the complex backscattering coefficient, which provides an important geophysical parameter for the integrated imaging of multiple sensor geophysical data sets (14). The estimated backscattering coefficient using the Born inversion formula derived in this paper is, however, valid only for the surface-scattering cases and not for the volume-scattering problems.

### Acknowledgements

This research is funded by the Natural Sciences and Engineering Research Council of Canada under operating grant No. A-7400. The author also gratefully acknowledges the encouraging and fruitful discussions with Dr. R. Keith Raney during the early stages of this research. Constructive criticisms of the anonymous reviewer is greatly appreciated. Numerical testing of the result is being carried out by J. S. Won.