Applications of Dense Media Radiative Transfer Theory for Passive Microwave Remote Sensing of Foam Covered Ocean

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Abstract—The effect of the foam covered ocean surface on the passive microwave remote sensing measurements is studied based on the electromagnetic scattering theory. In formulating an electromagnetic scattering model, we treat the foam as densely packed sticky air bubbles coated with thin seawater coating. The layer of foam covers the ocean surface that has air bubbles. We then use dense media radiative transfer (DMRT) theory with quasi-crystalline approximation (QCA) for densely distributed sticky moderate size particles to calculate the brightness temperatures of the foam-covered ocean surface. Results are illustrated for 19 GHz and 37 GHz and for both vertical and horizontal polarizations as a function of foam microstructure properties and foam layer thickness. Comparisons are also made with experimental measurements.

Index Terms—Dense media radiative transfer, electromagnetic wave scattering, microwave emissivity.

I. INTRODUCTION

O estimate the effect of the foam above the ocean surface on the passive microwave remote sensing measurements, various empirical microwave emissivity models were used [1]–[6]. Williams [1] measured emissivities of foam in a waveguide and found that at X-band, the emissivity of foam depends strongly on the thickness of the foam layer. Wilheit’s model [2] treats foam as having neither polarization nor viewing angle dependence. In Pandey’s empirical emissivity model [3], the effect of foam was taken into account by coupling theoretical expressions of specular ocean surface emissivity with empirical expressions from ocean tower observations and from the analysis of published measurements. Smith [4] described the brightness temperature over foam-covered ocean as a function of incidence angle and frequency. He related the emissivities of foam at the three channels (vertical polarization at 19 GHz and both polarizations at 37 GHz) to one another by linear regression. Stogryn [5] used a least squares fit of a polynomial to the data of measurements and derived an expression for the foam emissivity as a function of incidence angle and frequency. All these models are empirical fitting procedures using experimental data. The empirical models do not take into account the physical microstructure properties of foam and the foam layer thickness.

It is known that foam on the ocean surface can affect the brightness temperatures measured by microwave radiometers. However, there is relatively little known definitively concerning how foam affects such fundamental parameters like the drag coefficient or the impact of foam on the retrieval of the ocean surface wind vector from satellite-mounted microwave instruments. This gap in knowledge is due in large part to the difficulty in making measurements at high wind speeds, when significant foam coverage is present. Understanding how foam increases microwave emissivity, and developing quantitative emissivity models for a foam-covered water surface, would be of great help in evaluating radiometer performance at high wind speeds.

The subject of foam dynamics has attracted attention recently. Controlled experiments are performed for foam dynamics and microwave emissivity measurements [6]. The microstructures of foam are also studied [7]. In this paper, we apply the recently developed dense media radiative transfer (DMRT) theory [8] based on sticky particle model to analyze this problem, taking into account the physical microstructure properties of foam and the foam layer thickness.

We model the foam as densely packed sticky air bubbles coated with thin seawater coating. The model of medium with densely distributed coated particles of moderate size can be used. In the past, both theoretical and experimental studies [9], [10] of the propagation and scattering of waves in dense media show that the assumption of independent scattering is not valid and the correlated scattering effects and coherent near field interaction effects must be taken into account.

The quasi-crystalline approximation (QCA) accounts for the pair distribution function of the particle positions and coherent wave interactions [11]. The Percus–Yevick equation, which compares well with the Monte Carlo simulations [12], is used to describe the pair distribution functions [11]. The scattering results of QCA also compare well with the Monte Carlo simulations of exact solutions of Maxwell’s equations [10] of randomly distributed finite size spheres. Recently, we have extended the dense media theory to include cases when the particles have sticky force that make them adhere to form aggregates [8]. A key difference between sticky particle model and nonsticky particle model is the varied frequency dependence of scattering by changing the stickiness parameter $\tau$.

We will focus on the foam impact. By using the distorted Born approximation with QCA, we calculate numerical results.
of the complex effective propagation constants, scattering coefficients and phase matrix, of densely packed air bubbles coated with sea water. The brightness temperature of foam-covered ocean surface can be simulated. We describe the dynamic properties of foam in Section II. In Section III, the QCA theory for densely packed coated particles of moderate sizes is presented. For particles of moderate size, the QCA equations are formulated in terms of the \( T \)-matrix formulism and utilizing vector spherical waves as basis functions. Also, we discuss the pair distribution functions of sticky particles of multiple sizes based on the Percus–Yevick pair distribution functions. In Section IV, the DMRT theory is discussed. Derived from QCA, the DMRT theory describes the scattering, absorption and emission of a dense medium and relates the physical parameters of air bubbles coated with sea water in foam, to the brightness temperatures. The theoretical results of brightness temperatures with typical parameters of foam in passive remote sensing at 19 GHz and 37 GHz are illustrated numerically in Section V. Comparisons are also made with experimental measurements for vertical and horizontal polarizations [6].

II. DYNAMIC DESCRIPTION OF FOAM

In order to model the microwave radiometric properties of foam, it is critical to have an accurate model for the physical structure of the bubbles therein. Foam on a water surface is a dynamic system, and the defining physical properties such as mean bubble size and liquid water content are functions of the age of the foam. In general, the mean bubble size increases with age as bubbles coalesce and the water content decreases as water drains from the interstitial spaces [7]. In addition, the shape of the bubbles changes from nearly spherical for very young foams with relatively large liquid water content to polyhedrons for aged foams with small amounts of water in the interstitial spaces. Modeling the radiometric properties of foam is greatly simplified if it can be assumed that foam on the ocean surface is composed of nearly spherical bubbles. It is also necessary to have realistic values for the foam void fraction (i.e., the ratio of the volume of air to the total volume of the foam) and mean bubble size.

Fig. 1 is a video photograph of the bubble structure of foam on the ocean surface. The image was acquired using a partially submerged underwater CCD video camera equipped with a macro telecentric lens whose focal plane lay precisely at the front window of the camera housing. The overall thickness of the foam layer was on the order of 5 cm and it is estimated that this image was acquired in the bottom half of the foam layer. Because the foam is formed from bubbles rising to the surface from below and the lifetime of the foam layer was on the order of 4 s, this image is therefore of foam having a mean age of under 2 s. In agreement with previous data on the bubble structure of young water-based foams [7], the majority of bubbles are nearly spherical and the average thickness of the interstitial water layer between bubbles in the foam is seen to be on the order of 80 \( \mu \text{m} \). Furthermore, digital image analysis of 20 similar foam images showed that the mode of the bubble size distribution is approximately 500 \( \mu \text{m} \). Based on these numbers, the void fraction of the foam is on the order of 0.75 (i.e., 75% of the total volume of the foam is air). These values were used in formulating the physical model for the foam used in the electromagnetic scattering calculations.

III. QUASI-CRYSTALLINE APPROXIMATION FORMULATION FOR MEDIA WITH MODERATE SIZE COATED PARTICLES

Consider a distribution of \( N \) coated particles centered at \( \vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N \) in a volume \( V \). We assume that there are \( L \) species of coated particles in the medium. The physical and geometric structure of a spherical dielectric coated particle is shown in Fig. 2, where \( s_j \) denotes that the coated particle is of the \( j \)-th species. The random distributed coated particles need not be identical in size, permittivity and the thickness of the coating. However, interpenetration of particles is not allowed. Let the incident wave impinging in the direction of \( \vec{k}_i \). The Foldy-Lax multiple scattering equation can be written in matrix form as

\[
\tilde{w}^{(f)} = \sum_{l=1}^{N} \tilde{\sigma}(k \vec{k}_i \vec{k}_l) \tilde{w}^{(l)} + \epsilon(\vec{k}_i, \vec{k}_j) \tilde{\sigma}_{\text{vac}}.
\]
where \( \tilde{\mathbf{w}}^{(j)} \) and \( \tilde{\mathbf{w}}^{(l)} \) are field exciting coefficients of the \( j \)th particle and the \( l \)th particle, respectively; 
\[
\tilde{T}^{(l)}
\]
\( T \)-matrix of the \( l \)th particle; 
\( \overline{\sigma}_{\text{inc}} \) incident field coefficient.

Equation (1) can be interpreted as the field exciting the \( j \)th particle is the sum of incident wave and the scattered wave from all other particles. The matrix \( \tilde{\mathbf{u}}(k\mathbf{r};\mathbf{r}'j) \) is the matrix that describes the vector translation formula transforming spherical waves centered at \( \mathbf{r}' \) to spherical waves centered at \( \mathbf{r} \). By taking the conditional average by holding position \( \mathbf{r}' \) fixed, we have

\[
\overline{\mathbf{w}}(\mathbf{r}') = E_j \left( \tilde{\mathbf{w}}^{(j)} \right) = \sum_{l \neq j}^N \int d\mathbf{r}'' \tilde{\mathbf{u}}(k\mathbf{r};\mathbf{r}'j) p(\mathbf{r''}\mathbf{r}'j) \tilde{T}^{(l)} E_j \left( \tilde{\mathbf{w}}^{(l)} \right) + e^{i (\overline{\sigma}_{\text{inc}} - \overline{\sigma}' j)} \overline{\sigma}_{\text{inc}}.
\]

Equation (2) can be interpreted as the field exciting the \( j \)th particle is the sum of incident wave and the scattered wave from all other particles. The matrix \( \tilde{T}^{(l)} \) is the matrix that describes the vector translation formula transforming spherical waves centered at \( \mathbf{r}' \) to spherical waves centered at \( \mathbf{r} \). By taking the conditional average by holding position \( \mathbf{r}' \) fixed, we have

\[
\tilde{T}^{(l)} = \sum_{l \neq j}^N \int d\mathbf{r}'' \tilde{\mathbf{u}}(k\mathbf{r};\mathbf{r}'j) p(\mathbf{r''}\mathbf{r}'j) \tilde{T}^{(l)} E_j \left( \tilde{\mathbf{w}}^{(l)} \right) + e^{i (\overline{\sigma}_{\text{inc}} - \overline{\sigma}' j)} \overline{\sigma}_{\text{inc}}.
\]

Equation (3) can be interpreted as the field exciting the \( j \)th particle is the sum of incident wave and the scattered wave from all other particles. The matrix \( \tilde{T}^{(l)} \) is the matrix that describes the vector translation formula transforming spherical waves centered at \( \mathbf{r}' \) to spherical waves centered at \( \mathbf{r} \). By taking the conditional average by holding position \( \mathbf{r}' \) fixed, we have

\[
\overline{\mathbf{w}}(\mathbf{r}') = E_j \left( \tilde{\mathbf{w}}^{(j)} \right) = \sum_{l \neq j}^N \int d\mathbf{r}'' \tilde{\mathbf{u}}(k\mathbf{r};\mathbf{r}'j) p(\mathbf{r''}\mathbf{r}'j) \tilde{T}^{(l)} E_j \left( \tilde{\mathbf{w}}^{(l)} \right) + e^{i (\overline{\sigma}_{\text{inc}} - \overline{\sigma}' j)} \overline{\sigma}_{\text{inc}}.
\]

The conditional averaged exciting field coefficients obey the following homogeneous system of equations (Lorentz–Lorenz law). Let \( \overline{\mathbf{w}}(\mathbf{r};\mathbf{r}'j) \) be expanded into vector spherical waves with \( \overline{Y}^{(s_j)}(m) \) and \( \overline{Y}^{(s_j)}(N) \) as coefficients where \( \nu = 1, 2, \ldots, (M) \) and \( (N) \) represents the \( M \) and \( N \) vector spherical waves, respectively

\[
\overline{Y}^{(s_j)}(M) = -2\pi \sum_{s_l=1}^L n_{s_l} \sum_{n_{j p}}^L (2n_{j p} + 1) S_p(k; R_{s_j s_l}) \
\times \left[ \{a(1, n_{j p} - 1, n_{j p}) A(n_{j p} p, p - 1) B(n_{j p} p) \right. \
\times T^{(s_l)(M)} Y^{(s_l)(M)}(N) \
\left. + a(1, n_{j p} - 1, n_{j p} p - 1) B(n_{j p} p) \right. \
\times T^{(s_l)(N)} Y^{(s_l)(N)}(N) \right] 
\]

(5)

\[
\overline{Y}^{(s_j)}(N) = -2\pi \sum_{s_l=1}^L n_{s_l} \sum_{n_{j p}}^L (2n_{j p} + 1) S_p(k; R_{s_j s_l}) \
\times \left[ \{a(1, n_{j p} - 1, n_{j p} p - 1) B(n_{j p} p) \right. \
\times T^{(s_l)(M)} Y^{(s_l)(M)}(N) + a(1, n_{j p} - 1, n_{j p} p) \right. \
\times A(n_{j p} p) T^{(s_l)(N)} Y^{(s_l)(N)}(N) \right].
\]

(6)

In (5) and (6)

\[
S_p(k; R_{s_j s_l}) = -\frac{R_{s_j s_l}^2}{K^2 - K^2} \left[ h_p(kR_{s_j s_l}) j_p(kR_{s_j s_l}) \right. \
\times - K h_p(kR_{s_j s_l}) j_p(kR_{s_j s_l}) \right. \
\times + \int_{R_{s_j s_l}}^\infty d\mathbf{r}'' \left[ g_{s_j s_l}(r) - 1 \right] \times h_p(kr) j_p(kr)
\]

(7)

where \( A(n_{j p} p) \) and \( B(n_{j p} p) \) are coefficients defined in [11, p. 496], \( a(1, n_{j p} - 1, n_{j p}) \) and \( a(1, n_{j p} - 1, n_{j p} p - 1) \) are terms of Wigner 3-j symbols defined in [11, pp. 449–450]; \( K = K' + iK'' \) is the effective propagation constant; \( h_p \) and \( j_p \) are spherical Bessel function and its derivative; \( h'_p \) and \( j'_p \) are spherical Hankel function and its derivative. The scattering coefficients \( T^{(M)} \) and \( T^{(N)} \), for coated spheres of \( j \)th species with outer radius \( a_{s_l} \), inner radius \( b_{s_l} \), core wavenumber \( k_{s_l} \) and shell wavenumber \( k_{s_l} \), are those of Mie scattering \( T \)-matrix coefficients expressed by [14] in (8) and (9), shown at the bottom of the page.
where

\[ B_n^q = \frac{[\xi_n(j_n(\xi_n))'] j_n(\eta_q) - [\eta_q(j_n(\xi_n))'] j_n(\xi_n)}{[\xi_n(j_n(\xi_n))'] j_n(\eta_q) - [\eta_q(j_n(\xi_n))'] j_n(\xi_n)} \]  

(10)

and

\[ A_n^q = \frac{[\xi_n(j_n(\xi_n))'] [\eta_q(j_n(\xi_n))'] - [\eta_q(j_n(\xi_n))'] [\xi_n(j_n(\xi_n))']}{[\xi_n(j_n(\xi_n))'] [\eta_q(j_n(\xi_n))'] - [\eta_q(j_n(\xi_n))'] [\xi_n(j_n(\xi_n))']} \]  

(11)

and \( j_n(\cdot) \) is the spherical Neumann function. The symbols \( \rho_n, \xi_n, \eta_n \) are defined as \( \rho_n = k_n q, \xi_n = k_n b_n, \eta_n = k_n b_n \), respectively.

The ratios of exciting field coefficients \( Y_n^{(s_j)(M)} \) and \( Y_n^{(s_j)(N)} \) are determined by setting the determinant of (5) and (6) equal to zero. The determinant equation gives the solution for the effective propagation constant \( K \). They can be solved numerically by using Muller’s method to search the root \( K \). Therefore, the value of \( Y_n^{(s_j)(M)} \) and \( Y_n^{(s_j)(N)} \) can be determined within a single constant. The effective propagation constant \( K_n \) is complex, \( K_n = K_n' + iK_n'' \), and the extinction rate is \( \kappa_n = 2K_n'' \). In principle, both \( K_n \) and \( K_n'' \) can be computed by the Lorentz–Lorenz law. However, since \( K_n' \gg K_n'' \), calculating \( K_n'' \) in this manner may not have the desired accuracy. Also, the imaginary part \( K_n'' \) is affected by the particle-pair distribution functions that have sharp peaks that may be difficult to evaluate numerically. To ensure energy conservation, we proceed as follows. We use the real part \( K_n' \) as calculated from (5) and (6) of Lorentz–Lorenz law. We also use the relative values of \( Y_n^{(s_j)} \)'s as determined by (5) and (6). For the single constant, it is determined by Ewald–Oseen extinction theorem. The generalized Ewald–Oseen extinction theorem is obtained by balancing the incident wave term and the term of the same phase dependence that is a result of the integral in (4). We then use the coherent exciting field to calculate the absorption coefficient. The distorted Born approximation is applied to calculate the phase matrix elements and the scattering coefficient. The phase matrix is the bistatic cross section per unit volume. In passive remote sensing, we further integrate over the azimuthal angle \( \phi \)

\[ p_{n,\beta}(\theta, \phi') = \int_0^{2\pi} d\phi p_{n,\beta}(\theta, \phi; \phi', \phi = 0) \]  

(12)

where \( \alpha, \beta = v, h \), and \( P_{n,\beta} \) is given by [8, eqs. (37)–(40)]. Then, the extinction coefficient is calculated by the addition of absorption and scattering coefficients.

By employing the coherent exciting field on a particle \( \psi^{(s_j)(P)}(r') \) as solved by QCA, we can obtain the reflection coefficient which is given in [8] as

\[ R = \sum_{s_j=1}^{L} \frac{2\pi n_{s_j} 2}{k_{n_s}^2 (k_n^2 + k_{n_s}^2)} \sum_n (-1)^n \frac{(2n+1)}{n(n+1)} \]  

\[ \times \left\{ \frac{T_n^{(s_j)(M)} Y_n^{(s_j)(M)}(Y_n^{(s_j)(M)}(V)) P_n^1(\cos(\beta_i + \theta_i))}{\sin(\theta_i + \theta_t)} + T_n^{(s_j)(N)} Y_n^{(s_j)(N)}(Y_n^{(s_j)(N)}(V)) \frac{\cos(\theta_i + \theta_t)}{P_n^1(\cos(\theta_i + \theta_t))} \right\} \]  

(13)

where

\[ P_n^1 \]  

associated Legendre polynomial of \( n \)th order and degree 1;

\[ P_n \]  

Legendre polynomial;

\( \theta_i \) and \( \theta_t \) incident and transmitted angles, respectively.

The general expressions for the complex dielectric constant may then be calculated by the following expressions:

\[ \begin{align*}
\kappa_n &= \frac{2\pi}{k_n^2} F \sum_{s_j=1}^{L} \sum_{n=1}^{N} (2n+1) \left| Y_n^{(s_j)(M)}(V) \right|^2 \times \left( \operatorname{Re} T_n^{(s_j)(M)} - \left| T_n^{(s_j)(M)} \right|^2 \right) + \left| Y_n^{(s_j)(N)}(V) \right|^2 \\
&\quad \times \left( \operatorname{Re} T_n^{(s_j)(N)} - \left| T_n^{(s_j)(N)} \right|^2 \right) \\
&= \int_0^\pi d\theta \sin \theta \left( \rho_{n_\alpha}(\theta, \theta') + \rho_{n_\beta}(\theta, \theta') \right) \\
&\quad + \int_0^\pi d\theta \sin \theta \left( \rho_{n_\alpha}(\theta, \theta') + \rho_{n_\beta}(\theta, \theta') \right) \\
\end{align*} \]  

(14)

The scattering coefficient can be calculated by the following expressions:

\[ \begin{align*}
\kappa_n &= \int_0^\pi d\theta \sin \theta \left( \rho_{n_\alpha}(\theta, \theta') + \rho_{n_\beta}(\theta, \theta') \right) \\
&\quad + \int_0^\pi d\theta \sin \theta \left( \rho_{n_\alpha}(\theta, \theta') + \rho_{n_\beta}(\theta, \theta') \right) \\
\end{align*} \]  

(15)

The extinction coefficient is \( \kappa_n = \kappa_n + \kappa_n \) and the Albedo is \( \omega = \kappa_n / \kappa_n \). The effective propagation constant \( K_n = K_n' + iK_n'' \).

Physically, the air bubbles in the foam layer adhere to each other, which is shown in Fig. 1. To simulate the foam’s effect, we use the sticky particle model in which the particles are allowed to adhere together to form clusters to better represent the physical property of the foam. The sticky particle has a stickiness parameter \( \tau \). The smaller the \( \tau \) is, the more sticky the particles are. The Percus–Yevick approximation of the pair distribution function \( g(r) \) for the sticky spherical particles can be solved analytically using the factorization method of Baxter [15]. The calculations of \( g(r) \) are given by equations in [16]. In the following sections, we use particles of identical size but with different thicknesses of coating.

IV. DENSE MEDIA RADIATIVE TRANSFER THEORY

Consider thermal emission from a layered medium with coated dielectric particles embedded in a background medium, as indicated in Fig. 3. The layer consisting of coated particles (in region 1) covers a half space of ocean (region 2) embedded with air bubbles. The radiative transfer (RT) equations for passive remote sensing in region 1 are in the following form:

\[ \cos \theta \frac{dI_v}{dz} (\theta, z) = -\kappa_v I_v (\theta, z) + \kappa_v C T \\
+ \int_0^\pi d\theta \sin \theta \rho_{v,\alpha}(\theta, \theta') I_v (\theta', z) \\
+ \rho_{v,\beta}(\theta, \theta') I_v (\theta', z) \]  

(16)

\[ \cos \theta \frac{dI_h}{dz} (\theta, z) = -\kappa_h I_h (\theta, z) + \kappa_h C T \\
+ \int_0^\pi d\theta \sin \theta \rho_{h,\alpha}(\theta, \theta') I_v (\theta', z) \\
+ \rho_{h,\beta}(\theta, \theta') I_v (\theta', z) \]  

(17)
Fig. 3. Geometrical configuration for thermal emission from foam covered ocean. The foam layer is region 1 and is absorptive and scattering. Region 2 is air bubbles embedded in sea water and is absorptive.

where

\[ C \text{ equal to } K_b k^2 / (\lambda^2 k^2); \]

\[ K_b \text{ Boltzman's constant; } \]

\[ I_v, I_h \text{ vertical and horizontal specific intensities, respectively.} \]

The boundary conditions are the Fresnel type using the effective propagation constant for the dense medium. For the upper boundary, the air/region 1 interface at \( z = 0 \)

\[ R_v(\theta) = r_v(\theta) \]
\[ R_h(\theta) = r_h(\theta) \]

where \( r_v \) and \( r_h \) are the Fresnel reflectivities with effective propagation constant \( K \)

\[ r_v(\theta) = \left| \frac{k^2 \cos \theta - k' (k^2 - k'^2 \sin^2 \theta)^{1/2}}{k^2 \cos \theta + k' (k^2 - k'^2 \sin^2 \theta)^{1/2}} \right|^2 \]
\[ r_h(\theta) = \left| \frac{k' \cos \theta - (k^2 - k'^2 \sin^2 \theta)^{1/2}}{k' \cos \theta + (k^2 - k'^2 \sin^2 \theta)^{1/2}} \right|^2 \]

In the lower boundary, the region 1 and region 2 interface, \( z = -d \)

\[ R_{v1}(\theta) = r_{v1}(\theta) \]
\[ R_{h1}(\theta) = r_{h1}(\theta) \]

with (24) and (25), shown at the bottom of the page, where \( \varepsilon_2 \) is the permittivity of the media in region 2. For the case of foam covered ocean, region 2 consists of air bubbles embedded in the ocean background so that we will use the effective permittivity \( \varepsilon_{eff2} \) of medium 2 in this set of equations, instead of \( \varepsilon_2 \). Region 2 is assumed to be absorptive. To calculate \( \varepsilon_{eff2} \), a simple physical model based on induced dipoles is used [13]. Let \( \varepsilon_{w} \) denote the permittivity of the seawater, \( f_a \) the fractional volume occupied by the air bubbles. Then the effective permittivity \( \varepsilon_{eff2} \), is given by the Maxwell-Garnett mixing formula

\[ \varepsilon_{eff2} = \varepsilon_w \frac{1 + 2 f_a y}{1 - f_a y} \]

where

\[ y = \frac{\varepsilon_0 - \varepsilon_w}{\varepsilon_0 + 2 \varepsilon_w}. \]

Note that the effective permittivity \( \varepsilon_{eff2} \) here does not include scattering attenuation which is small due to the fact that the seawater is heavily absorptive.

After solving the eigenvalue problem and imposing on the boundary conditions, the brightness temperatures in the direction \( \theta_0 \), where \( \theta_0 = \sin^{-1}(k' \sin \theta / k) \) is related to \( \theta \) by Snell’s law, are given by

\[ T_{B_v}(\theta_0) = \frac{1}{C}(1 - R_v(\theta))I_v(z = 0, \theta) \]
\[ T_{B_h}(\theta_0) = \frac{1}{C}(1 - R_h(\theta))I_h(z = 0, \theta). \]

V. NUMERICAL SIMULATIONS OF BRIGHTNESS TEMPERATURE AND COMPARISON WITH EXPERIMENTAL MEASUREMENTS

In the following, we illustrate the numerical results of the brightness temperatures based on the sticky particle model. The effective propagation constant, scattering rate, extinction rate,
albedo, and the phase functions are first calculated by QCA. Consequently, these parameters are put into the RT equations to compute the brightness temperatures.

As described in the foregoing sections, the foam is modeled as densely packed sticky air bubbles with thin seawater coating and the background medium is air. In this model, region 0 is air, region 1 is foam layer consists of coated air bubbles embedded in a background of air, and region 2 consists of air bubbles and ocean as its background. The geometrical configuration is shown in Fig. 3.

We define the foam parameters as follows:

- $d$ foam layer thickness;
- $a_j$ outer radius of coated air bubbles of the $j$th species;
- $b_j$ inner radius of coated air bubbles of the $j$th species;
- $f_j$ fractional volume of coated air bubbles of the $j$th species

\begin{equation}
    f_j = \frac{4\pi}{3} r_j f_j^2
\end{equation}

- $f$ total fractional volume of air bubbles

\begin{equation}
    f = \sum_{j=1}^{N_s} f_j
\end{equation}

- $N_s$ number of species, we choose two in the numerical simulations;
- $f_w$ fractional volume of sea water in foam

\begin{equation}
    f_w = \sum_{j=1}^{N_s} f_j \left[ 1 - \left( \frac{b_j}{a_j} \right)^3 \right]
\end{equation}

- $\tau$ stickiness parameter for the sticky model;
- $\varepsilon_w$ permittivity of seawater;
- $T$ the temperature of the foam;
- $\theta$ observation angle;
- $f_0$ fractional volume of air bubble in medium 2 (used in (26) to calculate $\varepsilon_{eff2}$).

Note that $a_j - b_j$ is the thickness of coating of the $j$th species. And we assume that the permittivity of the sea water in this layer is the same as that in foam, also they have the same temperature.

According to the microstructure of foam, $f_w$ is about 4%. The diameter of coated air bubbles $2r_j$ ranges from 200 $\mu$m to several millimeters. The thickness of the coating of the air bubbles in foam varies. The foam extends to a layer thickness $d$ ranging from several millimeters to several centimeters. Note that the permittivity of seawater $\varepsilon_w$ is a function of the frequency and other physical parameters such as the temperature $T$ and salinity. To calculate the permittivity of seawater at microwave remote sensing frequencies, we set the temperature of 284 K and salinity of 20 per thousand, and the model of Klein and Swift [17] is applied. Based on this model, the permittivity of the sea water at 19 GHz and 37 GHz are $28.9541 + i36.8340$ and $13.2444 + i24.5221$, respectively.

First, we present the absorption coefficient dependence on the coating thickness for several $f_w$ in Fig. 4. It is clear that, with the same water fractional volume $f_w$, the absorption coefficient $\kappa$ for air bubbles with thinner coating is much larger than that of bubbles with thicker coating. Also, while keeping the coating thickness fixed, the absorption coefficient will increase with the increase of water fractional volume. On the other hand, we can see that the coated air bubble is more absorptive at 37 GHz than that at 19 GHz.

The following results show the brightness temperatures as a function of the observation angle, the thickness of the foam layer, and the size of the coated air bubbles in the foam layer. All the parameters for the simulation results shown in Figs. 5–8 are listed in Table I. For simplicity, we have used two species of coated air bubbles having identical size but with different
coating thickness and fractional volume, namely thicker coated one with small fractional volume and thinner coated one with larger fractional volume, in the foam description.

Fig. 5 shows the calculated brightness temperatures as a function of observation angle at 19 GHz and 37 GHz, for both horizontal polarization and vertical polarization, respectively. In Figs. 6–8, we plot the brightness temperatures of horizontal polarization and vertical polarization, at 19 GHz and 37 GHz also, as a function of thickness of the foam layer with different size of the coated air bubbles. The diameters of air bubbles are 500 µm, 500 µm, 1000 µm, and 1500 µm in Figs. 4–7, respectively. As the size of bubbles increases the scattering effects increases and the albedo also increases, which is reflected by the decrease of the corresponding saturated brightness temperatures. The observation angle is 53° for Figs. 5–7. Fig. 5 shows the polarization dependence as a function of viewing angles. We note that 19 GHz has a stronger polarization dependence than 37 GHz. The polarization difference are respectively 43 K and 32 K at 53° observation angle. Fig. 6 indicates that, once the brightness temperatures at 19 GHz increase to the values of that at 37 GHz, then saturations for 19 GHz take place. The frequency dependence is weak after saturation. The polarization dependence is also weak for a large thickness of foam layer of small coated air bubbles. From these three figures, we can conclude that as
TABLE II
NUMERICAL RESULTS FROM THE QCA-STICKY SIMULATION FOR FIGS. 5 AND 6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Albedo</th>
<th>Extinction Rate $\kappa_{a}/k$</th>
<th>Effective Permittivity $\varepsilon_{eff}$</th>
<th>$ka$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 GHz</td>
<td>0.0080</td>
<td>0.0165</td>
<td>1.1365 + i0.0176</td>
<td>0.0995</td>
</tr>
<tr>
<td>37 GHz</td>
<td>0.0377</td>
<td>0.0151</td>
<td>1.1306 + i0.01604</td>
<td>0.1937</td>
</tr>
</tbody>
</table>

TABLE III
NUMERICAL RESULTS FROM THE QCA-STICKY SIMULATION FOR FIG. 7

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Albedo</th>
<th>Extinction Rate $\kappa_{a}/k$</th>
<th>Effective Permittivity $\varepsilon_{eff}$</th>
<th>$ka$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 GHz</td>
<td>0.0407</td>
<td>0.0098</td>
<td>1.1395 + i0.0105</td>
<td>0.1990</td>
</tr>
<tr>
<td>37 GHz</td>
<td>0.1642</td>
<td>0.0140</td>
<td>1.1394 + i0.0150</td>
<td>0.0875</td>
</tr>
</tbody>
</table>

TABLE IV
NUMERICAL RESULTS FROM THE QCA-STICKY SIMULATION FOR FIG. 8

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Albedo</th>
<th>Extinction Rate $\kappa_{a}/k$</th>
<th>Effective Permittivity $\varepsilon_{eff}$</th>
<th>$ka$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 GHz</td>
<td>0.1147</td>
<td>0.0099</td>
<td>1.1481 + i0.0107</td>
<td>0.2604</td>
</tr>
<tr>
<td>37 GHz</td>
<td>0.2466</td>
<td>0.0200</td>
<td>1.1579 + i0.0210</td>
<td>0.5612</td>
</tr>
</tbody>
</table>

the thickness of the foam layer increases, the brightness temperatures of all four channels will increase correspondingly and then saturate at particular thickness of the foam layer. For vertical polarization, the saturation at 19 GHz occurs at 21.9 cm, 16.4 cm and 15.9 cm, for the three cases of coated air bubble size of 500 $\mu$m, 1000 $\mu$m, and 1500 $\mu$m, respectively. At 37 GHz, saturation of vertical polarization occurs at smaller layer thickness of 8.3 cm, 4.5 cm, and 4.0 cm, respectively. The saturation point of horizontal polarization is slightly different from that of vertical polarization. For layer thickness larger than saturation thickness, the difference between brightness temperatures of vertical polarization and horizontal polarization is a function of the size of coated air bubbles. The extinction rates, effective wavenumbers and albedo calculated from QCA for these cases are shown in Tables II–IV, respectively. It can be seen that the seawater is absorptive and absorption dominates over the extinction rates for small particle sizes. However, as the particle size increases, the albedo increases with the size of coated air bubbles. In these tables, the effective permittivity $\varepsilon_{eff}$ of region 1 is defined as $\varepsilon_{eff} = (K/k)^2$, where $K$ is the effective propagation constant and $k$ is the wave number in free space.

Next, in Fig. 9 we compare the simulation results with the experimental measurements in [6]. In the experiment, brightness temperatures of vertical polarization and horizontal polarization were measured at 19 GHz. The incidence angle was set at 53°. To reduce noise, Asher et al. [6] averaged the time series of brightness temperature for successive bubble plumes by using the tipping bucket to generate reproducible bubble plumes. Following the computational procedure described in the foregoing sections, we simulated the corresponding time series of brightness temperature. The experimental observation indicates that the brightness temperatures increases rapidly with a time interval of 1 s and then gradually decreases to the ocean values of brightness temperatures in about 8 s as the foam dissipates. The thickness of the foam layer increases from 0 to 2 cm, and then down to 0 which ends the simulation. The parameters for this figure are listed below.

\begin{align*}
&d \quad \text{increasing from 0 to 2 cm very quickly and then decreasing to 0 exponentially;}
&2a \quad 400 \mu m;
&2b \quad 200 \mu m–395 \mu m;
&f \quad 50\%;
&f_w \quad 4\%;
&f_{a} \quad \text{increasing from 0 to 50\% very quickly and then down to 0;}
&\tau \quad 0.1;
&\varepsilon_w \quad 28.9541+i36.8340;
&T \quad 284 \text{ K.}
\end{align*}

From Fig. 9, we see that the simulation results compare well with the experimental measurements. At a single time instant, the simulated brightness temperatures of vertical polarization and horizontal polarization agree well with the experimental measurements. Moreover, from the point of view of time series, the simulated brightness temperatures agree well with those of measurements. Also, the polarization dependence for the time series (corresponding to different thickness of foam layer) is well predicted by the sticky particle model.

VI. CONCLUSIONS

By formulating an electromagnetic scattering model of foam, we apply electromagnetic scattering theory and dense media radiative transfer theory to analyze the effects of foam on the passive microwave remote sensing measurements rigorously. We model the foam as densely packed sticky air bubbles coated with thin seawater coating. The DMRT for moderate size sticky particles based on QCA is applied to calculate the brightness temperatures of the foam-covered ocean surface. Numerical simulations show that the polarization and frequency dependences of brightness temperatures on microstructure properties such as foam layer thickness, foam air bubble size, coating thickness
and coated bubbles fractional volume. Simulation results are in good agreement with experimental measurements.

REFERENCES


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Kung-Hau Ding, photograph and biography not available at the time of publication.

Chi-Te Chen, photograph and biography not available at the time of publication.