DESIGN OF A SINGLE-LENS STEREO CAMERA SYSTEM

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Abstract—A camera system is designed that can obtain stereo information in a single shot and through a single lens. A single image obtained by this camera is equivalent to two images obtained by two aligned cameras with exactly the same optical properties. In this system, stereo is achieved by viewing the reflections of a scene in two mirrors that have a common axis. The angle between the mirrors may be changed to adjust the field of vision of the camera and to fixate at desired points in a scene.

Computer vision  Depth perception  Image disparity  Stereo camera  Correspondence process

I. INTRODUCTION

Depth perception by stereo disparity has been studied extensively in computer vision.1-4 Stereo disparity is a powerful cue which, without the presence of other monocular and binocular cues, can measure depth.5 Depth perception via stereo disparity is a passive method that does not require any special lighting or scanner to acquire the images. This method may be used to determine depths of points in indoor as well as outdoor scenes, and depths of points that are inches or miles away from the viewer.

A major step in stereo depth perception is the determination of correspondence between points in the images. This step is known as the correspondence process, and many algorithms for it have been developed.6-13 Another major step is the computation of depth values from the point correspondences.11-14 The objective of this paper is neither to propose a new correspondence algorithm nor to implement a new method that determines depth values from the correspondences. Rather, it is to propose a new camera system that can obtain stereo images free of unwanted geometric and intensity differences, thus simplifying the correspondence process. We will first determine the desired properties of an ideal stereo camera system and then design a system that can provide those properties.

A correspondence algorithm can produce more reliable matches if the underlying images have smaller intensity and geometric differences. Some geometric difference between stereo images is unavoidable, for it is actually the local geometric difference between stereo images that results in the perception of depth. Image geometric difference due to rotation of one camera with respect to another, however, does not have anything to do with the three-dimensional (3D) structure of a scene and only confuses the correspondence process. If the scene has Lambertian surfaces,15 there would be no difference in the intensities of corresponding points in the images. Differences in the optical properties of the two cameras, however, cause intensity difference between corresponding points in stereo images. These unwanted geometric and intensity differences should be removed from stereo images, or reduced as much as possible, to increase the correspondence reliability.

When stereo images are obtained by two cameras, it is possible that the focal lengths and zoom levels of the cameras are slightly different. The two cameras may have lenses that do not have exactly the same optical properties. The optical axes of the cameras may not lie in the same plane. These camera differences create unwanted geometric and intensity differences between stereo images, making the correspondence process less reliable. It is desired that a stereo camera system can obtain a pair of images in a single shot and through a single lens so that unwanted geometric and intensity differences between the images could be avoided. A single camera with multiple shots will remove some of the unwanted image differences; but in a dynamic scene, since the scene can change between the times the images are obtained, positions of corresponding points in obtained images will no longer relate to the depth values.

Stereo images obtained from two cameras that are not aligned require larger search areas to establish correspondence between points in the images. Larger search areas increase the probability of mismatch. Therefore, the cameras should be aligned so that corresponding points fall on the same scanline in the images, reducing the search area size. If stereo images are obtained with cameras that have parallel optical axes normal to the baseline, corresponding points will fall on the same scanline in the images, facilitating the correspondence process.

It is desired that a camera system could change its field of vision to see different parts of a scene and fixate at required points.16 When two cameras are used, this would mean letting the optical axes of the cameras converge. Although cameras with converging optical
axes are more useful than cameras with parallel optical axes, images obtained from the former are more difficult to match than images from the latter. This is because corresponding points in images from the former may not fall on the same scanline. Therefore, images obtained from cameras with converging optical axes should be transformed to appear as if obtained by cameras with parallel optical axes.

To summarize, an ideal stereo camera system has the following properties:

1. It obtains stereo images in a single shot and through a single lens.
2. It can adjust its field of vision.
3. It produces no unwanted geometric and intensity differences between images in a stereo pair.
4. It produces images with corresponding points lying on the same scanline.

In the following, after reviewing existing stereo camera systems, a new camera system is designed that has the properties listed above.

2. EXISTING STEREO CAMERA SYSTEMS

Stereo images are usually obtained either by displacing a single camera in a scene or using two cameras mounted on a platform separated by a small distance. Although these set-ups acquire stereo images in a simple manner, they rarely provide the desired properties mentioned above. Unwanted geometric and intensity differences between stereo images make the correspondence process unreliable. By devising a more accurate calibration process and using higher-quality cameras, the unwanted geometric and intensity differences between stereo images can be reduced. Some differences, however, will still remain between the images.

Nishimoto and Shirai proposed a single-lens camera system that can obtain stereo images. In this system, a glass plate is placed in front of the camera, and images are obtained with the plate at two different rotational positions (see Fig. 1(a)). When the glass plate is rotated, the optical axis of the camera shifts slightly, simulating two cameras with parallel optical axes. The obtained stereo images have very small disparities making the point correspondences easy. However, only coarse depth values can be obtained from the disparities. This camera system requires two shots from a scene and, therefore, should be used only in static environments. Otherwise, the scene will change during the time the images are obtained, and the positions of corresponding points will no longer relate to the depths of points in 3D.

Teoh and Zhang described a single-lens stereo camera system with the geometry shown in Fig. 1(b). Two mirrors, fixed to the body of the camera, make a 45° angle with the optical axis of the camera. A third mirror which can rotate is placed directly in front of the lens. The rotating mirror is made parallel to one of the fixed mirrors and an image is obtained. Then it is made parallel to the other fixed mirror and another image is obtained. Here also, although a single camera is used, the result is the same as using two cameras with parallel optical axes. Note that since two shots of a scene are required, the camera should only be used in static scenes.

Both of these cameras considerably reduce unwanted geometric and intensity differences between stereo images. But the cameras have parts that should be rotated when obtaining a pair of images. Exact rotation of the parts is a major design issue in these systems, and since two shots of a scene are required, the scene under study must not change during the time the images are obtained. In the following, a camera system is designed that can obtain stereo images in a single shot and through a single lens. Since the two images of a stereo pair are obtained at the same time, this camera can be used in dynamic scenes.

3. A SIMPLE STEREO CAMERA GEOMETRY

To derive the geometry of an ideal stereo camera, we will start with the geometry of a single pin-hole camera as shown in Fig. 2. Suppose point P with coordinates \((x, y)\) is the image of point P in the scene with coordinates \((X, Y, Z)\). Also, suppose \(O\) is the camera's lens center, \(h\) is the camera's piercing point with coordinates \((x_0, y_0)\) (this is the point where the principal ray \(OZ\) intersects the \(x-y\) plane), \(f\) is the focal length of the camera, \(XYZ\) is the world coordinate system with origin \(O\), and the \(X-Y\) plane is parallel to the
image plane. From the perspective rule we have

\[ \frac{X}{Z} = \frac{x - x_0}{f} \]  \hspace{1cm} (1) \\
\[ \frac{Y}{Z} = \frac{y - y_0}{f} \]  \hspace{1cm} (2)

Now, suppose two such cameras are separated horizontally by 2B with their optical axes parallel and normal to the baseline (the line connecting the lens centers of two cameras), as shown in Fig. 3. Suppose \( O_L \) and \( O_R \) are the left and right camera lens centers, respectively, and the world coordinate system is midway between the two cameras with the Z-axis parallel to the optical axes of the cameras. Therefore, the left and right camera lens centers are at \((-B, 0, 0)\) and \((B, 0, 0)\), respectively. Also, suppose \( h_L \) and \( h_R \) are the left and right camera piercing points with coordinates \((x_{L}, y_{L}, z_{L})\) and \((x_{R}, y_{R}, z_{R})\), respectively; \( P \) with coordinates \((X, Y, Z)\) is a point in the scene whose projections in
the left and right image planes are \( p_l \) and \( p_r \) with coordinates \((x_l, y_l)\) and \((x_r, y_r)\), respectively, and both cameras have the same focal length \( f \). Then, using relations (1) and (2), the relation between a point in the scene and its images in the left and right cameras can be correspondingly written as

\[
\begin{align*}
\frac{X + B}{Z} &= \frac{x_l - x_{l0}}{f} \\
\frac{Y}{Z} &= \frac{y_l - y_{l0}}{f}
\end{align*}
\]  

and

\[
\begin{align*}
\frac{X - B}{Z} &= \frac{x_r - x_{r0}}{f} \\
\frac{Y}{Z} &= \frac{y_r - y_{r0}}{f}
\end{align*}
\]  

From relations (4) and (6) we find

\[
y_l - y_{l0} = y_r - y_{r0}
\]

and if \( y_{l0} = y_{r0} \), then

\[
y_l = y_r.
\]

Relation (8) shows that if the piercing points of the two cameras appear on the same scanline in the images, corresponding points will lie on the same scanline in the images.

From equations (3) and (5) we find

\[
\begin{align*}
\frac{f}{Z} (X + B) + x_{l0} &= x_l \\
\frac{f}{Z} (X - B) + x_{r0} &= x_r
\end{align*}
\]  

The horizontal displacement between corresponding points in the images, known as the disparity, may be computed by subtracting equation (10) from equation (9):

\[
x_l - x_r = \frac{2Bf}{Z} + (x_{l0} - x_{r0}).
\]

From equation (11) the relation between the depth of a point in a scene and the disparity between its images may be written as

\[
Z = \frac{2Bf}{[(x_l - x_r) - (x_{l0} - x_{r0})]}.
\]

Relation (12) shows that when the geometry of Fig. 3 is used, depth is inversely proportional to image disparity, and for a given depth, the larger the baseline length the larger the disparity. Therefore, baseline length may be increased to increase the accuracy of measured depth.

Corresponding points in images obtained from two cameras will fall on the same scanline if they are displaced horizontally, have parallel optical axes normal to the baseline, and have exactly the same focal length and zoom level. This property simplifies the correspondence process. However, unless one is viewing an object that is far away, cameras with parallel optical axes are not very useful. It is more useful if cameras in a stereo system could fixate at desired points in a scene to bring to the field of view of both cameras a desired object. When using cameras with converging optical axes, however, corresponding points may not fall on the same scanline, complicating the correspondence process. In the following, we will show how images obtained from stereo cameras with con-

![Fig. 4. Relation between a camera system with parallel optical axes and a camera system with converging optical axes. This figure shows rotation of the left optical axis clockwise by \( x \) and rotation of the right optical axis counterclockwise by \( a \). The optical axes of the cameras and the baseline are in the same plane.](image-url)
Converging optical axes may be transformed to appear as if obtained by cameras with parallel optical axes.

4. A STEREO CAMERA GEOMETRY WITH CONVERGING OPTICAL AXES

The camera system shown by dotted lines in Fig. 4 has parallel optical axes. The camera geometry shown by solid lines is one with converging optical axes. When the optical axes of the cameras converge, corresponding points in the images may not fall on the same scanline. Therefore, although cameras with converging optical axes were useful, they complicate the correspondence process. Unless a procedure can be formulated that can transform the obtained images to appear as if obtained by cameras with parallel optical axes, cameras with converging optical axes are not very practical.

Consider the geometry of the left camera in a stereo camera system with converging optical axes as shown in Fig. 5. Suppose the image shown with solid lines is the one obtained by this camera, and the image shown by dotted lines is the one obtained if the left camera’s optical axis was normal to the baseline. Assuming the camera has been rotated by a rad in the clockwise direction about the ZL-axis, we determine the relation between points in the two images.

In the following formulas, the coordinates of points in the O coordinate system will be referred to as (X1, Y1, Z1), and coordinates of points in image planes A and B will be referred to as (x1, y1) and (x2, y2), respectively. The point with coordinates (x1, y1) in image plane B will be at

\[ X_L = x_1 \cos \alpha + y_1 \sin \alpha \]
\[ Y_L = y_1 \]
\[ Z_L = f \cos \alpha - x_1 \sin \alpha \]  \hspace{1cm} (13)

in the O coordinate system. The point with coordinates (x1, y1) in image A will be at

\[ X_L = x_1 \]
\[ Y_L = y_1 \]
\[ Z_L = f \]  \hspace{1cm} (14)

in the O coordinate system. Now, consider a point in the scene with coordinates (X1, Y1, Z1) whose projection in image B is point (x1, y1). Then, the equation of line O1P may be written as

\[ X_L = \frac{f \sin \alpha + x_1 \cos \alpha}{y_1} \]
\[ Y_L = \frac{f \cos \alpha - x_1 \sin \alpha}{y_1} \]  \hspace{1cm} (15)

The intersection of this line with plane A is obtained by letting ZL = f:

\[ X_L = \frac{f(f \sin \alpha + x_1 \cos \alpha)}{(f \cos \alpha - x_1 \sin \alpha)} \]
\[ Y_L = \frac{f y_1}{(f \cos \alpha - x_1 \sin \alpha)} \]
\[ Z_L = f \]  \hspace{1cm} (16)

To determine the coordinates of the same point in the local coordinate system of image A, we substitute values of X1 and Y1 from relation (14) into relation (16) to obtain

\[ x_1 = \frac{f(f \sin \alpha - x_1 \cos \alpha)}{(f \cos \alpha - x_1 \sin \alpha)} \]
\[ y_1 = \frac{f y_1}{(f \cos \alpha - x_1 \sin \alpha)} \]  \hspace{1cm} (17)

Fig. 5. Image plane A is parallel to the baseline and image plane B which makes an angle \( \alpha \) with image plane A, is obtained by rotating the left camera about the YL-axis clockwise by \( \alpha \). The intersections of rays connecting point O1 to points in image B with plane A produce points that would have been obtained if the camera was not rotated.
Equations (17) show the relation between points in images A and B.

In the same manner, the relation between points in the image of the right camera after rotation counterclockwise by \( \alpha \), and the coordinates of points in the image that would have been obtained if the camera was not rotated, may be obtained from

\[
x_r = \frac{f(-f \sin \alpha + x'_r \cos \alpha)}{(f \cos \alpha + x'_r \sin \alpha)}
\]

\[
y_r = \frac{f y'_r}{(f \cos \alpha + x'_r \sin \alpha)}
\]  \hspace{1cm} (18)

In the following, we assume the amount of rotation of a camera can be measured and is known. Therefore, given images of stereo cameras with converging axes we transform the images as if they were obtained by cameras with parallel optical axes using relations (17) and (18). This transformation is necessary in order to make corresponding points fall on the same scanline and simplify the correspondence process.

5. THE NEW CAMERA GEOMETRY

If two cameras with converging optical axes are separated horizontally as shown in Fig. 6(a), and two vertical mirrors are placed equidistant to the cameras, the reflections of the cameras in the mirrors produce two virtual cameras that fall on top of each other. If the cameras have slight rotational differences about the X or Z axes, the virtual cameras will not match, producing double images. One way to align the cameras is to shift and rotate one with respect to the other until perfectly matched images are obtained. This camera alignment is required so that the optical axes of the cameras converge, and optical axes and the baseline of a stereo camera system fall in the same plane.

Our objective in using the mirrors is, however, not to align the cameras but rather to design a stereo camera system using the geometry of Fig. 6(a). Imagine placing a real camera where the images of perfectly matched cameras were formed. Also suppose the polished sides of the mirrors are reversed so that the images of the real camera in the two mirrors become the virtual cameras as shown in Fig. 6(b). With this geometry, taking an image from the real camera with the mirrors present would be the same as taking two images from the virtual cameras without the mirrors. In this way, with a single shot a stereo image can be obtained that is equivalent to two images obtained from two aligned cameras. This is the basic principle behind the proposed stereo camera system. Since a

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**Fig. 6.** (a) If two hinged mirrors are placed in front of two perfectly aligned cameras such that the mirrors make the same angle with the optical axes of the cameras, then virtual images of the cameras in the mirrors fall exactly on top of each other as shown here. (b) Reversing the polished sides of the mirrors and placing a camera where the virtual camera was formed, produces two virtual cameras as shown here. Taking two images from the cameras shown in (a) without the mirrors would be the same as taking a single image from the camera in (b) with the mirrors present.
single image obtained in the new system contains stereo information, we will call it a stereo image. The left half of a stereo image corresponds to the image obtained from the right virtual camera, while the right half of the image corresponds to the image obtained from the left virtual camera.

Consider the geometry shown in Fig. 7. The real camera is positioned at point O' facing the common axis of the mirrors. Suppose the reflections of the camera in the two mirrors are virtual cameras with centers at O_x and O_y. Then an image obtained from point O' in the presence of the mirrors is the same as two images obtained from two aligned cameras at points O_x and O_y without the mirrors. Suppose the distance of the real camera lens center to the mirrors' common axis is d and the angle between the mirrors is (\pi - 2\alpha); then, the baseline length of the two virtual cameras is

\[ 2B = 2d \sin 2\alpha. \]  

(19)

Using the geometry of Fig. 3 for a two-camera system, and assuming X'Y'Z' is the new world coordinate system with its origin at the real camera lens center, and X', Y', and Z' axes are parallel to X, Y, and Z axes, respectively, the relation between corresponding points in the XYZ and X'Y'Z' coordinate systems may be written as

\[ Z = Z' + d(1 + \cos 2\alpha); \quad X = X'; \quad Y = Y' \]  

(20)

Fig. 7. The geometry of the new camera system. A camera is placed at point O with its optical axis intersecting the common axis of two mirrors. The distance between point O' and the mirrors' common axis is d. The reflections of the camera in the two mirrors are two virtual cameras that have the geometry shown with solid lines in Fig. 4.
or
\[ Z' = Z - d(1 + \cos 2\alpha); \quad X' = X; \quad Y' = Y. \] (21)

Substituting the value of \( Z \) from relation (12) into (21), we obtain
\[ Z' = \frac{2df \sin \alpha}{[(x_1-x_0)-(x_0-x_0)]} - d(1 + \cos 2\alpha). \] (22)

In relation (22) \( x_1 \) and \( x_0 \) are the positions of corresponding points on the same scanline in the left and right halves of a stereo image, after transformation according to relations (17) and (18).

We assume that parameters \( \alpha \) and \( d \) are given, and determine parameters \( f \) and \( (x_0-x_0) \) by calibrating the camera. Having the coordinates of at least two points in the scene and their correspondences in the images, these parameters can be determined from relation (22). Knowing \( f, (x_0-x_0), d, \alpha \), and given the positions of corresponding points \( x_1 \) and \( x_0 \) from the same scanline in the images, we can determine the depth of points in the scene using relation (22).

6. PROPERTIES OF THE NEW CAMERA SYSTEM

The field of vision of the camera is equal to \( 4\alpha \) as shown in Fig. 8(a), and is maximum when \( 4\alpha = (\pi - 2\alpha) \) or \( \alpha = \pi/6 \). When \( \alpha > \pi/6 \) the reflection of a scene in one mirror will appear in the other causing multiple images of the scene in the same mirror. If the width of the mirrors is large enough, the field of vision decreases by decreasing \( \alpha \), as shown by comparing Figs 8(b) and (c). When \( \alpha \) becomes zero, the field of vision of the camera becomes minimum and no point in the scene will be visible in both halves of a stereo image.

Note that by increasing \(\alpha\), the field of vision of the camera increases while simultaneously the distance between corresponding points increases. Since increased distance between corresponding points means more accurate measurement of depth, we see that by increasing \(\alpha\) the same effect as increasing the baseline length in a two-camera system is obtained. This is an obvious result since by increasing \(\alpha\), the single-camera model simulates a two-camera model with a larger baseline length.

The field of vision of the camera decreases as the sizes of the mirrors decrease. This can be concluded by comparing Figs 8(b) and (d). From the geometry of Fig. 8(e) we see that the width of each mirror should be at least \( 2d \sin \alpha \) in order for points in infinity to remain in the field of vision of the camera:
\[ w \geq 2d \sin \alpha. \] (23)

Or, for a fixed \( w \), the angle between the mirrors should be at least equal to \( (\pi - 2\alpha) \), where \( \alpha = \sin^{-1}(w/2d) \), in order for points in infinity to remain visible in both halves of a stereo image. We, therefore, see that \(\alpha\) should at most be equal to the smaller of \(\pi/6\) and \( \sin^{-1}(w/2d) \). For instance, when \( w = 2\) in. and \( d = 4\) in. we find \(\alpha \leq 14^\circ\), and when \( w = 3\) in. and \( d = 2\) in. we find \(\alpha \leq 30^\circ\).

Fig. 8. The field of vision of the camera is equal to \(4\alpha\). (b), (c) By decreasing \(\alpha\), the field of vision decreases. (b), (d) By decreasing the width of the mirrors, the field of vision decreases. (e) In order to see points in infinity, \(\alpha\) should be at most equal to \(\sin^{-1}(w/2d)\). (f) The depth of a fixation point depends on parameter \(\alpha\).
Considering the geometry of Fig. 8(f) and assuming the camera is viewing at a point P in front of it, then if rays from the virtual cameras to point P make an angle $\beta$ with lines $O'O_k$ and $O'O_k$ as shown in Fig. 8(f), we find that the image of point P will appear at the centers of the left and right image halves when

$$\tan \beta = \frac{w - 2d \sin \alpha}{2d \cos \alpha} \quad (24)$$

This shows that given $\alpha$, we can determine the fixation point of the camera by computing $\beta$. Knowing $\beta$, we can determine the depth of the fixation point from

$$Z' = \frac{d \sin 2\alpha \cos (\alpha - \beta)}{\sin (\alpha - \beta)} - d(1 + \cos 2\alpha). \quad (25)$$

Assuming $d = 4$ in. and $w = 2$ in., Fig. 9 shows the depths of fixation points in the scene as a function of parameter $\alpha$. When $\alpha$ changes between 6 and 14 deg, the camera can fixate at points that are at infinity to points that are next to the camera. These values of $\alpha$ ensure that both images cover the same areas in the scene at required depth values. If $\alpha$ is less than 6 deg, although points very far in the scene will be visible in both image halves, the common areas covered by the image halves may not be very large. When $\alpha = 0$ no points in the scene will be visible in both image halves. When $\alpha$ is greater than 14 deg, the camera will fixate at points between the camera and the mirrors.

The mirrors used in the design of this camera system should have a homogeneous reflection property. Mirrors with dielectric surfaces should be avoided because they attenuate the intensity of reflected light. In a dielectric mirror, the intensity of specular reflections depends on the angle of incidence of the light, and since points that are off-center in the scene will have different angles with the mirrors, they will have different intensities in the left and right image halves.

The mirrors should be installed in front of the camera such that the optical axis of the camera makes an angle either slightly smaller or larger than 90° with the common axis of the mirrors as shown in Fig. 10. Otherwise, the camera itself will appear in the obtained images. Note that rotating the camera about the $X'$

<table>
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<th>$\alpha$</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>$\infty$</td>
<td>103</td>
<td>30</td>
<td>10</td>
<td>5</td>
<td>2</td>
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Fig. 9. Depth of a fixation point as a function of parameter $\alpha$ when $d = 4$ in and $w = 2$ in. $\alpha$ is in deg and $Z'$ is in in. When $\alpha < 6°$, as $\alpha$ decreases the common area covered by both image halves becomes smaller. When $\alpha$ becomes larger than 14 deg, mostly areas that are between the camera and the mirrors will be seen in the image halves.
axis (see Fig. 7) will not change any of the above formulas. Only the virtual cameras will be rotated by the same amount. To install the mirrors symmetric with respect to the optical axis of the camera, they should be adjusted such that the images of any point in the plane formed by the real camera’s lens center and the common axis of the mirrors appear equidistant to the image of the mirrors’ axis in a stereo image.

Examples of stereo images obtained by the proposed camera are shown in Figs 11(a) and 12(a). The mirrors were fixed to the camera in such a way that the image of the mirrors’ common axis appeared vertical in obtained images. The obtained images are reflected about the image of the mirrors’ axis and transformed using relations (17) and (18), before carrying out the correspondences and measuring the depth values from the correspondences. Images 11(a) and 12(a) were digitized as shown in Figs 11(b) and 12(b) and then reflected about the mirrors’ common axis to obtain images 11(c) and 12(c). After transforming the left half of images 11(c) and 12(c) according to relation (17) and transforming the right half of images 11(c) and 12(c) according to relation (18), images of Figs 11(d), 11(e), 12(d), and 12(e) were obtained. These are the images which appear as if obtained by cameras with parallel optical axes. In this experiment, images 11(a) and 12(a) were obtained by a regular camera and the images were then digitized with a scanner to obtain images 11(b) and 12(b). This step is not needed if a digital camera which is connected to a frame grabber is used to obtain the images. The hinged mirrors shown in Fig. 10 work like an adapter which can be attached to a regular camera to transform it to a stereo camera.

Formulations derived in this paper are based on the assumption that the optical axes of the camera make the same angle with the mirrors. This, however, is not
a requirement and is used only for convenience. If the optical axis of the camera makes angles $\alpha$ and $\beta$ with lines normal to the left and right mirrors, respectively, we should replace $\alpha$ in relations (17) and (18) by $\alpha$ and $\beta$, respectively. Formula (22) which determines depths of points from disparities of images obtained by transformation of the left and right halves of a stereo image using formulas (17) and (18) will remain unchanged. Figure 13(a) shows a stereo image in which the right mirror was normal to the optical axis of the camera, but the left mirror was rotated by 6 deg. Digitization and reflection of this image about the mirrors' common axis are shown in Figs 13(b) and (c), respectively. The images to be used in the correspondence process are shown in Figs 13(d) and (e). These images are obtained by transforming the left half of image 13(c) using relation (17) with $\beta = 6^\circ$ and the right half of image 13(c) using relation (18) with $\alpha = 6^\circ$.

Except for the fact that the new camera system obtains stereo information in a single shot, it provides the same property as two perfectly aligned cameras with exactly the same optical properties. This camera eliminates inter-camera distortions; however, intra-camera distortions still exist between stereo image
Fig. 12. (a) Stereo image of an indoor scene obtained with $x = 8\text{"}$. (b)–(e) Same as in Figs 11(b)–(e), respectively.
Fig. 13. (a) A stereo image with $\alpha_1 = 6^\circ$ and $\alpha_2 = 0^\circ$. (b), (c) Same as in Figs 11(b) and (c). (d), (e) Left and right images obtained by transforming the left half of (c) using relations (17) with $\alpha = 6^\circ$ and the right half of (c) using relation (18) with $\alpha = 0^\circ$. 
halves and should be removed by a process known as the decalibration. The decalibration process transforms an image obtained by a regular camera to one obtained by a pin-hole camera.

7. CONCLUSIONS

A camera system that can obtain stereo information in a single shot and through a single lens was introduced. This camera system has the following properties:

(1) A single image obtained by this camera is equivalent to two images obtained from two perfectly aligned cameras with exactly the same optical properties.

(2) Inter-camera distortions no longer exist since a single camera is used to obtain the images, but intra-camera distortions may still exist which should be reduced by a decalibration process.

(3) From the angle between the mirrors, the depth of the camera fixation point can be determined from relations (24) and (25).

(4) The angle between the mirrors should be larger than $2\pi/3$ to avoid the reflection of a scene in one mirror appearing in the other.

(5) In order for points in infinity to be visible in both halves of a stereo image, it is required that the angle between the mirrors be larger than $(\pi - 2\alpha)$, where $\alpha = \sin^{-1}(w/2d)$, $w$ is the width of the mirrors, and $d$ the distance of the camera lens center to the common axis of the mirrors.

(6) The field of vision of the camera increases with the decrease of the angle between the mirrors.

(7) Decreasing the angle between the mirrors has the same effect as increasing the baseline length in a two-camera system. Therefore, as the angle between the mirrors changes, depth perception accuracy changes also.

This camera system provides the desirable properties mentioned in the Introduction and is useful in applications where the field of vision of the camera should be changed to fixate at a required depth in a scene.

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