Tree Size Distribution Functions of Four Boreal Forest Types for Biomass Mapping

Wenjun Chen

ABSTRACT. To accurately map forest biomass, which is essential for many socioeconomic and environmental studies, reliable information of tree size distribution within a stand is needed. Using measurements at 87 stands of trembling aspen (*Populous tremuloides* Michx.), jack pine (*Pinus banksiana* Lamb.), black spruce (*Picea mariana* [Mill.] BSP.), and mixed forests along a 1,000-km boreal forest transect in western Canada, we investigated three methods for estimating tree size distribution: parameter prediction method for diameter-frequency distribution (PPM-N_k/N), PPM for diameter-basal area distribution (PPM-B_k/B), and parameter recovery method for diameter-frequency distribution (PRM). For diameter-frequency distribution estimation, the PPM-N_k/N and the PRM were more accurate than PPM-B_k/B. For biomass estimation, the PPM-B_k/B and the PRM were found to have mean estimation errors <3% and root mean square errors <10%, relative to the "true" values given by summation overall trees, but the PPM-N_k/N didn't perform as well. Three distribution functions (i.e., the Weibull, the Johnson's S_B, and the lognormal) were used for each method but were found to make little difference in method performance. An additional test indicated that the PPM-B_k/B and the PRM performed equally well for a balsam fir (*Abies balsamea* [L.] Mill) stand in eastern Canada. FOR. SCI. 50(4):436–449.

Key Words: Tree size distribution, biomass mapping, boreal forest, parameter prediction method, parameter recovery method.

BIOMASS IS ONE of the most important renewable natural resources, providing food, fiber, and fuel for human use. Will climate change, air pollution, and natural and human disturbances (e.g., deforestation, forest fire, harvesting, and insect-induced mortality) degrade the sustainability of biomass production? To address this question, mapping biomass distribution in a timely and consistent fashion is essential. During the 1980s and 1990s, terrestrial ecosystems took up about one-third to one-half of carbon dioxide (CO_2) released from burning fossil fuels, substantially reducing the growth rate of the atmospheric CO_2 concentration (Houghton et al. 2001). Will this carbon sink continue to exist in the future? Biomass information is one of the key data sets required for quantifying the terrestrial carbon sink. Specifically, foliage, sapwood, and fine root biomass are needed for estimating autotrophic respiration (Goetz et al. 1999, Liu et al. 1999, Baldwin et al. 2001). Biomass information is also needed for quantifying fire carbon emission (Stocks and Kauffman 1997, Amiro et al. 2001), for quantifying postdisturbance effects on net

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primary productivity (Gower et al. 1999, Chen et al. 2002), and on the carbon cycle (Kurz et al. 1992, Apps et al. 1999, Kurz and Apps 1999, Chen et al. 2000a, b, c).

Unfortunately, most of the current information regarding the spatial distribution of biomass contains large uncertainties due to various reasons, especially at broader spatial scales. One key reason is that the effect of tree size variation within a stand on biomass estimation is often neglected or not incorporated properly. Baskerville (1965a) compared seven methods of calculating biomass of foliage, branches, cones, stem wood, stem bark, roots, and the whole tree for a balsam fir stand (*Abies balsamea* [L.] Mill). The "true" values of biomass were calculated by the summation of overall trees, and errors were estimated as the deviations from the true values expressed as a percentage. Methods that neglect tree size distribution (i.e., direct use of mean height and diameter at breast height [dbh]) could result in an error as high as 60% in biomass estimates (Baskerville 1965a). With a stand table in which trees were divided into 10 dbh classes, Baskerville (1965a) found that biomass estimation errors were reduced to –2.6–2.9%. Because of the lack of tree size distribution functions developed specifically for Canadian forest species, simplistic assumptions have often been made in previous large-scale biomass mapping studies (Penner et al. 1997). In developing the biomass data from Canada's Forestry Inventory 1991(CANFI91), Penner et al. (1997) used a hypothetical 1-ha stand that assumes dbh and height are normally distributed variables with a variance equal to some specific value (e.g., a coefficient of variance of 20%). Where a height range was available, it was used to approximate the variance of the height distribution in which the variance equals one-fourth of the data range. Extensive tree size distribution studies have indicated that dbh and height are not normally distributed (Bliss and Reinker 1964, Bailey and Dell 1973, Hafley and Schreuder 1977, Kamziah et al. 1999, Li et al. 2002), and thus substantial error may exist in the CANFI91 biomass data.

While there is a clear need to fill the information gap of tree size distribution functions for Canadian forest species, the summation method or the direct construction of a stand table from measurements is often not practical, especially at a large spatial scale. On the other hand, stand-level attributes (e.g., average dbh or height, basal area, volume) often have much higher availability because they can be derived indirectly using other means, such as remote sensing techniques (Nilsson 1996, Magnussen and Boudewyn 1998). These stand-level attributes may also be obtained using yield tables (Plonski 1981) if spatial information of species, age class, and site index is available. Therefore, developing an alternative method to construct the tree size distribution from stand-level attributes provides one of the best alternatives for the purpose of mapping biomass at a large spatial scale. Many approaches have been proposed and used to construct the tree size distribution from stand-level attributes, such as the parameter prediction method (PPM), the parameter recovery method (PRM), and the imputation method. In PPM, parameters are estimated directly for the selected probability density function (PDF) by fitting the observed data; the fitted parameters are then regressed against stand-level attributes (Smalley and Bailey 1974, Feduccia et al. 1979, Hyink 1980). The PRM recovers parameters of a PDF from stand-level attributes and allows for a consistent mathematical relationship between the stand- and tree-level attributes (Hyink and Moser 1983, Reynolds et al. 1988, Vanclay 1994). The imputation method involves "replacing missing or nonsampled measurements for any unit in the population with measurements from another unit of similar characteristics" (Ek et al. 1997, Maltamo and Kangas 1998). Using the imputation method requires a large database of reference stands, which is often difficult to obtain (Maltamo and Kangas 1998). Therefore, this study will focus on the two more practical methods at present (i.e., PPM and PRM). In this study, we aim to develop tree size distribution functions for four major Canadian forest types using the two methods and available field data. These functions can then be used for mapping biomass distribution at broader spatial scales in future studies.

Materials and Methods
Data Source

We used dbh measurements collected from 87 stands across a 1,000-km boreal forest transect from southern Saskatchewan to northern Manitoba by the Boreal Forest Transect Case Study (Price and Apps 1995, Halliwell and Apps 1997a and b). Among the 87 stands, 18 were trembling aspen (*Populous tremuloides* Michx.), 25 were jack pine (*Pinus banksiana* Lamb.), 34 were black spruce (*Picea mariana* [Mill.] BSP.), and 10 were mixed stands. In each stand, three plots (occasionally two plots) were usually sampled. At each plot, an average of approximately 50 trees (ranging from 22 to 289 trees) were selected using either the fixed-area sampling method (plot size = 25 m^2) or point sampling method (plot size varies). The dbh of each selected tree was then measured. Dead trees were also measured but were not included in this analysis. Also excluded from the analysis were three stands where <20 trees were measured. Table 1 lists the arithmetic mean dbh, d_a, and the number of trees measured for each stand, n.

Computation of Observed Diameter-Frequency Distribution

Conventionally, diameter class, k, is defined in absolute scale (e.g., 0–1 cm for $k = 1$, >1–2 cm for $k = 2$, etc.). Because the stands sampled across the boreal forest transect were at different development stages, the diameter ranges differed substantially among stands. Consequently, the conventional diameter scale would be too coarse for very young stands and too fine for old stands for the purpose of this analysis, and the resultant stand tables would not be comparable. To avoid this deficiency, we define diameter class as a relative term, i.e., $k = 1$ if $d_{ji}/d_{ai} = 0$–0.2, $k = 2$ if $d_j/d_a = 0.2$–0.4, . . . , and if $d_{ji}/d_{ai} = 2.8$–3, when d_{ji} is the dbh of tree j in stand i, and d_{ai} is the arithmetic mean dbh of stand i. Trees with dbh three times above d_{ai} are rare and thus are grouped into the last diameter class in this study.

Table 1. Arithmetic mean diameter at the breast height (d_{ai}) of a stand and the corresponding number of trees sampled (n_i) for trembling aspen, jack pine, black spruce, and mixed stands along a 1,000-km boreal forest transect in Canada (Halliwell and Apps 1997a). Dead trees were excluded

Trembling aspen		Jack pine		Black spruce		Mixed	
d_{ai}	n_i	d_{ai}	n_i	d_{ai}	n_i	d_{ai}	n_i
1.3	289	1.9	251	1.6	124	6.6	40
2.4	145	2.8	105	2.7	117	7.7	34
2.7	115	3.1	59	2.9	73	8.7	38
4.6	67	3.2	24	3.5	73	12.2	31
4.7	37	3.6	33	3.5	29	14.7	146
4.9	46	5.4	81	4.0	35	15.9	22
6.4	56	7.3	69	5.0	32	17.9	71
6.5	36	7.6	34	5.0	40	19.7	43
9.4	54	7.6	82	5.4	24	21.9	45
12.4	73	8.4	27	5.9	41	22.5	46
12.5	37	8.5	22	6.0	23		
12.6	36	10.7	29	6.1	43		
16.9	45	10.8	26	6.4	45		
17.4	41	13.0	23	6.5	39		
18.0	61	14.0	63	6.9	42		
20.1	36	14.8	27	7.1	24		
20.1	23	15.3	127	7.3	56		
20.3	81	15.6	28	8.0	53		
		15.7	49	8.0	104		
		15.9	61	8.1	26		
		17.5	43	8.2	48		
		18.0	75	8.4	69		
		18.2	40	8.5	53		
		18.2	51	8.9	40		
		21.9	33	9.0	36		
				9.6	41		
				9.8	63		
				10.1	55		
				10.1	22		
				10.9	26		
				11.4	59		
				14.0	34		
				14.3	71		
				15.3	26		

When the fixed-area sampling method is used, the estimate of tree number per hectare in diameter class k at stand i is:

$$N_{ki} = \sum_{j=1}^{n_i} \frac{I(d_{ji}/d_{ai})}{A_i},$$

$$I(d_{ji}/d_{ai}) = \begin{cases} 1, & \text{if } d_{ji}/d_{ai} \text{ is in class } k \\ 0, & \text{if } d_{ji}/d_{ai} \text{ is not in class } k \end{cases} \quad (1)$$

where n_i is the number of trees measured in stand i ($i = 1, 2, \ldots, p$, where p is the total number of stands sampled), and A_i is the sample area in hectares for stand i (Halliwell and Apps 1997a, Green and Clutter 2000). When the point sampling method is used, the estimates of tree number per hectare in diameter class k at stand i is:

$$N_{ki} = \sum_{j=1}^{n_i} \frac{\text{BAF}_i}{b_{ji}} I(d_{ji}/d_{ai}),$$

$$I(d_{ji}/d_{ai}) = \begin{cases} 1, & \text{if } d_{ji}/d_{ai} \text{ is in class } k \\ 0, & \text{if } d_{ji}/d_{ai} \text{ is not in class } k \end{cases} \quad (2)$$

where BAF_i is the basal area factor for stand i (i.e., the basal area in m^2/ha represented by each sample tree), and b_{ji} is the basal area (m^2) of tree j in stand i (Halliwell and Apps 1997a, Green and Clutter 2000). The frequency of stems per hectare in diameter class k at stand i, the diameter-frequency distribution, is then given by:

$$\frac{N_{ki}}{N_i} = N_{ki} / \sum_{k=1}^{15} N_{ki}, \quad (3)$$

where N_i is the total number of trees per hectare in stand i.

Computation of Observed Diameter-Basal Area Distribution

Because large trees in a stand are much more important in determining stand biomass than small trees, an accurate representation of the distribution of the number of trees in a stand does not always guarantee an accurate estimation of the stand's biomass. For example, an overestimation of one tree in diameter class of $d_{ji}/d_{ai} = 2.8–3$ could cause an error in stand biomass as large as an overestimation of 1,000 trees in dbh class of $d_{ji}/d_{ai} = 0–0.2$. To avoid this potential problem, we also calculated the diameter-basal area distribution. The calculation of diameter-basal area distribution is similar to that of diameter-frequency distribution, except that the diameter classes are normalized by the quadratic mean dbh of the concerned stand i, d_{qi}. When the fixed-area sample method is used, the estimate of basal area per hectare in diameter class k at stand i is:

$$B_{ki} = \sum_{j=1}^{n_i} b_{ji} \frac{I(d_{ji}/d_{qi})}{A_i},$$

$$I(d_{ji}/d_{qi}) = \begin{cases} 1, & \text{if } d_{ji}/d_{qi} \text{ is in class } k \\ 0, & \text{if } d_{ji}/d_{qi} \text{ is not in class } k \end{cases} \quad (4)$$

When the point sampling method is used, the estimate of basal area per hectare in diameter class k at stand i is:

$$B_{ki} = \sum_{j=1}^{n_i} \text{BAF}_i \, I(d_{ji}/d_{qi}),$$

$$I(d_{ji}/d_{qi}) = \begin{cases} 1, & \text{if } d_{ji}/d_{qi} \text{ is in class } k \\ 0, & \text{if } d_{ji}/d_{qi} \text{ is not in class } k \end{cases} \quad (5)$$

The basal area frequency in diameter class k at stand i, the diameter-basal area distribution, is then given by:

$$\frac{B_{ki}}{B_i} = B_{ki} / \sum_{k=1}^{15} B_{ki}, \quad (6)$$

where B_i is the total basal area per hectare of stand i.

Parameter Prediction Method (PPM)

Three commonly used diameter-frequency distribution functions—the Weibull function (Bailey and Dell 1973), the Johnson's S_B function (Hafley and Schreuder 1977), and the lognormal distribution (Bliss and Reinker 1964)—are fitted
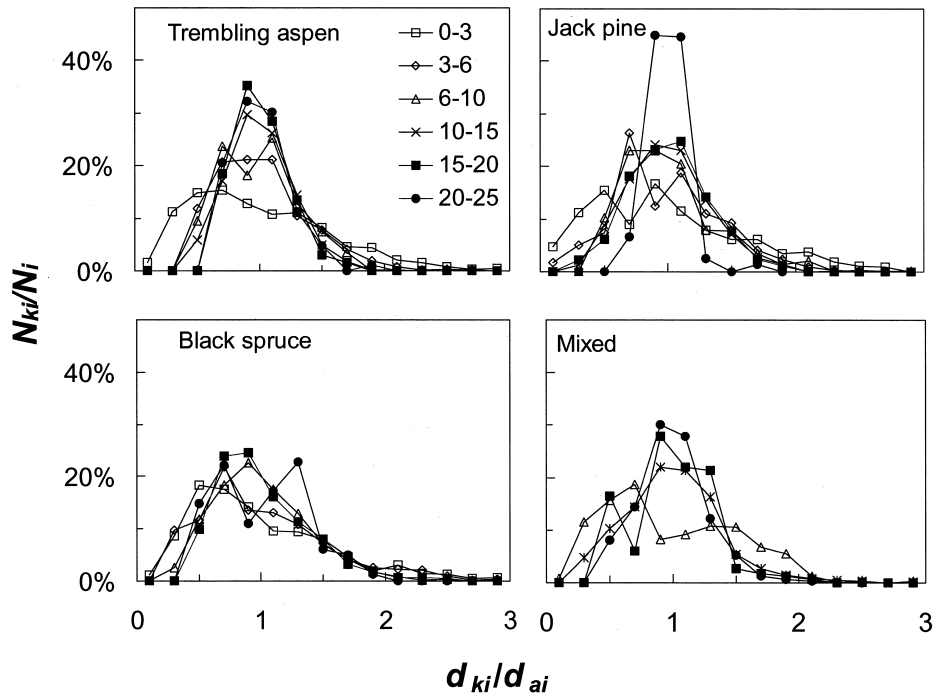
Figure 1. The distribution of the number of trees normalized by the total tree number (N_{ki}/N_i) against dbh normalized by the arithmetic mean dbh of stand i (d_{ki}/d_{ai}) for trembling aspen, jack pine, black spruce, and mixed forests along a 1,000-km boreal forest transect (Halliwell and Apps 1997a). For clarity, results are shown as mean values of d_a ranges of 0–3, 3–6, 6–10, 10–15, 15–20, and 20–25 cm.

to the observations for the four boreal forest types. Following the notation of Bailey and Dell (1973), the Weibull PDF for variable x is given by:

$$f(x) = \frac{c}{b}\left[(x-a)/b\right]^{c-1}\exp\{-[(x-a)/b]^c\};$$

$$x \geq a, b > 0, c > 0, \quad (7)$$

where a is the location parameter, b is the scale parameter, and c is the shape parameter. In a diameter class context, the parameter a can be interpreted as the smallest d/d_a (i.e., 0), which reduces the Weibull function to two parameters. With $a = 0$, the cumulative distribution function of the Weibull can be written as:

$$F(x) = 1 - \exp\{-[(x)/b]^c\}. \quad (8)$$

If $x = b$, the value of $F(b)$ will be about 0.63, and so the parameter b can be interpreted as the diameter such that 63% of all trees are smaller. If the shape parameter $c < 1$, the Weibull PDF is a reversed J. When $c = 1$, the curve becomes an exponential distribution. For $1 < c < 3.6$, the PDF is unimodal and positively skewed (long tail on the right). As c is increased above 3.6, the PDF becomes progressively more negatively skewed (long tail on the left) and approaches a spike over a single point.

Following the notation of Hafley and Schreuder (1977), we have the Johnson's S_B distribution as:

$$f(x) = \frac{\delta}{2\pi}\frac{\lambda}{(x-\varepsilon)(\lambda+\varepsilon-x)}$$

$$\exp\left\{-0.5\left[\gamma + \delta\ln\left(\frac{(x-\varepsilon)}{(\lambda+\varepsilon-x)}\right)\right]^2\right\};$$

$$\varepsilon \leq x \leq \varepsilon + \lambda, \delta > 0, -\infty < \gamma < \infty, \lambda > 0, \quad (9)$$

where ϵ is the lower limit, λ is the range, γ is the shape parameter, and δ is the scale parameter. In this study, we set ϵ to 0 and λ to 6 based on observational data to reduce the Johnson's S_B distribution to two parameters. Increases in γ and δ both result in higher Kurtosis (peakedness) but cause different responses in skewness, with more positive skewness as δ increases and more negative skewness as γ increases.

Using the notation of Bliss and Reinker (1964), we have the general formulas for the PDF of the lognormal distribution as follows:

$$f(x) = \frac{1}{(x-\theta)\sigma\sqrt{2\pi}}\exp\{-[\ln((x-\theta)/\mu)^2/(2\sigma^2)]\};$$

$$x \geq \theta, \mu > 0, \sigma > 0, \quad (10)$$

where θ is the location parameter (=0 in this study), σ is the shape parameter, and μ is the scale parameter. If σ increases, the PDF becomes more peaked, while an increase in μ results in more negative skewness.

Note that the observed diameter-frequency distribution is cumulative within each diameter class, and so corresponding

Table 2. Percentage (fraction in the bracket) of stands for which the null hypothesis that the observed and fitted distributions are the same cannot be rejected at the 0.05 significance level of the goodness-of-fit χ^2 test for the Weibull, the Johnson's S_B, and the lognormal distribution functions

	Weibull	Johnson's S_B	Lognormal
Trembling aspen	78 (14/18)	83 (15/18)	78 (14/18)
Jack pine	68 (17/25)	76 (19/25)	72 (18/25)
Black spruce	69 (22/32)	78 (25/32)	78 (25/32)
Mixed	60 (6/10)	60 (6/10)	60 (6/10)
Overall	68 (59/87)	74 (65/87)	72 (63/87)

cumulative distribution functions should be used in the fitting. Unlike the Weibull function, cumulative distribution functions for the Johnson's S_B function and the lognormal function do not exist in a simple, analytic form. Consequently, we calculated the cumulative distribution functions for the Johnson's S_B function and the lognormal function in a discrete format for stand i:

$$F_i(k) = \sum_{m=1}^{L} \left[f(d_{ki}/d_{ai} - 0.1(L + 1 - 2m)/L) \frac{0.2}{L} \right], \quad (11)$$

where d_{ki}/d_{ai} equals the middle value of diameter range k (e.g., in diameter class $k = 1$, $d_{ki}/d_{ai} = 0.1$), and L is the number of subclasses divided for each diameter class ($L = 4$ is used in this study, as a result of trade-off between computation efficiency and the error in $F_i(k)$ caused by a smaller L value). The least-square error technique is used for estimating the parameters.

To complete the PPM, the estimated parameters of distribution functions from the observations are then regressed against stand-level attributes (Smalley and Bailey 1974, Feduccia et al. 1979, Hyink 1980). Smalley and Bailey

(1974) correlated the parameters of the Weibull function with average total height of dominant and codominant trees, stand age, initial stand density, and surviving stand density for shortleaf pine (*Pinus echinata* Mill.) plantations in Tennessee, Alabama, and Georgia Highlands. Because the most widely available stand attribute is the mean dbh, and other stand attributes, such as height, age, and density, are related to dbh, in this study we focused on the relationships between distributional parameters and d_{ai}.

Parameter Recovery Method (PRM)

PRM solves for parameters of a distribution function using stand-level attributes (Hyink and Moser 1983, Reynolds et al. 1988, Vanclay 1994). The underlying assumption of this method is that the tree size distribution of a stand can be described by the distribution function for which we intend to recover its parameters. For two-parameter distribution functions investigated in this study, two known stand attributes such as basal area (B) and volume (V) are required. For stand i, if B and V are given, the two parameters (θ_1 and θ_2) of a distribution function can be determined by minimizing the estimation error between the known B and V and their estimates, $\hat{B}$ and $\hat{V}$:

$$\left| \frac{B - \hat{B}}{B} \right| + \left| \frac{V - \hat{V}}{V} \right| \to 0 \qquad (12)$$

where

$$\begin{cases} \hat{B} = \sum_{k=1}^{15} [F(k, \theta_1, \theta_2) B(d_k) N] \\ \hat{V} = \sum_{k=1}^{15} [F(k, \theta_1, \theta_2) V(d_k) N] \end{cases} . \qquad (13)$$
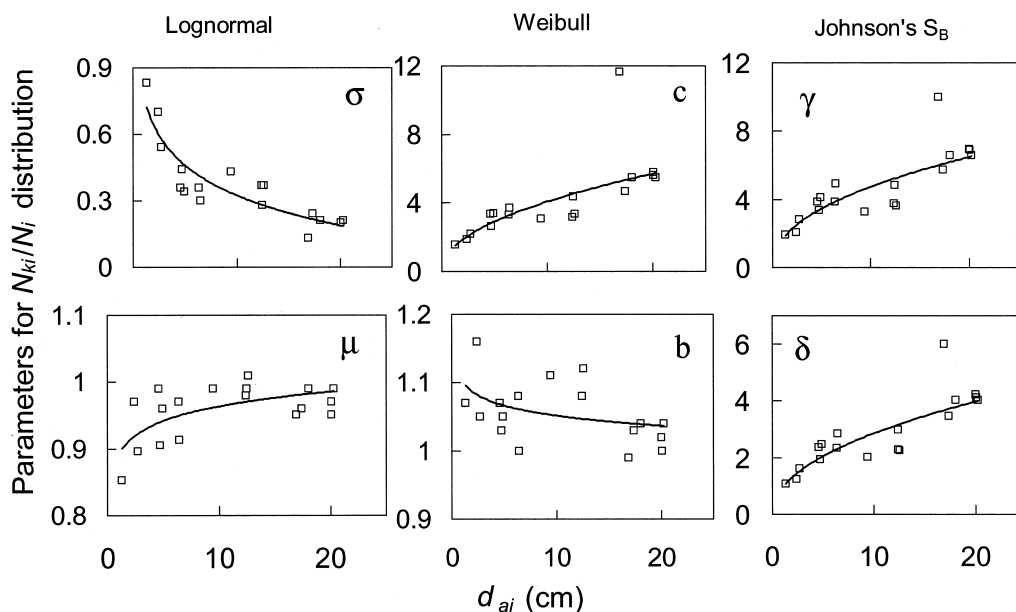

Figure 2. The relationships between d_{ai} and parameters of diameter-frequency distribution functions (the lognormal, the Weibull, and the Johnson's S_B) for trembling aspen stands along the Canadian boreal forest transect. Details of these relationships, R^2 value, and the corresponding significance level are listed in Table 3.

The original formula for calculating B and V as a function of dbh, $B(d_k)$ and $V(d_k)$, is used here following Halliwell and Apps (1997a).

Goodness-of-Fit Tests

We used the Pearson χ^2 test to test the goodness of fit of distribution functions to the observations of diameter-frequency distribution (Reynolds et al. 1988). If the observed number in an original dbh class is less than five, the class is merged with its nearest class until the five-tree criterion is met. For the two-parameter Weibull, Johnson's S_B, and lognormal distribution functions, the degrees of freedom are $k_{\chi i} - 3$. If the calculated χ_i^2 is less than a critical value at a significance level ($\alpha = 0.05$), then we cannot reject the null hypothesis that the model is equal to the observed distribution. However, as pointed out by Reynolds et al. (1988), even if the test rejects the null hypothesis, it just means that the model is not a perfect representation of the real system. Yet no model is going to be perfect, and a model could be the best available even though it is not perfect. In addition, the χ_i^2 test is also not proper for testing the goodness of fit when the diameter-frequency distribution is estimated in a complicated way (e.g., PPM and PRM) or for testing biomass estimates. Consequently, we use two other types of goodness-of-fit tests, namely the Kolmogorov-Smirnov test (KS_i) and an error index proposed by Reynolds et al. (1988), for these purposes.

We used KS_i to rank the goodness of fit of various forms of PPM and PRM for estimating diameter-frequency distribution. Reynolds et al. (1988) proposed a "weighted" error index to account for the difference in errors associated with prediction targets (e.g., biomass) by one-tree estimation error in a large dbh class and in a small dbh class. For biomass calculation, a proper "weight" is biomass itself, and we calculated the error index for stand i, E_i, as the normalized difference between the estimated biomass M_i and the true biomass computed by every-tree summation M_{ti} (Baskerville 1965a) for the four forest types:

$$E_i = \frac{M_i - M_{ti}}{M_{ti}} \times 100\%. \tag{14}$$

Table 3. Relationships between parameters of distribution function (i.e., the Weibull, the Johnson's S_B, and the lognormal) for the diameter-frequency distribution and the most common stand attribute, the arithmetic mean dbh d_{ai}, for trembling aspen, jack pine, black spruce, and mixed forest stands along a Canadian boreal forest transact. The form of relationship is αd_{ai}^β for the Weibull and the Johnson's S_B functions, and is $\alpha + \beta \ln(d_{ai})$ for the lognormal function. R^2 values stand for the fraction of variation explained by a relationship

|	α Mean (STD)	β Mean (STD)	R^2 (Significance level)	α Mean (STD)	β Mean (STD)	R^2 (Significance level)	n
		b			c		
Weibull							
Trembling aspen	1.1016 (0.0284)	−0.0202 (0.0115)	0.1606 (0.1)	1.3340 (0.54074)	0.4854 (0.1693)	0.7296 (0.01)	18
Jack pine	1.1039 (0.0565)	−0.0197 (0.0221)	0.0329 (ns)	1.3879 (0.5068)	0.3951 (0.1557)	0.3683 (0.01)	25
Black spruce	1.0250 (0.0597)	0.0024 (0.0293)	0.0002 (ns)	1.4425 (0.6477)	0.3631 (0.1828)	0.1861 (0.05)	34
Mixed	1.1625 (0.1659)	−0.0243 (0.0541)	0.0250 (ns)	0.2672 (0.3871)	0.9534 (0.3978)	0.6531 (0.05)	10
		δ			γ		
Johnson's S_B							
Trembling aspen	0.9502 (0.2432)	0.4784 (0.1036)	0.7884 (0.01)	1.6724 (0.4298)	0.4521 (0.1061)	0.7521 (0.01)	18
Jack pine	0.9742 (0.3428)	0.4086 (0.1372)	0.3831 (0.01)	1.7234 (0.5993)	0.3841 (01.326)	0.3569 (0.01)	25
Black spruce	1.0474 (0.4385)	0.3566 (0.1663)	0.1806 (0.05)	1.9699 (0.7970)	0.3148 (0.1640)	0.1520 (0.05)	34
Mixed	0.2038 (0.2812)	0.9054 (0.4383)	0.5595 (0.05)	0.4334 (0.5026)	0.8107 (0.4343)	0.5165 (0.05)	10
		μ			σ		
Lognormal							
Trembling aspen	0.8916 (0.1772)	0.0314 (0.0102)	0.4035 (0.01)	0.7755 (0.0526)	−0.0046 (0.0233)	0.8160 (0.01)	18
Jack pine	0.8909 (0.5541)	0.0515 (0.306)	0.1113 (0.1)	0.8393 (0.1367)	−0.2042 (0.0588)	0.3440 (0.01)	25
Black spruce	0.8061 (0.3299)	0.0587 (0.0446)	0.1093 (0.1)	0.7438 (0.1370)	−0.1592 (0.0691)	0.1990 (0.05)	34
Mixed	0.6177 (0.3427)	0.1405 (0.0880)	0.2713 (ns)	1.6756 (0.3726)	−0.4875 (0.1408)	0.6001 (0.01)	10
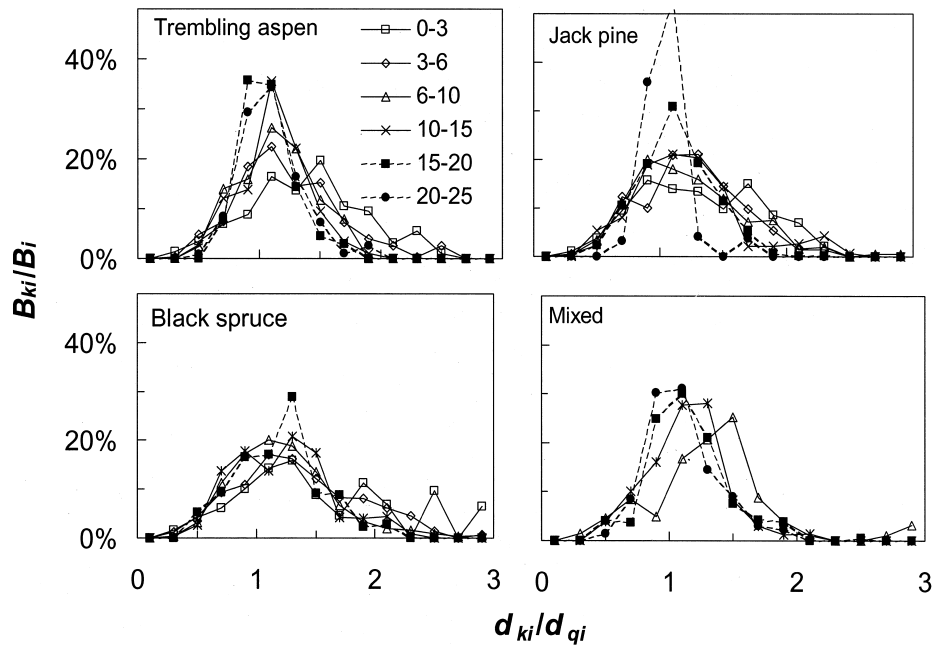
Figure 3. The distributions of normalized basal area, B_{ki}/B_i, against normalized diameter, d_{ki}/d_{qi}, for trembling aspen, jack pine, black spruce, and mixed stands along a 1,000-km boreal forest transect (Halliwell and Apps 1997a). For clarity, results are shown as mean values of d_{ai} ranges of 0–3, 3–6, 6–10, 10–15, 15–20, and 20–25 cm.

The biomass of stem, branch, and foliage and the sapwood volume of a tree were calculated using the biomass equations developed by Gower et al. (1997) for black spruce, jack pine, and trembling aspen, respectively, for northern and southern study areas of the Boreal Ecosystem-Atmosphere Study (BOREAS) based on site measurements. The equations developed for the northern BOREAS study area were used for calculating tree biomass in Manitoba, whereas those for the southern BOREAS study area were used for trees in Saskatchewan. For the mixed forests, tree biomass was calculated as the average of black spruce, jack pine, and trembling aspen equations.

Four types of methods for estimating stand biomass are assessed in this study: directly using a mean stand attribute, PPM for diameter-frequency distribution (PPM-N_{ki}/N_i), PPM for basal area-diameter distribution (PPM-B_{ki}/B_i), and PRM for diameter-frequency distribution. For the first type, d_{ai} or d_{qi} was used in biomass calculation, which implies that all trees have the same diameter of the value d_{ai} or d_{qi}. We also test the normal diameter-frequency distribution with $\sigma = 0.2$, as used by Penner et al. (1997). For the last three types, all three distribution functions of the Weibull, the Johnson's S_B, or the lognormal are tested. To compare the overall performance of each method, we further computed the arithmetic mean estimation error (MEE) and the root mean square error of estimation (RMSE).

Results and Discussion

Functions for Diameter-Frequency Distribution

Interesting diameter-frequency distribution, N_{ki}/N_i, against d_{ki}/d_{ai}, for trembling aspen, jack pine, black spruce, and mixed stands was found (Figure 1). Two features are

common for all of the forest species types. First, the highest value of N_{ki}/N_i does not always occur around $d_{ki}/d_{ai} = 1$, as a normal distribution would suggest. Instead, it usually occurs at a $d_{ki}/d_{ai} < 1$, especially for young stands with a lower value of d_a. Second, the derivation of a tree's dbh from d_a within a stand varies with the value of d_a. For a young stand with a small d_{ai}, trees are more evenly distributed across the relative diameter range. As the stand grows, self-thinning eliminates many disadvantaged small trees, and so the stand becomes more uniform relative to the mean dbh, as indicated by the higher values of N_{ki}/N_i for diameter intervals d_{ki}/d_{ai} close to 1.

Based on the goodness-of-fit χ_i^2 test, the Johnson's S_B function can best describe the diameter-frequency distribution of the trembling aspen, jack pine, black spruce, and mixed forest stands along the boreal transect (Table 2). The null hypothesis that the observed and fitted distributions are the same cannot be rejected for 65 out of 87 stands (74%) when using the Johnson's S_B function ($\alpha = 0.05$). It is followed by the lognormal function at a nonrejection rate of 72%, with the Weibull function performing worst at a nonrejection rate of 68%, although the differences among the different methods are small. We also noted that these functions typically fit the diameter-frequency distribution better for shade intolerant forest species (e.g., aspen and jack pine) than for shade tolerant species (e.g., black spruce and mixed).

To predict the diameter-frequency distribution from stand attributes using PPM, the relationship between parameters of a distribution function (i.e., the Johnson's S_B, the Weibull, or the lognormal) and one of the most commonly available stand attributes, d_{ai}, was examined (Figure 2). For
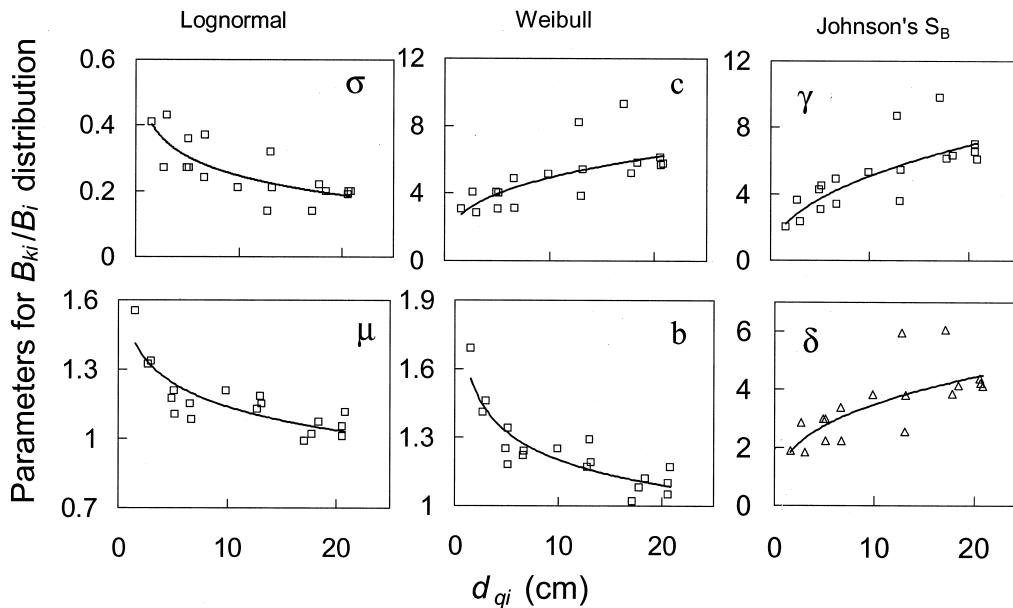
Figure 4. The relationships between d_{ai} and parameters of diameter-basal area distribution functions (the lognormal, the Weibull, and the Johnson's S_B) for trembling aspen stands along the Canadian boreal forest transect. Details of these relationships, R^2 value standing for fraction of variation explained, and the corresponding significance level are listed in Table 4.

each relationship, the coefficients (i.e., α and β) for estimating the parameters, R^2 standing for the fraction of variation explained by the relationship, and the corresponding significance level are listed in Table 3. For the Johnson's S_B and the Weibull functions, the power form ($\alpha\, d_{ai}\beta$) was found to be optimal, whereas the logarithmic form ($\alpha + \beta \ln(d_{ai})$) proved the best for the lognormal function. From Table 3, we can see that the relationships between parameters of the Johnson's S_B function and d_{ai} are significant at the 0.01 level for the trembling aspen stands. A relationship at the significance level of 0.01 also exists with the lognormal function, but the significance level is reduced to 0.1 with the Weibull function for its scale parameter b.

To test if there is a significant difference in coefficients α and β between different forest species types, Student's t-test was used, following the notation of Spiegel (1975). If t is larger than a critical value at a certain significance level, we reject the null hypothesis that there is no significant difference in the coefficient between two forest species types. The test results indicate that at the significance level of 0.01, we can reject the null hypothesis for all parameters between any two species types, i.e., there is a significant difference in parameters among forest species types. Consequently, we cannot merge any of two forest species types, and therefore four separate sets of equations were calculated and used (Table 3). It is worthwhile to point out that similar to the goodness-of-fit χ^2 test, the relationships between the distribution function parameters and d_{ai} are typically better for shade-intolerant forest types (e.g., aspen and jack pine) than for shade-tolerant types (e.g., black spruce and mixed).

Function for Diameter-Basal Area Distribution

The distributions of normalized basal area, B_{ki}/B_i, against normalized diameter, d_{ki}/d_{qi}, for trembling aspen, jack pine,

black spruce, and mixed stands are shown in Figure 3. The same grouping technique as with the diameter-frequency distribution was used for the purpose of clarity. Here we used quadratic mean diameter d_{qi} because basal area and d_{qi} are more closely related, although it may not make much difference because d_{qi} and d_{ai} are also closely related:

$$d_{qi} = \begin{cases} 1.1595d_{ai}^{0.9552}, & r^2 = 0.9996, \ n = 18, \ \text{trembling aspen} \\ 1.1541d_{ai}^{0.9659}, & r^2 = 0.9981, \ n = 25, \ \text{jack pine} \\ 1.1916d_{ai}^{0.9521}, & r^2 = 0.9982, \ n = 32, \ \text{black spruce} \\ 1.3398d_{ai}^{0.9161}, & r^2 = 0.9940, \ n = 10, \ \text{mix} \end{cases}$$

(15)

The distributions of B_{ki}/B_i are similar to that of N_{ki}/N_i, except that the higher values of B_{ki}/B_i generally occur in the diameter range of $d_{ki}/d_{qi} > 1$, especially for young stands with a lower value of d_{qi}. The skewness of B_{ki}/B_i toward $d_{ki}/d_{qi} > 1$ for young stands can be explained by the following two facts. First, trees with dbh $> d_{qi}$ play a much more important role in the distribution of B_{ki}/B_i than in that of N_{ki}/N_i. Second, there is a higher proportion of trees with dbh $> d_{qi}$ for young stands.

As with the diameter-frequency distribution, the parameters for the diameter-basal area distribution functions are closely related to one of the most commonly available stand attributes d_{qi} (Figure 4). The associated parameter estimation coefficients (i.e., α and β), R^2 value as fraction of variation explained by the relationships, and the corresponding significance level are listed in Table 4. In general, the relationships between d_{qi} and parameters for the

Table 4. Relationships between parameters of distribution function (i.e., the Weibull, the Johnson's S_B, and the lognormal) for the basal area-diameter distribution and the most common stand attribute, the quadratic mean dbh d_{qi} (cm), for trembling aspen, jack pine, black spruce, and mixed forest stands along a Canadian boreal forest transect. The form of relationship is αd_{qi}^{β} for the Weibull and the Johnson's S_B functions, and is $\alpha + \beta \ln(d_{qi})$ for the lognormal function. R^2 values stand for the fraction of variation explained by a relationship

	α Mean (STD)	β Mean (STD)	R^2 (Significance level)	α Mean (STD)	β Mean (STD)	R^2 (Significance level)	n
		b			c		
Weibull							
Trembling aspen	1.6572	−0.1395	0.7872	2.3714	0.3179	0.5643	
	(0.0623)	(0.0173)	(0.01)	(0.5810)	(0.0964)	(0.01)	18
Jack pine	1.5893	−0.0955	0.2602	2.9522	0.1689	0.1089	
	(0.1253)	(0.0340)	(0.05)	(0.8585)	(0.1233)	(0.1)	25
Black spruce	1.7075	−0.1193	0.2420	2.0965	0.2638	0.1590	
	(0.1309)	(0.0376)	(0.01)	(0.4986)	(0.1054)	(0.05)	34
Mixed	2.4238	−0.2438	0.6750	2.8419	0.2118	0.4745	
	(0.3859)	(0.0596)	(0.01)	(0.6291)	(0.0773)	(0.05)	10
		δ			γ		
Johnson's S_B							
Trembling aspen	1.5760	0.3449	0.6085	1.7998	0.4506	0.7060	
	(0.3806)	(0.0914)	(0.01)	(0.5275)	(0.1038)	(0.01)	18
Jack pine	2.1932	0.1574	0.1380	2.6113	0.2309	0.2240	
	(0.5507)	(0.1065)	(0.1)	(0.7375)	(0.1244)	(0.05)	25
Black spruce	1.3686	0.2888	0.1920	1.5491	0.3781	0.2262	
	(0.4069)	(0.1266)	(0.05)	(1.0303)	(0.2581)	(0.05)	34
Mixed	1.6368	0.2892	0.5180	1.2038	0.5265	0.7634	
	(0.4648)	(0.0970)	(0.05)	(0.4592)	(0.1223)	(0.01)	10
		μ			σ		
Lognormal							
Trembling aspen	1.4767	−0.1469	0.7250	0.4405	−0.0843	0.5910	
	(0.1561)	(0.0184)	(0.01)	(0.0403)	(0.0175)	(0.01)	18
Jack pine	1.4432	−0.1071	0.3890	0.3763	−0.0431	0.1500	
	(0.2083)	(0.0228)	(0.01)	(0.0510)	(0.0214)	(0.1)	25
Black spruce	1.4924	−0.1265	0.1880	0.5377	−0.0970	0.2110	
	(0.2778)	(0.0369)	(0.05)	(0.0683)	(0.0332)	(0.01)	34
Mixed	2.0655	−0.3222	0.8030	0.5095	−0.0948	0.7161	
	(0.2918)	(0.0456)	(0.01)	(0.0572)	(0.0211)	(0.01)	10

diameter-basal distribution functions are better than those for the diameter-frequency distribution. A similar Student's t test indicates that we can reject the null hypothesis for all parameters between any two species types at the 0.01 significance level, i.e., there is a difference in parameters for the diameter-basal area distribution among forest species types as well. Consequently, we cannot merge these formulas between two different forest species types, and so four separate sets of equations are calculated and reported (Table 4).

Goodness of Fit for Estimating Diameter-Frequency Distribution

The diameter-frequency distribution is directly estimated using PPM-N_{ki}/N_i and PRM. When the PPM-B_{ki}/B_i is used, we need first to convert the values based on their definitions:

$$\frac{N_{ki}}{N_i} = \frac{B_{ki}}{B_i} \bigg/ \left(\frac{d_{ki}}{d_{qi}}\right)^2 \tag{16}$$

The PRM performs best with the KS_i statistics typically being less 0.2, followed by PPM-N_{ki}/N_i (Figure 5). The PPM-B_{ki}/B_i gives the biggest KS_i statistics, suggesting that this method is generally not good for estimating the diameter-frequency distribution.

Biomass Estimation Error

The percentage errors in biomass estimates calculated with direct use of d_{ai} or d_{qi} for the four forest species types along the boreal forest transect were plotted in Figure 6. Again, the error was estimated as the percentage difference between the calculated value and the true value determined by summation of overall trees (Baskerville 1965a). Substantial underestimation occurred when d_{ai} was directly used in biomass calculation, with the maximum underestimation approaching 60%. On average, this method underestimates stem biomass, sapwood volume, branch biomass, and foliage biomass by 20.4, 22.7, 24.0, and 17%, respectively, as shown by MEE (Table 5). While MEE represents the bias in estimation of a method, the RMSE represents the magnitude
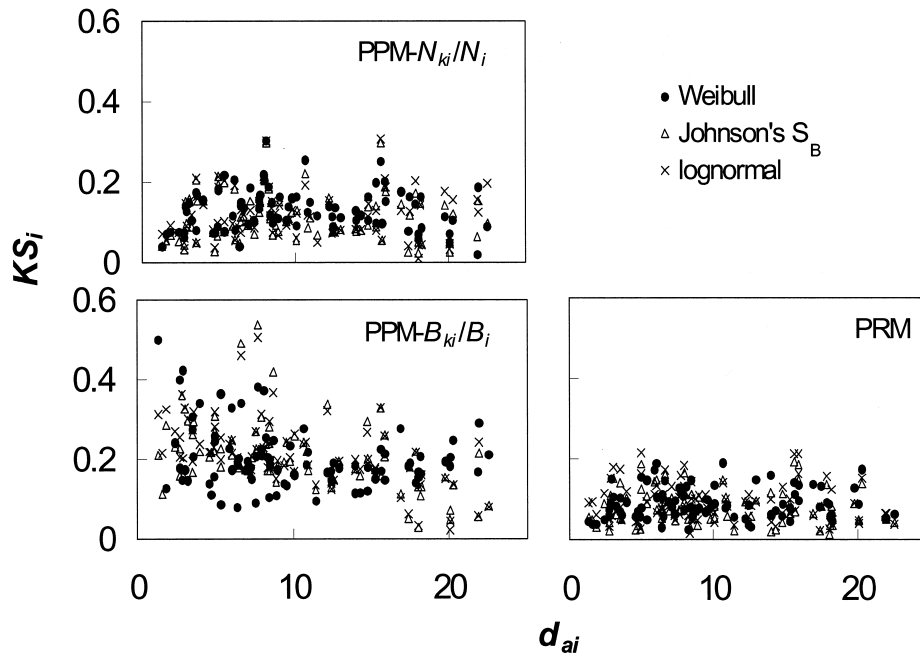
Figure 5. The Kolmogorov-Smirnov goodness-of-fit test statistics for stand *i*, *KS*_i, when the method of PPM-N_{ki}/N_i, PPM-B_{ki}/B_i, or PRM together with different distribution functions. All 87 stands of trembling aspen, jack pine, black spruce, and mixed stands along a 1,000-km boreal forest transect are included.
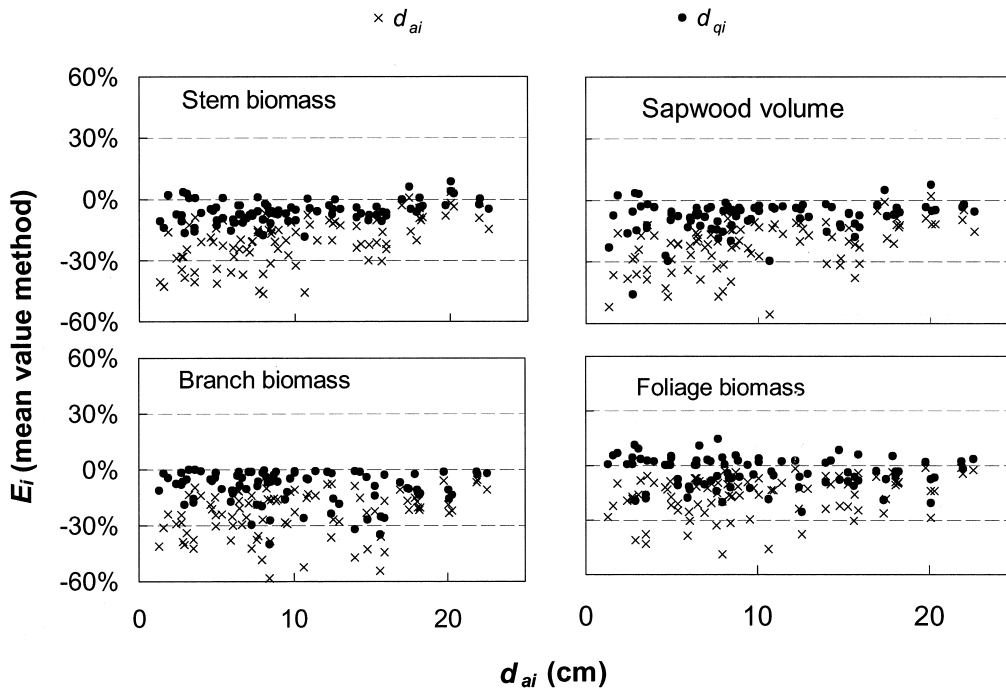

Figure 6. The percentage errors in biomass estimates (stem biomass, sapwood volume, branch biomass, and foliage biomass) calculated using the first type of methods (i.e., directly using d_{ai} or d_{qi}) for 87 stands of trembling aspen, jack pine, black spruce, and mixed stands along a 1,000-km boreal forest transect.

of the error. For the method of direct use of d_{ai}, RMSE for stem biomass, sapwood volume, branch biomass, and foliage biomass were, respectively, 23.3, 25.9, 26.9, and 20.1% (Table 5). The method of direct use of d_{qi} can significantly reduce MEE and RMSE to −6.1 and 10.4%, respectively,

for all biomass components considered (Table 5). The normal diameter-frequency distribution with $\sigma = 0.2$, as used by Penner et al. (1997), provides some improvements over the method of direct use of d_{ai} but is worse than the method of direct use of d_{qi}.

Table 5. Mean estimation error (MEE) and root mean square error (RMSE, in parentheses), both in percentage, of stem biomass, branch biomass, sapwood volume, foliage biomass and all components using different methods

	Methods	Stem	Sapwood	Branch	Foliage	All
	d_{ai}	−20.4 (23.3)	−22.7 (25.9)	−24.0 (26.9)	−17.0 (20.1)	−21.0 (24.0)
	d_{qi}	−6.1 (8.1)	−8.2 (11.3)	−9.5 (13.1)	−3.6 (8.9)	−6.9 (10.4)
	Normal	−15.0 (19.0)	−16.5 (20.8)	−17.3 (21.6)	−11.8 (15.5)	−15.1 (19.2)
PPM-N_k/N_i	Lognormal	3.9 (13.1)	5.7 (16.4)	5.0 (15.6)	3.0 (11.1)	4.4 (14.1)
	Weibull	−16.1 (19.4)	−16.6 (19.7)	−18.1 (21.6)	−15.2 (18.0)	−16.5 (19.7)
	Johnson's S_B	−3.6 (11.9)	−2.9 (12.6)	−3.9 (13.0)	−3.7 (10.0)	−3.5 (11.9)
PPM-B_k/B_i	Lognormal	1.7 (3.9)	2.5 (5.2)	2.5 (6.6)	0.6 (4.6)	1.8 (5.1)
	Weibull	−0.3 (3.1)	−0.4 (4.0)	−1.3 (5.8)	−0.3 (4.0)	−0.6 (4.2)
	Johnson's S_B	1.5 (3.7)	2.0 (4.7)	1.4 (5.7)	0.5 (4.2)	1.3 (4.6)
PRM	Lognormal	0.6 (4.9)	0.7 (5.1)	0.9 (5.4)	0.9 (4.7)	0.8 (5.0)
	Weibull	0.7 (4.9)	0.7 (5.1)	0.7 (8.0)	0.7 (4.7)	0.7 (5.7)
	Johnson's S_B	0.6 (4.9)	0.8 (5.1)	0.9 (8.0)	0.8 (4.7)	0.8 (5.6)

Surprisingly, the second type of method, PPM-N_{ki}/N_i, doesn't improve the biomass estimation significantly when compared with the first type of methods (Figure 7 and Table 5). The PPM-N_{ki}/N_i methods give RMSE ranging from 10 to 20% with different distribution functions (the Weibull, the Johnson's S_B, and the lognormal). This deficiency can be mainly attributed to the fact that the goal of the PPM-N_{ki}/N_i is the minimization of error in the diameter-frequency distribution instead of in the biomass distribution; yet a small estimation error in the number of largest trees in a stand may result in substantial error in biomass. We notice, however, the PPM-N_{ki}/N_i method with the Johnson's S_B and the lognormal distribution functions is less biased than the first type of method, although the same cannot be said for the PPM-N_{ki}/N_i method with the Weibull distribution function.

The results of the third and fourth types of methods generally support this argument (Figures 8 and 9 and Table 5). Because of the close relationships between basal area and biomass, the third type of method, the PPM-B_{ki}/B_i, has MEE < 3% (unbiased) and RMSE < 10% (Figure 8 and Table 5). The differences in using different distribution functions were not significant. The results from the fourth type of methods, the PRM, were similar, with MEE < 3% (unbiased) and RMSE < 10% (Figure 9 and Table 5). We noticed, however, occasionally an estimation error of approximately 30% could occur when using PRM. Investigation of these occasions indicates that they mainly happen when the observed tree size distribution does not follow closely the distribution functions.

To further test the applicability of the third and fourth types of methods, we applied them to the data set of a balsam fir stand in northwestern New Brunswick, Canada
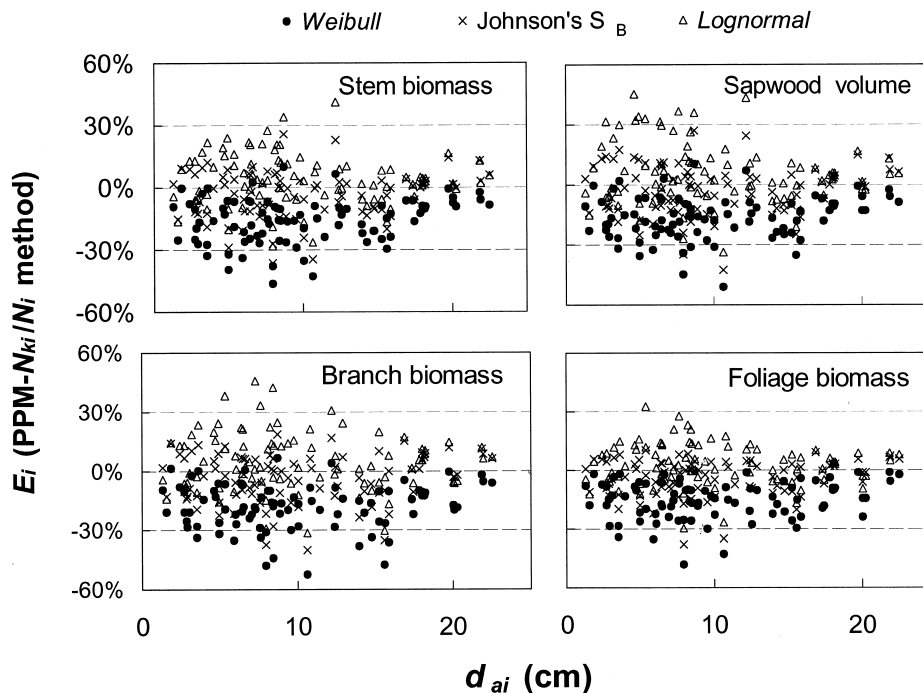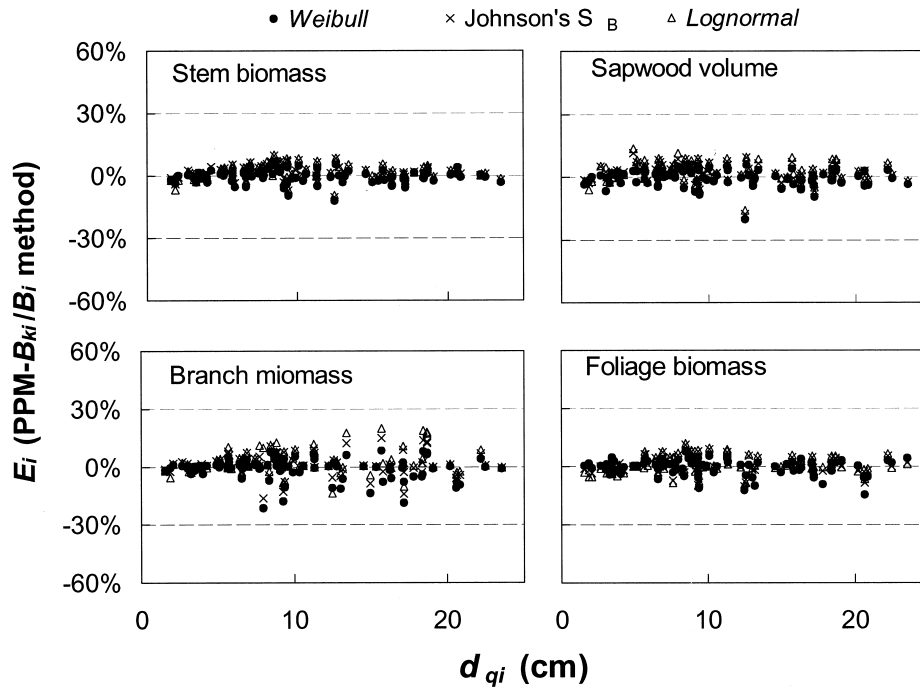

Figure 7. The percentage errors in biomass estimates (stem biomass, sapwood volume, branch biomass, and foliage biomass), calculated using the second type of method, i.e., PPM-N_{ki}/N_i, with either the Weibull, the Johnson's S_B, or the lognormal distribution functions.

Figure 8. The percentage errors in biomass estimates (stem biomass, sapwood volume, branch biomass, and foliage biomass), calculated using the third type of method, i.e., PPM-B_{ki}/B_i, with either the Weibull, the Johnson's S_B, or the lognormal distribution functions.
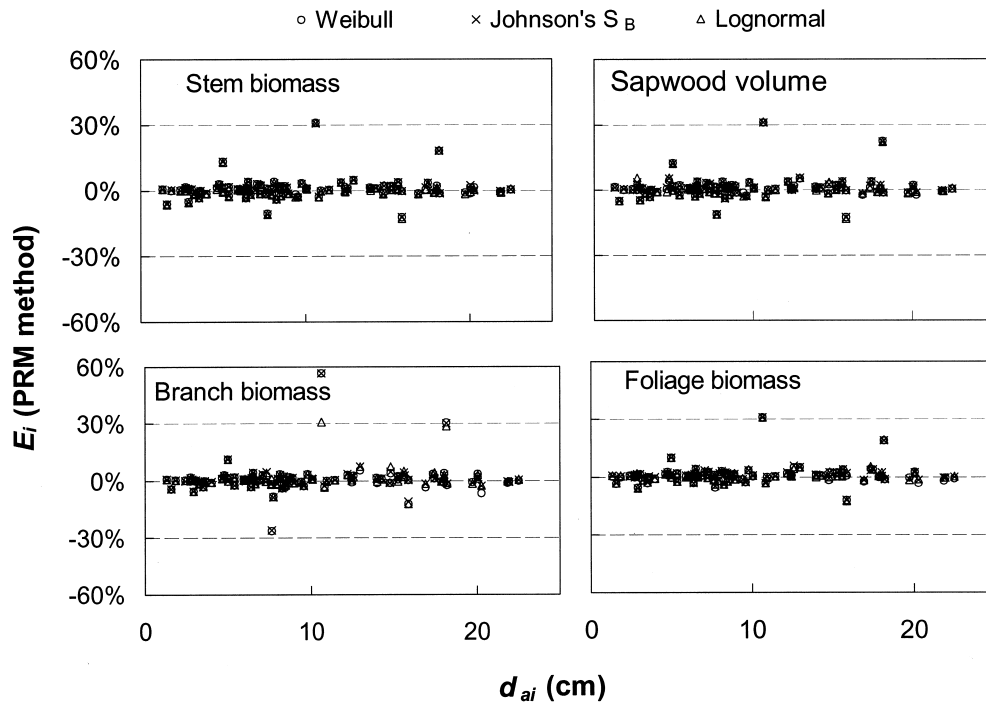

Figure 9. The percentage errors in biomass estimates (stem biomass, sapwood volume, branch biomass, and foliage biomass), calculated using PRM with either the Weibull, the Johnson's S_B, or the lognormal distribution functions.

(Baskerville 1965a). Biomass equations for stem, branch, and foliage developed for the same site were used (Baskerville 1965b). Because no distribution functions were developed specifically for balsam fir, black spruce distribution functions were used instead. The test results shown in Table

6 indicate that both PPM-B_{ki}/B_i and PRM work equally well for the balsam fir stand, despite the fact that the climate conditions were substantially different from the boreal forest transect. This result suggests that these two types of methods may also be applicable to forest stands outside the

Table 6. Test results of the four types of methods for estimating biomass components for a balsam fir stand in Northwest New Brunswick, Canada. Values shown are mean estimation errors (%)

	Methods	Stem	Branch	Foliage
	d_{ai}	−29.4	−11.9	−31.3
	d_{qi}	−11.4	−1.2	−13.1
PPM-N_{ki}/N_i	Normal	−23.7	18.9	−25.2
	Lognormal	−11.0	−11.9	−11.4
	Weibull	−31.6	−32.5	−32.8
	Johnson's S_B	−18.7	−19.3	−19.5
PPM-B_k/B_i	Lognormal	−1.0	−1.2	−1.1
	Weibull	−4.1	−4.5	−4.7
	Johnson's S_B	−1.6	−1.7	−1.8
PRM	Lognormal	1.5	1.6	1.9
	Weibull	2.1	2.1	2.2
	Johnson's S_B	1.7	1.8	1.8

boreal forest transect, although more tests are required to support this conclusion.

Conclusions

We conclude that:

1. The diameter-frequency distribution (d_{ki}/d_{ai}-N_{ki}/N_i) is not normal for most of the sampled stands of trembling aspen, jack pine, black spruce, and mixed forests along the boreal forest transect. Instead, the distribution is generally positively skewed because there are usually more trees with dbh $< d_{ai}$ within a stand, especially for the young stands. On the other hand, the diameter-basal area distribution (d_{ki}/d_{qi}-B_{ki}/B_i) is negatively skewed for most stands because the larger trees, although fewer in number, contribute more to a stand's total basal area.

2. The χ^2 test suggested that the Johnson's S_B function fit observed d_{ki}/d_{ai}-N_{ki}/N_i best. The null hypothesis that the observed and fitted distributions are the same cannot be rejected ($\alpha = 0.05$) at 65 out of 87 stands (74%) when using the Johnson's S_B function, at 72% when using the lognormal function, and at 68% when using the Weibull function. Nevertheless, the differences in goodness of fit among different functions are small.

3. Based on the Kolmogorov-Smirnov goodness-of-fit test statistics, PRM and PPM-N_{ki}/N_i perform well for estimating diameter-frequency distribution, but the PPM-B_{ki}/B_i is generally not as good.

4. Direct use of a mean value (d_{ai} or d_{qi}) is often biased with a substantial RMSE. The normal diameter-frequency distribution with $\sigma = 0.2$ provides some improvements over the method of direct use of d_{ai} but is worse than the method of direct use of d_{qi}. The second type of method, PPM-N_{ki}/N_i, failed to provide much improvement over the first type either, probably because the goal of PPM-N_{ki}/N_i is to minimize errors in tree number distribution; however, a small error in the number of largest trees within a stand may result in a substantial error in biomass estimates. The third

(PPM-B_{ki}/B_i) and fourth (PRM) types of method can provide unbiased (MEE $< 3\%$) and accurate (RMSE $< 10\%$) estimates, relative to the true values given by summation of overall trees, regardless of the distribution functions used (i.e., the Weibull, the Johnson's S_B, and the lognormal). Therefore, the uses of PPM-B_{ki}/B_i and PRM for future biomass mapping at broader spatial scales are recommended.

The test of a balsam fir stand in northwest New Brunswick, Canada (Baskerville 1965a) indicates that PPM-B_{ki}/B_i and PRM performed equally well despite the difference in climate conditions between the boreal forest transect and this site, although more tests are clearly needed.

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