Targeted Advertising Strategies on Television*

Technical Appendix: Proofs of Lemmas and Propositions

By

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Lemma 1

(i) Factoring the cost of improving the levels of recognition and accuracy, we obtain from (11) the following net profit function:

$$N\Pi_i = \tilde{a} \left[ R_i R_2 \left( \frac{1}{2} + \frac{p_j - p_i}{2t} \right) p_i + R_i \left( 1 - R_i \right) p_i - \frac{R_c}{A_i} \right] - TC(R_i, A_i) \quad i, j = 1, 2; i \neq j$$  \hspace{1cm} (A.1)

Differentiating (A.1) with respect to $p_i$ yields the following first order condition:

$$\frac{\partial N\Pi_i}{\partial p_i} = \tilde{a} \left[ R_i R_2 \left( \frac{1}{2} + \frac{p_j - 2p_i}{2t} \right) + R_i \left( 1 - R_i \right) \right] = 0 \quad i, j = 1, 2; i \neq j$$  \hspace{1cm} (A.2)

Solving for $p_1$ and $p_2$ yields the result stated in (12). Second order conditions hold as well, because $\partial^2 \Pi_i / \partial p_i^2 < 0$.

(ii) From (A.2) it follows that:

$$R_i R_2 \frac{p_j}{2t} = R_i R_2 \left( \frac{1}{2} + \frac{p_j - p_i}{2t} \right) + R_i \left( 1 - R_i \right).$$

Substituting into (A.1) yields:

$$N\Pi_i = \tilde{a} \left[ R_i R_2 \frac{p_i^2}{2t} - \frac{R_c}{A_i} \right] - TC(R_i, A_i).$$

Substituting for $p_i$ from (12) yields the result stated in (13).

Q.E.D.

The Effect of Direct Surveying of the Population on Recognition and Accuracy

(i) Surveying a sample of viewers who are predicted to belong to Segment 1.

Suppose an advertiser chooses to verify the exact type of $\gamma$ individuals who are predicted to be in Segment 1. Table A.1 depicts the distribution of the population after such a choice as a function of the error rates $\alpha$ and $\beta$, and the accuracy level $A$ (as expressed in the text).
The entries in the second column above can be explained as follows: A viewer who is predicted to belong to Segment 1 has probability $A$ of belonging, indeed, to this segment and probability $(1 - A)$ of belonging to Segment 0, because, by definition,

$$\text{Prob (actual segment = 1 | predicted segment = 1) = A.}$$

As a result, the new recognition and accuracy levels following the sampling of $y$ viewers become:

$$\hat{R} = \frac{(1 - \alpha)a - Ay}{a - Ay} < 1 - \alpha = R_{old},$$

$$\hat{A} = \frac{(1 - \alpha)a - Ay}{\beta(1 - a) + (1 - \alpha)a - y} = \frac{(1 - \alpha)a - \beta(a - (1 - A)y)}{\beta(1 - a) + (1 - \alpha)a - y} = \frac{(1 - \alpha)a}{\beta(1 - a) + (1 - \alpha)a} = A_{old}$$

The above measures apply to the population of $(1 - y)$ viewers who remain after the sampling. For the $y$ viewers who are surveyed directly, however, recognition and accuracy levels are equal to one. Based upon the survey, the type of each viewer can be determined perfectly, implying that $\tilde{R} = \tilde{A} = 1$ for those sampled. Hence, to obtain the new recognition and accuracy levels that result from the direct sampling, the following weighted averages have to be calculated:

$$R_{new} = (1 - y)\hat{R} + y \cdot 1 = \frac{[(1 - \alpha)a - Ay](1 - y)}{a - Ay} + y$$

$$A_{new} = (1 - y)\hat{A} + y \cdot 1 = A + (1 - A)y > A_{old}$$

While the new level of accuracy is unambiguously higher, the new level of recognition may actually decline as a result of the direct survey, if $\beta < (1 - \alpha)$. 

Table A.1

<table>
<thead>
<tr>
<th>Predicted segment</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual segment</td>
<td>(1 - $\beta$)$(1 - a)$</td>
<td>$\beta(1 - a) - (1 - A)y$</td>
</tr>
<tr>
<td>0</td>
<td>$\alpha a$</td>
<td>$(1 - \alpha)a - Ay$</td>
</tr>
</tbody>
</table>

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Table A.1

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<th>Predicted segment</th>
<th>0</th>
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</tr>
<tr>
<td>0</td>
<td>$\alpha a$</td>
<td>$(1 - \alpha)a - Ay$</td>
</tr>
</tbody>
</table>
(ii) **Surveying a sample of viewers who are predicted to belong to Segment 0.**

Suppose an advertiser chooses to verify the exact type of \( y \) individuals who are predicted to be in Segment 0. Table A.2 depicts the distribution of the population after such a choice as a function of the error rates \( \alpha \) and \( \beta \)

<table>
<thead>
<tr>
<th>Predicted segment</th>
<th>Actual segment</th>
<th>( 0 )</th>
<th>( 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( (1-\beta)(1-a) \left[ 1 - \frac{y}{(1-\beta)(1-a) + \alpha a} \right] )</td>
<td>( \beta(1-a) )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( 0 )</td>
<td>( \alpha a \left[ 1 - \frac{y}{(1-\beta)(1-a) + \alpha a} \right] )</td>
<td>( (1-\alpha)a )</td>
</tr>
</tbody>
</table>

**Table A.2**

Calculating the levels of recognition and accuracy that are applicable to the remaining population of size \( (1-y) \) we obtain:

\[
\hat{R} = \frac{(1-\alpha)a}{a - \frac{\alpha ay}{(1-\beta)(1-a) + \alpha a}} > (1-\alpha) = R_{old}
\]

\[
\hat{A} = \frac{(1-\alpha)a}{\beta(1-a) + (1-\alpha)a} = A_{old}.
\]

Calculating the weighted averages that incorporate also the perfect recognition and accuracy levels applicable to the \( y \) viewers who are surveyed directly, we obtain:

\[
R_{new} = \frac{(1-\alpha)a(1-y)}{a - \frac{\alpha ay}{(1-\beta)(1-a) + \alpha a}} + y > R_{old}
\]

\[
A_{new} = A_{old}(1-y) + y > A_{old}.
\]

Hence both recognition and accuracy levels increase when the \( y \) individuals are sampled from the population who is predicted to belong to Segment 0.
Proof of Lemma 2

(i) Because each advertiser's objective (11) is a concave function, we are guaranteed that there is a unique local solution to its maximization. The solution is expressed in (12). To guarantee that the market is fully covered at the asserted equilibrium, it should be that the consumer who is indifferent between the two advertisers (located at distance

\[ x = \frac{1}{2} + \frac{1}{3} \left[ \frac{(1-R_i)}{R_i} - \frac{(1-R_j)}{R_j} \right] \]

from advertiser \( i \) ) obtains a non negative payoff. Hence,

\[ v = \frac{t}{2} - \frac{t}{3} \left[ \frac{(1-R_i)}{R_i} - \frac{(1-R_j)}{R_j} \right] - p_i \geq 0. \]

Using the expression derived for \( p_i \) in (12) yields:

\[ \frac{v}{t} \geq \frac{1}{R_i} + \frac{1}{R_j} - \frac{1}{2}. \]

To guarantee that the above corresponds, indeed, to an equilibrium in the pricing sub-game, no firm should have an incentive to deviate. Small deviations are definitely unprofitable by first and second order conditions. It is large deviations that may destroy the existence of a pure strategy equilibrium in prices, as pointed out in d'Aspremont et al. (1979) and Gal-Or (1982). In particular, one has to consider the possibility that no firm finds it optimal to undercut the price of its competitor by an amount \( 't' \) in order to steal its entire customer base. By undercutting the competitor's price to such an extent, even the consumer who is located the farthest away from the deviating firm will buy its product as long as she is familiar with it (with probability \( \tilde{a}R_j \) for advertiser \( j \)). If firm \( j \) considers such a deviation its profits are:

\[ v_j^D = \tilde{a}R_j (p_i - t) - \frac{\tilde{a}R_j c}{A_j} - TC(R_j, A_j) = \tilde{a}R_j t \left[ \frac{2}{3} (\frac{1-R_i}{R_i}) + \frac{4}{3} (\frac{1-R_j}{R_j}) \right] - \frac{\tilde{a}R_j c}{A_j} - TC(R_j, A_j) \]

A comparison of the above expected profits with the expression obtained in (13) yields:

\[ v_j - v_j^D = \frac{\tilde{a}t \left[ 2(R_i - R_j) - 3R_i R_j \right]^2}{18 R_i R_j} \geq 0. \]

Hence, it is never worthwhile for \( j \) to consider the above mentioned large deviation.
The second type of large deviation to consider is monopoly pricing. The optimal price a monopolist would set in this market is \( \max\left\{ \frac{v}{2}, v-t \right\} \). To guarantee that a monopolist wishes to serve for the entire market, implying that the monopoly price is \( v-t \). We allow the deviating firm to consider a richer family of possible deviations. Specifically, the deviating firm charges the price \( v-wt \), where \( 0 < w \leq 1 \). With such a deviation, the market share of the deviating firm (firm \( j \)) is

\[
\max\left\{ 0, \frac{1}{3R_i} + \frac{2v}{3R_j} - \frac{v}{2t} + \frac{w}{2} \right\}
\]

of the group of consumers who are informed of both products (a proportion \( R_iR_j\alpha \) of the population) and the segment \( w \) of consumers who are only informed of product \( j \) (a proportion \( R_j(1-R_i)\alpha \) of the population.) Two cases may arise: one in which firm \( j \) gets a positive share of the segment of consumers who are fully informed, and the second is when he gets no share of this segment of consumers.

The former arises when \( \frac{v}{t} \leq \left( \frac{2}{3R_i} + \frac{4}{3R_j} + w \right) \) and the latter arises in the opposite case.

**Case 1**

\[
\frac{v}{t} \leq \left( \frac{2}{3R_i} + \frac{4}{3R_j} + w \right).
\]

The expected payoff of firm \( j \) when deviating in this case is:

\[
v_j^{DD} = \alpha t \left( \frac{v}{t} - w \right) R_j \left[ R_i \left( \frac{1}{3R_i} + \frac{2}{3R_j} - \frac{v}{2t} + \frac{w}{2} \right) \right] + \left( 1 - R_i \right) w - \frac{\alpha R_i c}{A_j} - TC(R_j, A_j).
\]

A comparison with (13) yields that \( v_j - v_j^{DD} \geq 0 \) provided that:

\[
- \left( \frac{v}{t} \right)^2 R_i + \frac{v}{t} \left[ \frac{2v}{3R_i} + \frac{4R_j}{3R_j} + wR_i + 2(1-R_i)w + wR_j \right] - 2w \left[ \frac{1}{3} + \frac{2R_i}{3R_j} + \frac{wR_j}{2} \right] + (1-R_i)w
\]

\[
= \left[ 2R_i + 4R_j - 3R_iR_j \right] \frac{R_i}{9R_iR_j} \leq 0.
\]

The discriminant of the above quadratic expression in \( \frac{v}{t} \) is:
\[ \Delta = -2(1 - R_j)(1 - w) \left[ \frac{10}{3} + \frac{8}{3} \frac{R_j}{R_j} + 2(1 - R_j)w - 2R_j \right]. \]

Since the discriminant is always negative there are no real valued roots that solve the quadratic expression in \( \frac{v}{t} \) as an equality. Hence, we are guaranteed that \( v_j - v_j^{DP} \geq 0 \), and as a result, the deviation is never profitable in this case.

**Case 2**

\[ \frac{v}{t} > \left( \frac{2}{3R_j} + \frac{4}{3R_j} + w \right). \]

The expected payoff of firm \( j \) when deviating in this case is:

\[ v_j^{DP} = \bar{a}t\left( \frac{v}{t} - w \right)R_j(1 - R_j)w - \frac{\bar{a}R_jC_j}{A_j} - TC(R_j, A_j). \]

A comparison with (13) yields that \( v_j - v_j^{DP} \geq 0 \) provided that:

\[ \frac{v}{t} \leq w + \frac{[2R_j + 4R_j - 3R_jR_j]^2}{18R_jR_j^2w(1 - R_j)}. \]

The RHS of the above inequality is a decreasing function of \( w \). It obtains its maximum when \( w = 1 \). Hence to guarantee that the above mentioned deviation is unprofitable for \( j \):

\[ \frac{v}{t} \leq 1 + \frac{[2R_j + 4R_j - 3R_jR_j]^2}{18R_jR_j^2(1 - R_j)}. \]

Combining the above restriction with those necessary to guarantee that the market is fully covered (both for a monopoly and a duopoly), we obtain:

\[ \max\left\{ 2, \left( \frac{1}{R_j} + \frac{1}{R_j} - \frac{1}{2} \right) \right\} \leq \frac{v}{t} \leq 1 + \frac{[2R_j + 4R_j - 3R_jR_j]^2}{18R_jR_j^2(1 - R_j)}. \]

The above interval is non-empty provided that:

\[ (R_j - R_j) < \frac{3R_jR_j[7R_j + 2R_j - 6R_jR_j]}{2(R_j + 2R_j)}. \]

(ii) Substituting symmetry yields the interval stated in the Lemma. Note that this interval is non-empty for all values of \( R \).

\[ \text{Q.E.D.} \]

A-6
Proof of Lemma 3

Because $A_i$ and $R_i$ are not independent, the objective $v_i$ can be written as $v_i(A_i(x_i, R_i), R_i)$, where $x_i$ is a variable independent of $R_i$. Optimizing $v_i$ with respect to $x_i$ and $R_i$ instead of $A_i$ and $R_i$ yields the following two first-order conditions:

$$\frac{\partial v_i}{\partial x_i} = \frac{\partial v_i}{\partial A_i} \frac{\partial A_i}{\partial x_i} = 0 \quad \text{and}$$

$$\frac{\partial v_i}{\partial R_i} = \frac{\partial v_i}{\partial A_i} \frac{\partial A_i}{\partial R_i} + \frac{\partial v_i}{\partial R_i} = 0 .$$

If $\frac{\partial A_i}{\partial x_i} \neq 0$ the above two conditions reduce to: $\frac{\partial v_i}{\partial A_i} = 0$ and $\frac{\partial v_i}{\partial R_i} = 0$ as stated in the lemma. We now derive the condition necessary for $\frac{\partial A_i}{\partial x_i} \neq 0$.

Rewrite the confusion matrix of Table 1 in terms of frequency of each cell in Table A.1

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>actual</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$l$</td>
<td>$b$</td>
</tr>
<tr>
<td>1</td>
<td>$c$</td>
<td>$d$</td>
</tr>
</tbody>
</table>

Table A.1.

Hence,

$$A = \frac{d}{d+b} \quad \text{and} \quad R = \frac{d}{c+d} . \quad (A.4)$$

From the above equations it follows that:

$$A = \frac{1}{1 + \frac{b}{c} \left( \frac{1 - R}{R} \right)} . \quad (A.5)$$

Let $x \equiv \frac{b}{c}$, then it follows that:

$$\frac{\partial A}{\partial x} = - \frac{\left( \frac{1 - R}{R} \right)}{\left[ 1 + x \left( \frac{1 - R}{R} \right) \right]^2} < 0 \quad \text{if} \quad 0 < R < 1 .$$
Hence $\frac{\partial A}{\partial x} \neq 0$ as required. The only remaining fact to establish is that $x$ and $R$ are independent. Because $R = \left[ \frac{1}{1 + \frac{c}{d}} \right]$, we need to demonstrate that the value of the ratio $\left( \frac{c}{d} \right)$ can be determined independently of the ratio $\left( \frac{b}{c} \right)$ for a fixed $c$ value. The sum of the frequencies in the table is 1, so it follows that $(l + b) = 1 - (c + d)$, which yields the following restriction on the values of ratios $\left( \frac{b}{c} \right)$ and $\left( \frac{c}{d} \right)$:

$$\frac{b}{c} \leq \frac{1}{c} - 1 - \frac{1}{(c/d)}.$$  

(A.6)

If at the equilibrium, the incidence of true negatives in Table A.1 is strictly positive (i.e. $l > 0$) the inequality (A.6) is not binding, and the value of $b$ can be selected independently of the value of $d$ (for a fixed $c$ value). As a result, $x$ and $R$ are, indeed, independent. Q.E.D.

**Proposition 1**

(i) Differentiating $v_i$ in (13) with respect to $R_i$ and $A_i$ yields:

$$\frac{\partial v_i}{\partial R_i} = \frac{a y_i}{2} \left\{ \frac{t R_j R_2}{2} \left[ 1 + \frac{2(1 - R_j)}{3R_j} + \frac{4(1 - R_j)}{3R_j} \right]^2 - \frac{R_j c}{A_i} \right\} + \frac{\partial^2 v_i}{\partial R_j^2} - \frac{\partial v_i}{\partial R_j} = 0,$$

$$\frac{\partial v_i}{\partial A_i} = \frac{\partial^2 v_i}{\partial A_i^2} - \frac{\partial^2 v_i}{\partial R_j^2} = 0.$$  

(A.7)

To guarantee an interior equilibrium we require that:

$$\frac{\partial^2 v_i}{\partial R_i^2} < 0, \quad \frac{\partial^2 v_i}{\partial A_i^2} < 0, \quad \text{and} \quad \left[ \frac{\partial^2 v_i}{\partial R_i^2}, \frac{\partial^2 v_i}{\partial A_i^2} \left( \frac{\partial^2 v_i}{\partial R_i^2} \right)^2 \right] > 0.$$

To derive the slopes of the reaction functions for the two competing advertisers we first differentiate (A.7) with respect to $R_j$ to obtain:
\[
\frac{\partial^2 v_i}{\partial R_i \partial R_j} = \frac{a \gamma_i}{2} \left\{ \frac{1}{2} \left[ 1 + \frac{2(1-R_j)}{3R_i} + \frac{4(1-R_j)}{3R_j} \right] \left( -T(R_i,R_j) - \frac{c}{A_i} \right) \right. \\
+ \frac{\tilde{a}R_i}{2} \left\{ \frac{2}{9} \left( 3 - \frac{2}{R_i^2} \right) - \frac{1}{3} \left( \frac{4}{3R_j} - 1 \right) + \frac{4}{3R_j} \left( \frac{4}{3R_j} - \frac{1}{3} \right) \right\},
\]

where
\[
T(R_i,R_j) \equiv \frac{4R_i^2 + 2R_j^2 + 3R_i^2 R_j + R_i R_j^2 - 4R_i R_j}{3R_i R_j}.
\]

Note that the numerator of \( T(R_i,R_j) \) is a convex function of \( R_j \), that attains its minimum when \( R_i^* = \left( R_j \left( 4 - R_j \right) \right) / \left( 2 \left( 4 + 3R_j \right) \right) \). Evaluating \( T(R_i,R_j) \) at \( R_i^* \) yields that
\[
T(R_i^*,R_j) > 0 \text{ for all } R_j. \quad \text{Hence the function } T() \text{ is positive everywhere, implying that the first term of (A.9) is strictly negative. Similarly, the second term of (A.9) is strictly negative as well because } 2 \left( 3 - \frac{2}{R_i^2} \right) \frac{2}{9} \text{ and } \left( \frac{1}{3} \left( \frac{4}{3R_j} - 1 \right) + \frac{4}{3R_j} \left( \frac{4}{3R_j} - \frac{1}{3} \right) \right) > \frac{13}{9}, \text{ given that } R_i^* \text{ and } R_j \text{ are fractions.}
\]

Hence \( \frac{\partial^2 v_i}{\partial R_i \partial R_j} < 0 \). Similarly, differentiating (A.8) with respect to \( R_j \) yields:
\[
\frac{\partial^2 v_i}{\partial A_i \partial R_j} = \frac{a \gamma_i R_i c}{2A_i^2} > 0.
\]

Differentiating (A.7) and (A.8) with respect to \( A_j \) yields:
\[
\frac{\partial^2 v_i}{\partial R_i \partial A_j} = \frac{\partial^2 v_i}{\partial A_i \partial A_j} = 0 \quad i, j = 1, 2, i \neq j.
\]

Total differentiation of (A.7) and (A.8) yields, therefore:
\[
D \left( \begin{array}{c} \frac{\partial R_i}{\partial A_i} \\ \frac{\partial R_i}{\partial R_j} \\ \frac{\partial v_i}{\partial A_i} \\ \frac{\partial v_i}{\partial R_j} \end{array} \right) = \left( \begin{array}{cccc} \frac{\partial^2 v_i}{\partial R_i \partial R_j} & 0 \\ \frac{\partial^2 v_i}{\partial A_i \partial R_j} & 0 \\ \frac{\partial^2 v_i}{\partial R_i \partial A_j} & 0 \\ \frac{\partial^2 v_i}{\partial A_i \partial A_j} & 0 \end{array} \right) \left( \begin{array}{c} \partial R_i \\ \partial A_i \end{array} \right),
\]

where
\[
D \equiv \begin{pmatrix}
\frac{\partial^2 v_i}{\partial R_i^2} & \frac{\partial^2 v_i}{\partial R_i \partial A_i} \\
\frac{\partial^2 v_i}{\partial R_i \partial A_i} & \frac{\partial^2 v_i}{\partial A_i^2}
\end{pmatrix},
\]

and \( D \) is negative definite to guarantee that second order conditions hold (specifically, \( |D| > 0 \), \( \frac{\partial^2 v_i}{\partial R_i^2} < 0 \), and \( \frac{\partial^2 v_i}{\partial A_i^2} < 0 \)).

From (A.12) it immediately follows that:
\[
\frac{\partial R_i}{\partial A_j} = \frac{\partial A_i}{\partial A_j} = 0.
\]

As well,
\[
\begin{pmatrix}
\frac{\partial R_i}{\partial R_j} \\
\frac{\partial A_i}{\partial R_j}
\end{pmatrix} = -\frac{1}{|D|} \begin{pmatrix}
\frac{\partial^2 v_i}{\partial A_i^2} & -\frac{\partial^2 v_i}{\partial R_i \partial A_i} \\
-\frac{\partial^2 v_i}{\partial R_i \partial A_i} & \frac{\partial^2 v_i}{\partial R_i^2}
\end{pmatrix} \begin{pmatrix}
\frac{\partial^2 v_i}{\partial R_i \partial R_j} \\
\frac{\partial^2 v_i}{\partial A_i \partial R_j}
\end{pmatrix}.
\]

Hence,
\[
\begin{pmatrix}
\frac{\partial R_i}{\partial R_j} \\
\frac{\partial A_i}{\partial R_j}
\end{pmatrix} = -\frac{1}{|D|} \begin{pmatrix}
\frac{\partial^2 v_i}{\partial A_i^2} & -\frac{\partial^2 v_i}{\partial R_i \partial A_i} & -\frac{\partial^2 v_i}{\partial R_i \partial R_j} & \frac{\partial^2 v_i}{\partial A_i \partial R_j} \\
-\frac{\partial^2 v_i}{\partial R_i \partial A_i} & \frac{\partial^2 v_i}{\partial R_i^2} & \frac{\partial^2 v_i}{\partial A_i \partial R_j} & -\frac{\partial^2 v_i}{\partial R_i \partial R_j} \\
-\frac{\partial^2 v_i}{\partial R_i \partial A_i} & \frac{\partial^2 v_i}{\partial R_i^2} & \frac{\partial^2 v_i}{\partial A_i \partial R_j} & \frac{\partial^2 v_i}{\partial R_i \partial R_j}
\end{pmatrix} \quad (A.13)
\]

If own effects always dominate cross effects, a condition necessary to guarantee stability of reaction functions (to be derived in the sequel), the signs of the slopes of the reaction functions are determined by the sign of the first term in each of the rows of the matrix on the RHS of (A.13).

Because \( \frac{\partial^2 v_i}{\partial A_i^2} < 0 \), \( \frac{\partial^2 v_i}{\partial R_i^2} < 0 \), \( \frac{\partial^2 v_i}{\partial R_i \partial A_i} < 0 \), \( \frac{\partial^2 v_i}{\partial R_i \partial R_j} > 0 \) and \( |D| > 0 \), it follows that:
\[
\frac{\partial R_i}{\partial R_j} < 0 \text{ and } \frac{\partial A_j}{\partial R_j} > 0.
\]

Note that in the first order conditions (A.7) and (A.8) the behavior of advertisers who are not direct competitors of Advertisers 1 and 2 is reflected in \(\tilde{a}\). Because \(\tilde{a}\) is a decreasing function of the average error rates of the non-competing advertisers as measured by \(M_1\), the result follows. Q.E.D.

**Proof of Lemma 4**

As was pointed out in the discussion following Lemma 2, a firm is more likely to consider implementing monopoly pricing in the second stage, the higher its recognition level is in comparison to that of its competitor. Let Firm \(j\) consider a deviation to \(R_j > R\). This deviation will cause the firm to abandon the segment of consumers who are fully informed, if either one of the conditions stated in part (i) of Lemma 2 is violated. Specifically, if either

\[
R_j > R + \frac{3RR_j[7R_j + 2R - 6RR_j]}{2(R_j + 2R)}
\]  

(A.14)

or

\[
\frac{v}{t} > 1 + \frac{[2R + 4R_j - 3RR_j]^2}{18RR_j(1 - R)}. \tag{A.15}
\]

The former inequality reduces to the following quadratic inequality in \(R_j\):

\[
R_j^2(2 + 18R^2 - 21R) - 2R_jR(3R - 1) - 4R^2 > 0. \tag{A.16}
\]

The discriminant of the above quadratic expression is equal to \(4R^2(1 - R)(1 - 9R)\). Hence, it is negative for all \(R > \gamma\). In addition the quadratic expression is concave (i.e. the coefficient of \(R_j^2\) is negative) if \(R > 0.1046\). Hence if \(R > \gamma\), inequality (A.16) can never hold, which implies that a deviation of the kind described in (A.14) is not feasible.

The RHS of inequality (A.15) is a decreasing function of \(R_j\). Hence, if the inequality does not hold for the biggest possible value of \(R_j\) (i.e. \(R_j = 1\)), it does not hold for smaller values as well. Substituting \(R_j = 1\) in the RHS of (A.15) yields that if:
Firm $j$ will never find it optimal to deviate to monopoly pricing in order to take advantage of the poorly informed consumers. In addition, to guarantee full coverage of the market, Lemma 2 requires that

$$\frac{v}{t} \geq \max \left\{ 2, \frac{1}{R} + \frac{1}{R_j} - \frac{1}{2} \right\}. \quad (A.18)$$

The RHS of the above inequality is a decreasing function of $R_j$. Hence, if it holds for the smallest value of $R_j$ (i.e., $R_j = R$) it holds for all $R_j \geq R$. To guarantee, therefore, full coverage:

$$\frac{v}{t} \geq \max \left\{ 2, \frac{2}{R} - \frac{1}{2} \right\}. \quad (A.19)$$

Combining (A.17) with (A.19) yields the region specified in the Lemma. This region is non-empty if $R \geq 0.4665$.

Q.E.D.

**Proof of Proposition 2**

Evaluating (A.7) and (A.8) at the symmetric equilibrium yields the following two conditions:

$$H(R, A) = \frac{\partial v_i}{\partial R_i} \bigg|_{\text{symmetry}} = a\gamma_i R \left\{ \frac{tR}{2} \left[ \frac{2}{R} - 1 \right] - \frac{c}{A} \right\} + \tilde{a} \left\{ \frac{tR}{2} \left[ \frac{2}{3R} - 1 \right] - \frac{c}{A} \right\} - \frac{\partial TC}{\partial R_i} = 0, \quad (A.20)$$

and

$$G(R, A) = \frac{\partial v_i}{\partial A_i} \bigg|_{\text{symmetry}} = \frac{\tilde{a} R c}{A^2} - \frac{\partial TC}{\partial A_i} = 0, \quad (A.21)$$

where $\tilde{a} = a \left[ \gamma_0 - \gamma_1 (1-R) - \gamma_2 M_1 \right]$.

To guarantee stability of reaction functions the following conditions should hold:

$$\frac{\partial H}{\partial R} < 0, \quad \frac{\partial G}{\partial A} < 0, \quad \frac{\partial H}{\partial R} \frac{\partial G}{\partial A} - \frac{\partial H}{\partial A} \frac{\partial G}{\partial R} > 0. \quad (A.22)$$

To derive the comparative statics relationships, we partially differentiate the first order conditions (A.20) and (A.21) with respect to the parameters of the model to obtain:

$$\frac{\partial H}{\partial c} = \left[ \frac{a\gamma_i R}{2A} + \frac{\tilde{a}}{A} \right] < 0; \quad \frac{\partial G}{\partial c} = \frac{\tilde{a} R c}{A^2} > 0, \quad (A.23)$$
\[
\frac{\partial H}{\partial t} = \frac{a\gamma_1 R^2}{4} \left[ \frac{2}{R} - 1 \right]^2 + \frac{\tilde{a} R}{2} \left[ \frac{2}{R} - 1 \right] \left[ \frac{2}{3R} - 1 \right] > 0; \quad \frac{\partial G}{\partial t} = 0, \quad (A.24)
\]

\[
\frac{\partial H}{\partial a} = \frac{\gamma_1 R}{2} \left[ \frac{t R}{2} \left[ \frac{2}{R} - 1 \right] - \frac{c}{A} \right] + \left[ \gamma'_0 - \gamma_1 (1 - R) - \gamma_2 M_1 \right] \left[ \frac{t R}{2} \left[ \frac{2}{R} - 1 \right] \left[ \frac{2}{3R} - 1 \right] - \frac{c}{A} \right] > 0; \quad (A.25)
\]

\[
\frac{\partial G}{\partial a} = \frac{R c}{A} \left[ \gamma'_0 - \gamma_1 (1 - R) - \gamma_2 M_1 \right] > 0,
\]

The signs of \(\frac{\partial H}{\partial t}\) and \(\frac{\partial H}{\partial a}\) result because, at the interior equilibrium, (A.20) holds, implying that \(\frac{\partial H}{\partial t}\) and \(\frac{\partial H}{\partial a}\) are positive.

Total differentiation of (A.20) and (A.21) with respect to any parameter \(w\) yields the following system:

\[
\frac{\partial H}{\partial R} \frac{\partial R}{\partial w} + \frac{\partial H}{\partial A} \frac{\partial A}{\partial w} = -\frac{\partial H}{\partial w}
\]

\[
\frac{\partial G}{\partial R} \frac{\partial R}{\partial w} + \frac{\partial G}{\partial A} \frac{\partial A}{\partial w} = -\frac{\partial G}{\partial w}
\]

As a result,

\[
\begin{pmatrix}
\frac{\partial R}{\partial w} \\
\frac{\partial A}{\partial w}
\end{pmatrix} = -\frac{1}{D} \begin{pmatrix}
\frac{\partial G}{\partial A} & -\frac{\partial H}{\partial A} \\
-\frac{\partial G}{\partial R} & \frac{\partial H}{\partial R}
\end{pmatrix} \begin{pmatrix}
\frac{\partial H}{\partial w} \\
\frac{\partial G}{\partial w}
\end{pmatrix}, \quad \text{where} \quad D \equiv \frac{\partial H}{\partial R} \frac{\partial G}{\partial A} - \frac{\partial H}{\partial A} \frac{\partial G}{\partial R} > 0. \quad (A.26)
\]

Given the stability condition, we know that cross effects are always dominated by own effects, implying that in (A.26),

\[
\text{sgn} \left\{ \frac{\partial R}{\partial w} \right\} = \text{sgn} \left\{ -\frac{\partial G}{\partial A} \frac{\partial H}{\partial A} \frac{\partial H}{\partial w} \right\} = \text{sgn} \left\{ \frac{\partial H}{\partial w} \right\}, \quad \text{sgn} \left\{ \frac{\partial A}{\partial w} \right\} = \text{sgn} \left\{ -\frac{\partial G}{\partial R} \frac{\partial H}{\partial R} \frac{\partial G}{\partial w} \right\} = \text{sgn} \left\{ \frac{\partial G}{\partial w} \right\}. \quad (A.27)
\]

The equality in (A.27) follows from (A.22) because \(\frac{\partial G}{\partial A} < 0\) and \(\frac{\partial H}{\partial R} < 0\).

(i), (ii) Hence, from (A.23) – (A.25) it follows that:
\[
\frac{\partial R}{\partial c} < 0, \quad \frac{\partial A}{\partial c} > 0, \quad \frac{\partial R}{\partial t} > 0, \quad \frac{\partial A}{\partial t} > 0, \quad \frac{\partial R}{\partial a} > 0, \quad \frac{\partial A}{\partial a} > 0.
\]

**Derivation of the Second Order Condition of the Content distributor's Revenue Maximization Problem.**

From (17),

\[
\frac{\partial^2 TR}{\partial c^2} = \frac{8R(c)}{A(c)} a \gamma_R R'(c) \left[ 1 + \eta^R_e - \eta^A_e \right] + \frac{2\alpha R(c)}{c A(c)} \left[ \eta^R_e - \eta^A_e \right] \left[ 1 + \eta^R_e - \eta^A_e \right] + \frac{2c \alpha \gamma_R R(c) R'(c)}{A(c)}.
\]

Note that the first two terms of the above derivative are negative, because \( R'(c) < 0, \) \( \eta^R_e < 0, \) and \( \eta^A_e > 0 \) from Proposition 2, and \( \left( 1 + \eta^R_e - \eta^A_e \right) > 0 \) from the first order condition (17). To guarantee that \( \frac{\partial^2 TR}{\partial c^2} < 0, \) the sum of the last two terms should be either negative, or if positive sufficiently small, so that the first two terms dominate in determining the sign of \( \frac{\partial^2 TC}{\partial c^2}. \) A sufficient condition that \( \frac{\partial^2 TR}{\partial c^2} < 0 \) is that:

\[
\left( \frac{\partial \eta^R_e}{\partial c} - \frac{\partial \eta^A_e}{\partial c} \right) < 0 \quad \text{and} \quad R'(c) < 0.
\]

**Two Part Tariff Pricing Rules and Maximization of Channel Profits**

Suppose that the content distributor charges advertisers 1 and 2 a fixed sign up fee \( F \) and a variable charge \( 'c' \) per viewed commercial. Its maximization becomes:

\[
\max \left\{ \frac{2c \alpha R}{A} + 2F \right\}
\]

subject to \( v_i - F \geq 0 \) \hspace{2cm} (IR)

\[
\frac{\partial v_i}{\partial R_i} = \frac{\partial v_i}{\partial A_i} = 0.
\]

The content distributor wishes to maximize its proceeds subject to individual rationality constraints (IR) guaranteeing the participation of each advertiser, and incentive compatibility constraints (IC) which are implied by the optimizing behavior of the advertisers. Given its
monopoly in delivering messages, it can set \( v_i = F \). At the symmetric equilibrium, the above optimization reduces to:

\[
\begin{align*}
\max W &= \tilde{a}t(2 - R)^2 - 2TC(R, A) \\
\text{s.t.} & \quad R \text{ and } A \text{ satisfy (A.7) and (A.8).}
\end{align*}
\] (A.28)

Note that objective (A.28) coincides with the joint profits of the advertisers and distributor. Acting as a common agent for both advertisers, the two part tariff scheme facilitates the distributor to internalize the aggregate net payoff generated in the channel. While the aggregate channel profits are strictly decreasing with \( A \), changes in \( R \) have two counteracting effects on channel profits. On the positive side, higher recognition reduces clutter in advertising and increases the de-facto size of segment 1 viewers \( \tilde{a} \). On the negative side, higher recognition intensifies product market competition as reflected by the term \( (2 - R)^2 \). It also increases the costs incurred by the advertisers. The level of recognition that maximizes the channel profit, trades off those two counteracting effects. Specifically, this optimal level solves the condition.

\[
\frac{\partial W}{\partial R} = 2 \left[ \frac{(2 - R)^2}{2} \tilde{a} \gamma_1 - (2 - R)\tilde{a} - \frac{\partial TC}{\partial R} \right] = 0
\] (A.29)

Note that the optimality of \( A = 0 \) is implied by the fact that only profits in the industry where advertisers 1 and 2 compete are considered. If profits of advertisers who wish to deliver messages to segment 0 of viewers were also considered, higher levels of accuracy would be optimal. Higher accuracy levels chosen by advertisers 1 and 2 increase the de-facto size of segment 0 viewers, thus increasing the profitability of advertisers who wish to target commercials to this group.

The content distributor cannot choose \( R \) and \( A \) directly. Instead, the variable charge 'c' per viewed commercial determines indirectly the values of \( R \) and \( A \). To obtain the optimal charge 'c', we derive the total derivative of \( W \) with respect to \( c \) as follows:

\[
\frac{dW}{dc} = \frac{\partial W}{\partial R} \frac{\partial R}{\partial c} + \frac{\partial W}{\partial A} \frac{\partial A}{\partial c} = 2 \left[ \frac{(2 - R)^2}{2} \tilde{a} \gamma_1 - (2 - R)\tilde{a} - \frac{\partial TC}{\partial R} \right] \frac{\partial R}{\partial c} - 2 \frac{\partial TC}{\partial A} \frac{\partial A}{\partial c},
\] (A.30)

where from Proposition 2 \( \frac{\partial R}{\partial c} < 0 \) and \( \frac{\partial A}{\partial c} > 0 \). To guarantee the existence of a unique maximizing solution, assume that \( \frac{\partial^2 W}{\partial c^2} < 0 \).
Next, we derive a condition that guarantees that the content distributor can implement the levels of $A$ and $R$ that maximize channel profits. In order to implement $A = 0$, it is necessary from (A.8) that $c = 0$. Hence, we investigate when such an outcome is consistent with (A.30). Evaluating (A.7) at $c = 0$ yields that following first order condition that advertisers follow in determining $R$.

$$\frac{1}{2} \left[ \frac{a\gamma_l}{2} (2 - R)^2 - (2 - R) \tilde{a} \left( 1 - \frac{2}{3R} \right) - 2 \frac{\partial TC}{\partial R} \right] = 0.$$

Designate by $R(0)$ the value of recognition that advertisers choose when $c = 0$, according to the above incentive compatibility constraint. Substituting into (A.30) yields:

$$\frac{dW}{dc} \bigg|_{c=0} = \left[ \frac{2(2 - R(0))\tilde{a}}{3R(0)} - \frac{\partial TC}{\partial R} \bigg|_{R(0)} \right]. \quad (A.31)$$

If the RHS of (A.31) is negative, we are guaranteed that $c = 0$ is indeed an equilibrium. In this case, the content distributor can implement the "first best" outcome that maximizes channel profits. Otherwise, if the RHS of (A.31) is positive, the equilibrium is characterized by lower levels of recognition and higher levels of accuracy than those that maximize aggregate channel profits, since $c > 0$ at the equilibrium.

It is noteworthy that the two part tariff scheme yields a lower variable fee per viewed commercial than the linear scheme considered in the main text. The individual rationality constraint $v_i - F = 0$ implies that there is a negative tradeoff between $c$ and $F$. Hence, if $F > 0$, lower variable charges can be expected in comparison to linear pricing. As a result, non-linear pricing yields higher levels of recognition and lower levels of accuracy than linear pricing does (since $\frac{\partial R}{\partial c} < 0$ and $\frac{\partial A}{\partial c} > 0$).

Nonexistence of Asymmetric Equilibria for a Specific Example

Assume that $TC(R, A) = \frac{1}{2} R^2 + \frac{1}{2} A^2$, $\gamma_0 = \gamma_l$, and $M_1 = 0$. Substituting into the first order conditions with respect to $R_i$ and $A_i$ symmetry as well as the specific cost function and values of the parameters, yields the following system of two equations in two unknowns.

$$H(R, A) = \frac{a\gamma_l}{12} \left( 20 - 28R + 9R^2 \right) - \frac{3a\gamma_l R}{2A} - R = 0$$
\[ G(R, A) = \frac{a \gamma_i R^2 c}{A^2} - A = 0. \]

It is easy to show that the stability conditions (A.22) hold. According to Vives (page 47), stability of reaction function guarantees a unique solution to the above system. Moreover, the comparative statics results reported in Proposition 2 hold as well (i.e. \( \frac{\partial R}{\partial c} < 0, \frac{\partial A}{\partial c} > 0, \frac{\partial R}{\partial a} > 0, \frac{\partial A}{\partial a} > 0. \))

In their paper, Chen and Iyer 2002 demonstrate the possible existence of asymmetric equilibria, whereby one firm chooses maximum addressability (recognition in our model) and the other chooses partial addressability. They show, in particular, that when the marginal cost of improved addressability is zero, only asymmetric equilibria can exist. Next, we demonstrate that such asymmetric equilibria cannot arise in the above example when \( c = 0. \) Specifically, it can never be the case that one of the firms chooses maximum recognition while its competitor chooses a smaller level of recognition (i.e. \( R_i = 1 \) and \( R_j < 1. \)) When \( c = 0, A_i = A_j = 0 \) and the equilibrium can be derived by focusing on differentiating the objective (13) with respect to \( R_i \) and \( R_j \) only. Considering the possibility that \( R_i = 1, \) we evaluate those partial derivatives at \( R_i = 1 \) and \( c = 0 \) to obtain the following expressions:

\[
\begin{align*}
\frac{\partial v_i}{\partial R_i} & = \frac{a \gamma_i t}{4} \left[ R_j \left( 1 + \frac{4(1-R_j)}{3 R_j} \right) - 1 \right], \\
\frac{\partial v_j}{\partial R_j} & = \frac{a \gamma_i t}{4} \left[ R_j \left( 1 + \frac{2(1-R_j)}{9 R_j} \right) - 1 \right].
\end{align*}
\]

To guarantee the existence of an asymmetric equilibrium it should be that there are values of \( R_j \) such that \( \frac{\partial v_j}{\partial R_j} |_{R_j=1} = 0, \frac{\partial^2 v_j}{\partial R_j^2} |_{R_j=1} < 0, \) and \( \frac{\partial v_i}{\partial R_i} |_{R_i=1} > 0. \) In the sequel, we demonstrate that the above three requirements are inconsistent for any value of \( R_j \in (0,1). \)

Further simplification of the partial derivatives yields:
\[
\frac{\partial v_j}{\partial R_j} \bigg|_{R_j=1} \equiv \frac{a\gamma t}{36} \left[ 2R_j + 5 - \frac{4}{R_j^2} \right] - R_j = 0
\]

(A.32)

\[
\frac{\partial^2 v_j}{\partial R_j^2} \bigg|_{R_j=1} = \frac{a\gamma t}{36} \left[ 2 + \frac{8}{R_j^3} \right] - 1 < 0.
\]

The second order condition holds for \( R_j \in (0,1) \) only if

\[ R_j > \left[ \frac{8a\gamma t}{36 - 2a\gamma t} \right]^{\frac{1}{2}} \equiv \hat{R}_j \text{ and } a\gamma t < 3.6. \]  

(A.33)

Substituting the above threshold value of \( \hat{R}_j \) back into the expression obtained for \( \frac{\partial v_j}{\partial R_j} \bigg|_{R_j=1} \) yields:

\[
\frac{\partial v_j}{\partial R_j} \bigg|_{R_j=1, R_i=\hat{R}_j} = \frac{5a\gamma t \left( \hat{R}_j \right)^{\frac{1}{2}} - 12a\gamma t}{36\hat{R}_j^2} < 0.
\]

The above derivative is always negative since \( 0 < \hat{R}_j < 1 \). Hence, to obtain the first order condition \( \frac{\partial v_j}{\partial R_j} \bigg|_{R_j=1} = 0 \), it is necessary that \( R_j < \hat{R}_j \), which contradicts (A.33).

Next, we consider the possibility that a corner solution arises in the second stage game, namely one of the firms chooses monopoly pricing. Since we are investigating the possibility that \( R_j < R_i = 1 \) at the equilibrium, firm \( i \) is likely to be the one that charges the higher price. Restricting attention to the case that \( v > 2t \), implies that the monopoly price is \( \hat{R}_j \). Hence, we investigate whether \( p_i = (v-t) \) and \( R_j < R_i = 1 \) are consistent with an equilibrium when \( c = 0 \). At the second stage pricing game, firm \( j \) chooses \( p_j \) to maximize

\[
\pi_j \equiv \left\{ R_j \left[ R_j \left[ \frac{1}{2} + \frac{v-t-p_j}{2t} \right] + (1-R_j) \right] p_j - \frac{1}{2}R_j^2 \right\},
\]

yielding the "best response" price for \( j \):

\[
p_j = \begin{cases} 
\frac{v + t(1-R_i)}{2R_i} & \text{if } 2t < v \leq 4t + \frac{2t(1-R_i)}{R_i} \\
\frac{v - 2t}{R_i} & \text{if } 4t + \frac{2t(1-R_i)}{R_i} < v. 
\end{cases}
\]
The above "best response" implies that when $v$ is sufficiently small (i.e. $v \leq 4t + \frac{2t(1 - R)}{R}$), firm $i$ retains a positive market share even when the consumer is familiar with both products. In contrast, for big values of $v$ (bigger than $4t + \frac{2t(1 - R)}{R}$) firm $i$ sells only to consumers who are familiar with its product and not with that of firm $j$. We consider both of those possible best responses in what follows. As well, since firm $i$ chooses the monopoly price it should be that its second stage payoff function is everywhere increasing. Specifically, $\frac{\partial \pi_i}{\partial p_i \mid p_i = v-t} > 0$.

Case 1

$2t < v \leq 4t + \frac{2t(1 - R)}{R}$

Substituting the optimal prices back into the objective functions of the firm yields:

$v_i = aR_i \left[ R_j \left( 1 - \frac{v}{4t} + \frac{1}{2} \left( 1 - \frac{R}{R_i} \right) \right) + (1 - R) \right] (v-t) - \frac{1}{2} R_i^2.$

$v_j = aR_j \left[ R_i \left( \frac{v}{9t} - \frac{1}{2} \left( 1 - \frac{R_i}{R} \right) \right) + (1 - R) \right] \left[ \frac{v}{2} + t \left( 1 - \frac{R}{R_i} \right) \right] - \frac{1}{2} R_j^2.$

Note that since the price chosen by firm $i$ corresponds to a corner solution the value of $p_i$ is determined independent of the values of $R_i$ and $R_j$. In contrast, since the solution for $p_j$ is interior, the value of $p_j$ does depend upon the level of recognition selected by firm $i$ in Stage 1. Optimizing the payoffs with respect to $R_i$ and $R_j$ and evaluating the derivatives at the asserted equilibrium $R_i = 1$ yields:

$$\frac{\partial v_i}{\partial R_i \mid R_i = 1} = \frac{(v-t)\gamma_i a}{2} \left[ 2 - \frac{R_j}{2} \left( \frac{v}{t} - 1 \right) - \frac{R_j^2}{2} \left( 1 + \frac{v}{2t} \right) \right] - 1$$

$$\frac{\partial v_j}{\partial R_j \mid R_j = 1} = \frac{v^2 \gamma_j a (1 + 2R_j)}{16t} - R_j.$$

If at the equilibrium $R_j < R_i = 1$, then $\frac{\partial v_i}{\partial R_i \mid R_i = 1} > 0$, $\frac{\partial v_j}{\partial R_j \mid R_j = 1} = 0$, and $\frac{\partial^2 v_j}{\partial R_j^2 \mid R_j = 1} < 0$. Define $y = \gamma_i at$ and $x = \frac{v}{t}$ and use this notation in (A.34) to obtain the following conditions:
\[
\frac{\partial v_i}{\partial R_i} \bigg|_{R_i=1} = \frac{(x-1)y}{2} \left[ 2 - \frac{R_i}{2}(x-1) - \frac{R_i^2}{2} \left( 1 + \frac{x}{2} \right) \right] - 1 > 0
\] (A.35)

\[
\frac{\partial v_j}{\partial R_j} \bigg|_{R_j=1} = 0,
\frac{\partial^2 v_j}{\partial R_j^2} \bigg|_{R_j=1} < 0 \Rightarrow R_j = \frac{yx^2}{2(8-yx^2)}, \text{ where } yx^2 < \frac{16}{3}.
\] (A.36)

And finally, to guarantee that the price \((p-t)\) is the optimal choice of firm 1 it should be that:

\[
\frac{\partial \pi_i}{\partial p_i} \bigg|_{p_i=n-v-t} = \left[ \frac{1}{2} + \frac{\left( \frac{v}{2} - 2(v-t) \right)}{2t} - \frac{1-R_i}{R_j} \right] > 0.
\]

Using the notation \(x \equiv \frac{v}{t}\) in the above yields:

\[
R_j < \frac{4}{(3x-2)}.
\] (A.37)

Conditions (A.35) – (A.37) are incompatible for all values of \(x \in [2,4]\). Recall that those are the values considered in Case 1.

**Case 2**

\[
4t + \frac{2t(1-R_i)}{R_j} < v
\]

In this case both firms choose a corner solution in the second stage, namely \(p_i = (v-t)\) and \(p_j = (v-2t)\), implying that firm \(j\) dominates the market of all the consumers who are familiar with both products. Substituting those prices back into the objective functions of the firms yields:

\[
v_i = \bar{a}R_i \left( 1-R_j \right)(v-t) - \frac{1}{2} R_i^2
\]

\[
v_j = \bar{a}R_j \left( v-2t \right) - \frac{1}{2} R_j^2
\]

Differentiating with respect to \(R_i\) and \(R_j\) and evaluating the derivatives at \(R_i = 1\) yields the following two conditions:

\[
\frac{\partial v_i}{\partial R_i} \bigg|_{R_i=1} = \frac{(1-R_j)(x-1)y}{2}(2+R_j) - 1 > 0
\] (A.38)
\[ \frac{\partial v_j}{\partial R_j\mid_{R_i=1}}, \frac{\partial^2 v_j}{\partial R_j^2\mid_{R_i=1}} < 0 \implies R_j = \frac{(x-2)y}{2[1-(x-2)y]}, \text{ where } (x-2)y < \frac{x}{2}. \quad (A.39) \]

and finally to guarantee that \( p_t = (v-t) \) is, indeed, optimal:

\[ \frac{\partial \pi_i}{\partial p_j\mid_{p_j=(v-t)}} = \frac{1}{2} + \frac{(v-2t)-2(v-t)}{2t} + \frac{1-R_j}{R_j} > 0, \]

which reduces to the following constraint on \( R_j \):

\[ R_j < \frac{2}{(x+1)}. \quad (A.40) \]

Once again, conditions (A.38) – (A.40) are incompatible for all values of \( x > 4 \) (Case 2).

Finally, it is impossible that both firms would choose the monopoly price at the asymmetric equilibrium where \( R_j < R_i = 1 \). If \( p_j = v-t \) it should be that:

\[ \frac{\partial \pi_j}{\partial p_j\mid_{p_j=v-t}} = \frac{1}{2} + \frac{(v-2t)-2(v-t)}{2t} + \frac{1-R_i}{R_i} > 0. \]

However, when \( R_i = 1 \) the above inequality contradicts the assumption that \( v > 2t \), which is necessary in order to guarantee that the monopoly price is indeed \( (v-t) \).

The above derivations indicate that the type of asymmetric equilibria derived in Chen and Iyer 2002 can never exist in our model when \( c = 0 \). The derivations do not preclude, however, the possible existence of other types of asymmetric equilibria.