Cooperative Transmission Scheme in MIMO Relay Broadcast Channels

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Abstract—We consider relay broadcast channels (RBCs) with multiple antennas at all nodes. A practical precoding, relaying and combining scheme is proposed. Under an overall power constraint, we derive the optimal power allocation solution in a closed form. The conditions for an effective RBC scheme are identified with a qualitative analysis to the relationship between the relay gain and the relative strengths of the direct and relay links. Simulation results are shown for various channel configurations, which verify the analysis and conclude that the first hop in the relay chain is a determining factor to the overall performance of the RBC.

I. INTRODUCTION

Traditional wireless systems, such as cellular and WiFi networks, are based on centralized processing, where one base station (BS) communicates with multiple users directly. Relaying technique, introduced by van der Meulen [1], has emerged in recent years as a powerful approach to improve the reliability and throughput of wireless networks. Two types of relay have been studied: dedicated relay and non-dedicated relay. For the first type, a set of nodes, usually known as relay stations (RSs), are inserted into the networks with the sole function of relaying [2]. For the second type, mobile users can help each other by relaying information for their peers besides receiving their own data [3], [4]. The relay broadcast channel (RBC) is a typical case for the second type of relay systems. An RBC is based on a broadcast channel (BC) where a BS transmits to multiple users simultaneously. Relay mechanism is introduced into the BC such that users can benefit from each other by some forms of cooperation. The capacity of RBCs has attracted much attention in recent years [5]–[7]. It has been shown that the capacity region of the RBCs is significant bigger than that of the BCs. However, most of the published results on the RBCs have only considered the channel capacity of multiple-input single-output (MISO) systems, where only the BS is equipped with multiple antennas while the users have single antennas only. Furthermore, the capacity results only reveal theoretical limit on the achievable spectral efficiency of the system. Realistic techniques in realizing the capacity promise are still an open problem, which limits the practical value of the RBCs.

In this paper, we will consider multiple-input multiple-output (MIMO) RBCs, where all nodes have multiple antennas. A practical precoding, relaying and combining scheme will be proposed. For simplicity, we will assume a two-user system. Instead of considering time-division (TD) broadcast schemes as in [8], [9], we assume the BS transmits simultaneously to both users in the same frequency band. Precoding is used as in the BCs to steer the signals for the two users in different subspaces to avoid interuser interference. The user with better conditions will act as a relay to the other user besides receiving its own signal. Based on the proposed scheme, an optimal power allocation solution will be derived and the performance of the RBC will be analyzed. Simulation results will be shown to support the analysis. We find that the first hop in the relay chain, to a great extent, determines the overall performance of the RBC. In particular, the diversity order of the system depends largely on the amount of the weaker user’s information that reaches the stronger user (the relay).

II. SYSTEM MODEL AND ZERO-FORCING

We consider a flat fading MIMO RBC with one source $S$ (the BS) and two destinations $E_1$ and $E_2$ (the mobile users) as illustrated in Fig. 1. Assume that there are $n_S$ transmit antennas at the source and $n_{E_k}$ ($k = 1, 2$) receive antennas at destinations. We use an $n_{E_k} 	imes n_S$ matrix $H_k$ to represent the MIMO channel relating the source $S$ and the $k$-th destination $E_k$ ($k = 1, 2$). Assume that $E_1$ is in a better position than $E_2$ such that it can act as a relay for $E_2$ besides receiving its own data. Assume there are $n_R$ transmit antennas at $E_1$ and the relay channel from $E_1$ to $E_2$ is represented by an $n_{E_2} 	imes n_R$ matrix $H_3$. We impose the practical constraint that the relay channel $H_3$ is orthogonal to $H_1$ and $H_2$ (e.g. through TD). For ease of presentation, we refer to $H_1$ and $H_2$ as the BC, while $H_3$ as the relay channel (RC). It is clear that the introduction of orthogonal RC leads to spectrum expansion. We do not take this into account in this paper and assume there are enough vacant time slots/spectrum for relay purpose.

The entries of $H_k$ are assumed to be independent complex Gaussian variables with zero mean and variance $\rho_k$ ($k = 1, 2, 3$). We use a quasistatic fading model, such that $H_k$ remains constant within one frame and changes independently between frames. We apply a singular value decomposition (SVD) to $H_k$ ($k = 1, 2, 3$)

$$H_k = U_k \Sigma_k V_k^H,$$

where $\Sigma_k$ is a diagonal matrix containing the singular values of $H_k$, $U_k$ and $V_k$ are the left and right singular matrices of $H_k$, respectively. We denote the largest singular value
where $\mathbf{H}_k$ as $\sigma_{k1}$ and the second largest one as $\sigma_{k2}$, etc; and the left and right singular vectors corresponding to $\sigma_{k1}$ as $\mathbf{u}_{k1}$ and $\mathbf{v}_{k1}$, respectively. Denote the signal transmitted from $S$ by an $n_S \times 1$ vector $\mathbf{x}_S$. For $E_k$ ($k = 1, 2$), the received vector from the BC can be expressed as an $n_{E_k} \times 1$ vector

\[ \mathbf{y}_k = \mathbf{H}_k \mathbf{x}_S + \mathbf{n}_k, \tag{2} \]

where $\mathbf{n}_k$ is the additive white Gaussian noise (AWGN) vector. The entries of $\mathbf{n}_k$ are also assumed to be i.i.d. complex Gaussian with zero mean and variance $\sigma_n^2$. With proper power normalization, we can assume $\sigma_n^2 = 1$ without loss of generality. Assume that $x_1$ and $x_2$ are the data intended for $E_1$ and $E_2$, respectively. Then

\[ \mathbf{x}_S = \sqrt{\kappa_1}\mathbf{w}_{S1}x_1 + \sqrt{\kappa_2}\mathbf{w}_{S2}x_2, \tag{3} \]

where $\kappa_k$ and $\mathbf{w}_{Sk}$ ($k = 1, 2$) are the power scaling factor and the transmit beamforming vector for destination $E_k$, respectively. Note that all beamforming and combining vectors in this paper have a norm of unity. Substituting (3) into (2), we get

\[ \mathbf{y}_k = \mathbf{H}_k \mathbf{w}_{S1}\sqrt{\kappa_1}\mathbf{x}_1 + \mathbf{H}_k \mathbf{w}_{S2}\sqrt{\kappa_2}\mathbf{x}_2 + \mathbf{n}_k. \tag{4} \]

Assume $E_1$ relays a signal vector $\mathbf{x}_R$ to $E_2$, the received vector by $E_2$ from the RC is

\[ \mathbf{y}_R = \mathbf{H}_3 \mathbf{x}_R + \mathbf{n}_3, \tag{5} \]

where $\mathbf{n}_3$ is an $n_{E2} \times 1$ vector representing the AWGN at $E_2$ in the RC.

\[ \begin{array}{c}
\xymatrix{ & E_1 \ar@{-}[ld] \ar@{-}[rd] & \\
S \ar@{-}[r] & \mathbf{x}_S & \mathbf{x}_R \ar@{-}[l] \ar@{-}[r] & \mathbf{y}_R \\
& H_1 \ar@{-}[u] & \mathbf{y}_1 \ar@{-}[u] \ar@{-}[l] & \mathbf{E}_1 \\
& H_2 & \mathbf{y}_2 \ar@{-}[l] \ar@{-}[r] & \mathbf{E}_2 \\
& H_3 & \\
} 
\end{array} \]

Fig. 1. Block Diagram of RBC

Assume each node has full CSI knowledge on both BC and RC. This might be impractical for fast mobile environments. However, for low to moderate mobile channels, ancillary channels can be used to convey the CSI reliably. We adopt the zero-forcing criterion in designing all the beamforming and combining vectors, such that interference is completely eliminated, which simplifies the signal processing at the receivers compared to other interference cancellation algorithms. We use the BC precoding of the “coordinated transmit-receive processing” [10]. $E_k$ extracts its data from $\mathbf{y}_k$ by a linear combiner $\mathbf{w}_{E_k}$ ($k = 1, 2$)

\[ \mathbf{y}_E^k = \mathbf{w}_{E_k}^H \mathbf{y}_k \]

where

\[ \begin{align*}
\mathbf{y}_E^1 &= \mathbf{w}_{E1}^H \mathbf{y}_1 \\
&= \mathbf{w}_{E1}^H \mathbf{H}_1 \mathbf{w}_{S1}\sqrt{\kappa_1}\mathbf{x}_1 + \mathbf{w}_{E1}^H \mathbf{H}_1 \mathbf{w}_{S2}\sqrt{\kappa_2}\mathbf{x}_2 + n'_1, \tag{6}
\end{align*} \]

\[ \begin{align*}
\mathbf{y}_E^2 &= \mathbf{w}_{E2}^H \mathbf{y}_2 \\
&= \mathbf{w}_{E2}^H \mathbf{H}_2 \mathbf{w}_{S1}\sqrt{\kappa_1}\mathbf{x}_1 + \mathbf{w}_{E2}^H \mathbf{H}_2 \mathbf{w}_{S2}\sqrt{\kappa_2}\mathbf{x}_2 + n'_2, \tag{7}
\end{align*} \]

where $n'_k \triangleq \mathbf{w}_{E_k}^H \mathbf{n}_k$ and $\mathbf{w}_{E_k} = \mathbf{u}_{k1}$ is the left singular vector corresponding to the largest singular value of $\mathbf{H}_k$. Thus, we have

\[ \mathbf{w}_{E_k}^H \mathbf{H}_k = \sigma_{k1} \mathbf{v}_{k1}^H. \tag{8} \]

The transmit beamforming vector $\mathbf{w}_{S2}$ is chosen in the orthogonal space of $\mathbf{v}_{21}$ such that $\mathbf{w}_{E2}^H \mathbf{H}_1 \mathbf{w}_{S2} = 0$ and $|\mathbf{w}_{E2}^H \mathbf{H}_2 \mathbf{w}_{S2}|$ is maximized. Similarly, $\mathbf{w}_{S1}$ is chosen in the orthogonal space of $\mathbf{v}_{21}$ such that $\mathbf{w}_{E1}^H \mathbf{H}_2 \mathbf{w}_{S1} = 0$ and $|\mathbf{w}_{E1}^H \mathbf{H}_1 \mathbf{w}_{S1}|$ is maximized.

**Proposition 1:** Consider the following optimization problem: given arbitrary vectors $y$ and $z$,

\[ \begin{align*}
\text{maximize} & \quad |z^H y| \\
\text{subject to} & \quad y^H y = 0, \quad \|y\| = 1. \tag{9}
\end{align*} \]

The optimal solution is

\[ \begin{align*}
\hat{y} &= \left( \mathbf{I} - \frac{\mathbf{y y}^H}{\|y\|^2} \right) \mathbf{z} \tag{10} \\
\hat{y} &= \frac{\mathbf{z}}{\|z\|} \tag{11} \\
z^H x &= \|z\|. \tag{12}
\end{align*} \]

The proof of Proposition 1 is straightforward and we omit the detail for brevity. Using Proposition 1, the optimal values for $\mathbf{w}_{S1}$ and $\mathbf{w}_{S2}$ can be easily obtained.

Define

\[ \sqrt{D_k} \triangleq \mathbf{w}_{E_k}^H \mathbf{H}_k \mathbf{w}_{S_k}, \quad k = 1, 2. \tag{13} \]

We obtain

\[ \mathbf{y}_E^k = \sqrt{D_k} \sqrt{\kappa_k} x_k + n'_k, \quad k = 1, 2. \tag{14} \]

We note that in (4), generally, $\mathbf{H}_k \mathbf{w}_{S_k} \neq 0$, which means the received vector $\mathbf{y}_R$ also carries information intended for $E_2$. In order to extract that information, we apply another combining vector $\mathbf{w}_{R1}$ to $\mathbf{y}_1$ and get

\[ \begin{align*}
\mathbf{y}_1^R &= \mathbf{w}_{R1}^H \mathbf{y}_1 \\
&= \mathbf{w}_{R1}^H \mathbf{H}_1 \mathbf{w}_{S1}\sqrt{\kappa_1}\mathbf{x}_1 + \mathbf{w}_{R1}^H \mathbf{H}_1 \mathbf{w}_{S2}\sqrt{\kappa_2}\mathbf{x}_2 + n''_1, \tag{15}
\end{align*} \]

where $n''_1 \triangleq \mathbf{w}_{R1}^H \mathbf{n}_1$ and

\[ \sqrt{R_1} \triangleq \mathbf{w}_{R1}^H \mathbf{H}_1 \mathbf{w}_{S2}. \tag{16} \]

This time we choose $\mathbf{w}_{R1}$ such that $\mathbf{w}_{R1}^H \mathbf{H}_1 \mathbf{w}_{S1} = 0$ and $R_1$ is maximized. Again Proposition 1 can be used to solve $\mathbf{w}_{R1}$. We consider the amplify-and-forward relay scheme, such that the relayed vector

\[ \mathbf{x}_R(t) = \mathbf{v}_R \sqrt{\kappa_3} \mathbf{y}_1^R(t - \tau), \tag{17} \]
where $\mathbf{v}_R$ is the transmit beamforming vector for the RC and $\tau$ denotes the processing delay in $E_1$. When the context is clear, we will omit the time index.

In the RC, we use a normal beamforming such that $\mathbf{v}_R = \mathbf{v}_{31}$ is the right singular vector corresponding to the largest singular value of $\mathbf{H}_3$ and the combining vector $\mathbf{u}_R = \mathbf{u}_{31}$ is the corresponding left singular vector. Now the relay component received by $E_2$ after combining is

$$y_R^2 = \mathbf{u}_R^H \mathbf{y}_R = \sqrt{R_2} \sqrt{\kappa_3} y_1^R + n'_3,$$  

where $\sqrt{R_2} \triangleq \mathbf{u}_R^H \mathbf{H} \mathbf{y}_R = \sigma_{31}$ is the largest singular value and $n'_3 \triangleq \mathbf{u}_R^H \mathbf{n}_3$ is the equivalent noise after combining.

Using (17) in (20), we get

$$y_2^R = \sqrt{R_1 R_2} \sqrt{\kappa_3} x_2 + \sqrt{R_2} \sqrt{\kappa_3} n'_2 + n'_3.$$  

Normalize $y_2^R$ by its noise variance

$$\tilde{y}_2^R = \frac{y_2^R}{\sqrt{1 + R_2 \kappa_3}} = \sqrt{\alpha} x_2 + \tilde{n}_2$$  

where

$$\alpha \triangleq \frac{R_1 R_3 \kappa_2 \kappa_3}{1 + R_2 \kappa_3}$$  

and the equivalent noise

$$\tilde{n}_2 \triangleq \frac{\sqrt{R_2} \sqrt{\kappa_3} n'_2 + n'_3}{\sqrt{1 + R_2 \kappa_3}}$$  

has a unit variance.

We perform a maximal ratio combining on $y_2^E$ (after proper delay) and $\tilde{y}_2^R$ to arrive at

$$\hat{y}_2 = x_2 + \hat{n},$$  

where $\hat{n}$ is zero-mean and has variance $1/(D_2 R_2 + \alpha)$. Finally, the equivalent SNRs for $E_1$ and $E_2$ are, respectively,

$$\gamma_1 = D_1 \kappa_1$$  

$$\gamma_2 = D_2 \kappa_2 + \alpha.$$  

### III. Optimal Power Allocation

Now we consider the power allocation to minimize the BER, which is equivalent to maximize the SNRs in (26) and (27). Considering user fairness, we set an equal SNR constraint

$$\gamma_1 = \gamma_2.$$  

We also consider a total transmit power constraint

$$\mathcal{E} [\|x_2\|^2] + \mathcal{E} [\|x_R\|^2] = P_t,$$  

where $\mathcal{E} [\cdot]$ represents the expectation operator and $P_t$ denotes the total transmit power. Although a total power constraint is less practical than an individual power constraint on each node, it gives an indication on the minimum power consumption overall the network and hence an optimal way to determine the transmit power for each node whenever possible.

Now the power allocation can be described as the following optimization problem

$$\begin{align*}
\text{maximize} & \quad D_1 \kappa_1 \\
\text{subject to} & \quad D_1 \kappa_1 = D_2 \kappa_2 + \frac{R_1 R_2 \kappa_2 \kappa_3}{1 + R_2 \kappa_3} \quad \kappa_1 + \kappa_2 + \kappa_3 (1 + R_1 \kappa_2) = P_t.
\end{align*}$$

After a lengthy mathematical derivation, we can obtain a closed-form solution as follows.

$$\begin{align*}
a & \triangleq (D_1 + D_2)(D_2 + R_1) R_2 + D_1 D_2 R_1 R_2 \kappa_1 \kappa_2 \kappa_3 \quad \kappa_1 + \kappa_2 + \kappa_3 + \kappa_3 (1 + R_1 \kappa_2) = P_t. \quad (33)
c & \triangleq R_2 (D_1 D_2 R_2 + D_2^2 R_2) + 2D_2 R_1 R_2 + D_1 R_1^2 R_2 \kappa_1 \kappa_2 \kappa_3 \quad (34)
d & \triangleq D_1 R_1 R_2 (D_2 R_2 - D_2 R_1 + \kappa_3 R_2) \quad -[(D_1 + D_2) + (D_1 R_1 + D_1 R_2 + D_2 R_2 + R_1^2 \kappa_3 \kappa_3)] \quad (35)
\end{align*}$$

Now let us take a closer look at the condition simplifies to

$$D_1 R_1 R_2 (D_2 R_2 - D_2 R_1 + \kappa_3 R_2) \quad -[(D_1 + D_2) + (D_1 R_1 + D_1 R_2 + D_2 R_2 + R_1^2 \kappa_3 \kappa_3)] \quad (36)$$

Eq. (36) shows that if and only if $d > a^2$, the relay mechanism is activated. After some mathematical manipulation, this condition simplifies to

$$D_1 R_1 (D_2 - D_2) P_t - D_2 (D_1 + D_2) > 0.$$  

At high SNRs, it is approximated as

$$R_2 > D_2.$$  

Informally, this implies that the relay mechanism will not be activated unless the relay channel $E_1 \rightarrow E_2$ is better than the direct transmit channel $S \rightarrow E_2$.

Intuitively, for an effective relay, among the information received by $E_2$, a considerable amount should arrive via the relay of $E_1$ compared to the amount from $S$ directly. This requires both the direct channel $S \rightarrow E_1$ and the relay channel $E_1 \rightarrow E_2$ are strong enough compared to the direct channel $S \rightarrow E_2$. The $E_1 \rightarrow E_2$ channel requirement is formulated in (39) and (40). Now let us take a closer look at the $S \rightarrow E_1$ channel requirement. To this end, we assume the RC is perfect, or equivalently, $\rho_3 \rightarrow \infty$. With this assumption, the power allocation in (36)-(38) reduces to

$$\begin{align*}
\kappa_3 & \rightarrow 0, \quad (41) \\
\kappa_2 & \rightarrow \frac{D_1}{D_1 + D_2 + R_1} P_t, \quad (42) \\
\kappa_1 & \rightarrow \frac{D_1}{D_1 + D_2 + R_1} P_t. \quad (43)
\end{align*}$$

The corresponding equivalent SNR is

$$\gamma^* = \frac{D_1 (D_2 + R_1)}{D_1 + D_2 + R_1} P_t.$$  

(44)
This sets an upper limit for the gain of the relay mechanism. Note that (44) is a monotonically increasing function with respect to \( R_1 \). When \( R_1 \ll D_2 \), (44) reduces to the BC case where

\[
\gamma_{BC} = \frac{D_1 D_2}{D_1 + D_2} P_t. \tag{45}
\]

Therefore, a necessary condition for an effective relay is that \( R_1 \) must be comparable to or larger than \( D_2 \), which we formulated as

\[
R_1 \geq D_2. \tag{46}
\]

This formulates the requirement on the \( S \rightarrow E_1 \) channel.

Now let us see how easy/hard the requirements (40) and (46) can be met. \( \sqrt{R_1} = \sigma_{31} \) is the largest singular value of \( H_3 \). Using (8) in (15), we have

\[
\sqrt{D_2} = \sigma_{21} v_{21} w_{S2}. \tag{47}
\]

If \( w_{S2} \) can be chosen as an arbitrary unit-norm vector, then \( \sqrt{D_2} \) can achieve its maximum of \( \sigma_{21} \), the largest singular value of \( H_2 \). However, there is a constraint that \( w_{S2} \) has to lie in the orthogonal space of \( v_{11} \). Therefore,

\[
\sqrt{D_2} \leq \sigma_{21}, \tag{48}
\]

where the equality holds when \( v_{21} \) happens to lie in the orthogonal space of \( v_{11} \), i.e. \( v_{21}^H v_{11} = 0 \). Statistically speaking, the largest singular value of \( H_k \), \( \sigma_k \), is in proportion to \( \sqrt{\rho_k} \). Therefore, the requirement (40) will be satisfied with a high probability when \( \rho_3 > \rho_2 \). From (18) and considering the constraint \( v_{11}^H w_{S2} = 0 \), we get

\[
\sqrt{R_1} = \sqrt{\rho_1} \sum_{i=2}^{n_E1} \sigma_{1i} u_{1i} v_{11}^H w_{S2} \leq \sigma_{12}. \tag{49}
\]

Assuming both equalities in (48) and (49) hold, Eq. (46) can be seen as a comparison between the largest singular value of \( H_2 \) (\( \sigma_{21} \)) and the second largest singular value of \( H_1 \) (\( \sigma_{12} \)). Therefore, the requirement (46) becomes that \( \rho_1 \) has to be significantly larger than \( \rho_2 \).

In short, to achieve a significant relay gain, both \( S \rightarrow E_1 \) and \( E_1 \rightarrow E_2 \) have to be significantly stronger than \( S \rightarrow E_2 \). Furthermore, the requirement on \( S \rightarrow E_1 \) channel is more critical as it is the first hop of the relay chain, which limits the overall gain. This may seem to be in contradiction to the dedicated relay case, where even a weak relay may also help due to the spatial diversity gain. However, for an RBC, the relay node is also a user node. If the relay channel is weak, the spatial diversity gain is so small that the system would be better off to use all the power in the BC.

IV. NUMERICAL RESULTS

We provide the BER simulation results in several situations with different channel gains and antenna numbers. We set \( \rho_1 = 0 \) dB as the reference channel gain. Uncoded QPSK is used as the modulation scheme. In Fig. 2, we show the results with \( n_S = 4, n_{E1} = n_{E2} = n_R = 2 \) and \( \rho_2 = 0 \) dB. Different \( \rho_3 \) values are illustrated. The SNR \( \gamma_T \) is defined to be the average total transmit power per user

\[
\gamma_T = \frac{P_t}{2}. \tag{50}
\]

For comparison, we provide the results of the BC scheme where the relay mechanism is removed, or equivalently, \( \kappa_3 = 0 \) [see (45)]. We also include the results with perfect relay, i.e. \( \rho_3 = \infty \) [see (44)].

From Fig. 2, we find that the benefit of relay is small when destinations \( E_1 \) and \( E_2 \) have comparable channel quality, \( \rho_1 = \rho_2 \). This situation nearly does not change when we increase the relay channel quality \( \rho_3 \). Even with a perfect relay, the SNR gain is only 0.8 dB compared to the BC at BER = 10^{-6}. This
is consistent with our analysis in Section III because with a high probability the requirement (46) is not met when $\rho_1 \approx \rho_2$.

To increase the probability of (46) being met, a direct way is to increase $\rho_1$, or equivalently, decrease $\rho_2$. In Fig. 3, we show the BER results when $\rho_2 = -6$ dB while the other parameters are unchanged. Now the benefit of relay is a more than 3 dB gain in SNR for perfect relay and about 2.5 dB for $\rho_3 = 0$ dB at $\text{BER} = 10^{-6}$. We also include the result when $\rho_3$ is 3 dB lower than $\rho_2$. In that case, the requirement (40) is only met with a low probability. The relay gain is reduced to 1.4 dB at $\text{BER} = 10^{-6}$. An interesting observation is that the curves for all $\rho_3$ values are almost parallel at high SNRs. This indicates that the diversity order is to a great extent determined by the first hop ($S \rightarrow E_1$) in the relay chain.

Another way to increase the probability of meeting the requirement (46) is to increase $\sigma_{12}$ relative to $\sigma_{21}$. To this end, an effective way is to increase the size of $H_1$. In Fig. 4, we show the simulation results with $n_{E1} = 4$ and the other parameters unchanged from Fig. 3. With the increased antenna number, the relay gain is raised to over 6 dB with perfect relay and about 5 dB with $\rho_3 = 0$ dB. Again, when $\rho_3$ is 3 dB lower than $\rho_2$, the relay gain is significantly reduced to about 3 dB. However, the diversity order is the same for all values of $\rho_3$. This again confirms the conjecture that the first hop in the relay chain determines the overall diversity order and the relay gain limit.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{BER simulation results of RBCs. $\rho_3 = 0$ dB, $\rho_2 = -6$ dB, $n_S = n_{E1} = 4$, $n_{E2} = n_R = 2$}
\end{figure}

V. CONCLUSION

We have proposed a practical precoding, relaying and combining scheme for the MIMO RBCs. A closed-form solution to the optimal power allocation has been obtained, which minimizes the BER subject to the equal BER and total transmit power constraints. The analytical and simulation results have shown that the RBC scheme is effective in situations where the first hop in the relay chain is strong enough. The first hop link determines the diversity order of the RBC and it also puts an upper limit to the overall relay gain.

Only a fixed relay direction has been considered in this paper. More flexible relay mechanism is worth of consideration in order to achieve a bigger overall relay gain. For instance, users can determine the relay direction by taking into account the instantaneous link strengths or eigen subchannel gains. More complicatedly, users can cooperate by always doing a bi-directional relaying. In this paper, the beamforming and combining vectors have been determined in sequence, which is not optimal. A joint optimization of the beamforming and combining vectors might bring a further boost to the relay gain. Finally, the full CSI assumption is not realistic in some circumstances and schemes based on partial CSI is to be investigated.

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