On the optimization of decode-and-forward schemes for two-way asymmetric relaying

Zhiyong Chen†‡, Hui Liu‡ and Wenbo Wang‡

†Department of Electrical Engineering, University of Washington, Seattle, WA 98195-2500, USA
‡Key Laboratory of Universal Wireless Communications for Ministry of Education, Beijing University of Posts and Telecommunications, Beijing 100876, China

Email: {zychen, huiliu}@u.washington.edu, wbwang@bupt.edu.cn

Abstract—In this paper, we consider a two-way asymmetric relaying channel with unmatched traffics from two sources. Aiming at maximizing the sum rate under a minimum rate constraint, we address the optimization problem (in terms of time and power allocation) for two decode-and-forward (DF) strategies, namely, the decode-and-forward with joint modulation (DF-JM) and the decode-and-forward with network-superposition coding (DF-NSC). Closed-form solutions under different relay settings are presented. Numerical results are provided to illustrate the performance gains due to the optimal resource allocation, as well as the achievable rate gaps between two different DF strategies.

I. INTRODUCTION

Considering a two-way relay channel where two sources exchange information through a relay node, the typical operations at the relay node involve decoding, remodulation and encoding, and broadcasting of two messages to their destinations [1]-[5]. Upon receiving the broadcast signals from the relay node, each destination decodes its intended message with its own signal as side information. As such, the two-way relay can in general be regarded as two separate stages, i.e., the multiple-access (MAC) stage and the broadcasting with side information (BCSI) stage. Although the channel capacities of individual stages are well understood, the end-to-end behaviors of the relay system remain a topic of interest.

Taking advantage of the two-way structure, a number of strategies are devised for relay channel, most of which target at increasing the overall spectral efficiency. Among others, the most studied schemes are the decode-and-forward with a bitwise XOR (DF-Xor) [3][6], the decode-and-forward with superposition coding (DF-SC) [5]-[6], and the amplify-and-forward (AF) [5] methods. The DF-Xor in particular, is effective when dealing with symmetric data rates from the two sources. Specifically, the bitwise XOR requires the messages from the two sources to have the same length (i.e., symmetric relaying) [2]. The DF-SC approach on the other hand, can handle asymmetric data by splitting the transmit power of the relay node into two independently coded sequences [6].

Note that in most practical applications, the traffics are asymmetric by nature, where is considered in this paper. Our aim is to maximize the end-to-end data rate through the relay channel subject to certain resource constraints (e.g., time and power). In addition to the total data rate, other performance measures such as the quality of service (QoS) requirements must be satisfied. For example, the data rate of one direction should be larger than a given threshold. As a matter of fact, such behavior corresponds to a scenario where a base station communicates with a user through the relay node. While it is desirable to maximize the downloading speed from the base station to the user, a minimum uploading rate must be guaranteed nevertheless.

Two recently developed asymmetric relay schemes, namely, the decode-and-forward with joint modulation (DF-JM) [7] and the decode-and-forward with network-superposition coding (DF-NSC)[8][9], are analyzed and optimized in this paper. In the DF-JM, the relay node combines the two decoded messages into a new sequence and effectively handles the unmatched data rates through optimal constellation labeling maps. The underlying idea behind DF-NSC is to split the longer message into two parts, one of which is XORed with the shorter message while the remaining part is separately encoded - the relay node applies superposition coding on top of the bitwise XOR to better utilize the BCSI channel. Both of these schemes require full decoding at the relay node. The end-to-end achievable rate of the two-way relay channel depends on both time split and power allocation at the relay node. Therefore, it is important to understand how the resource allocation could affect the overall system performance, and whether there exists an optimal operating point that maximizes the total rate for the asymmetric relaying. In [10], the optimal power allocation problem is investigated for the two-way symmetric channel. The more complicated asymmetric relaying optimization, especially the one with QoS constraint, remains an open issue as of today.

The main contribution of this paper is the formulation of the aforementioned problem and the derivation of analytical solutions for both DF schemes. Specifically, we attempt to maximize the end-to-end asymmetric rates by adjusting the time split between MAC and BCSI and the transmit power at the relay node. Obviously, the optimal resource allocation depends on a number of system parameters, including the channel gains and the transmit powers at both the source and the relay nodes. Other factors such as the QoS constraints also must be considered. Whenever it is possible, closed-form solutions for the two DF strategies are derived. In some special cases, it turns out that the optimal resource allocation is convex
problem and can be obtained by a simple iterative search. Extensive numerical results included in this paper illustrate and validate the theoretical results.

II. SIGNAL MODEL AND ASYMMETRIC DF STRATEGIES

A. Signal model

We consider a two-way relay channel with two sources (i.e., $S_1$ and $S_2$) and one relay node, as depicted in Fig. 1. The half-duplex system is assumed, i.e., each node cannot transmit and receive simultaneously. The communication from one source to another source takes place in two transmission time slots. During the first slot, $S_1$ and $S_2$ simultaneously transmit their messages to the relay node, as in MAC. The relay node then processes, reformats if necessary, and broadcasts the resulting signals to $S_1$ and $S_2$, which become destinations during the second slot. As a result, the channel of the second time slot is equivalent to BCSI. The rest of the notation is as follow:

- $Z_1$, $Z_2$ and $Z_r$ are the zero mean complex Gaussian noise of variances $\sigma^2_1 = \sigma^2_2 = \sigma^2_r = 1$ at sources $S_1$, $S_2$, and the relay node, respectively.
- $h_i$ is the channel coefficient of the link between the source/destination $S_i$ and the relay node. We assume the channels are reciprocal.
- $P_i$ denotes the transmit power of the $i$-th node, $i = 1, 2, r$.
- $t_1$ and $t_2$ are the transmit time durations for the first time slot and the second time slot, i.e., $t_1 + t_2 = 1$.
- $R_{ij}$ is the achievable rate from node $i$ to node $j$.

During MAC, source messages $W_1$ and $W_2$ are respectively modulated and transmitted to the relay node by $S_1$ and $S_2$ simultaneously. The signal received at the relay node is thus given by $Y_r = \sqrt{P_1}h_1X_1 + \sqrt{P_2}h_2X_2 + Z_r$, where $X_1$, and $X_2$ are the transmit signals of sources $S_1$, and $S_2$, respectively.

With DF relaying protocol applied, the relay node first decodes the messages $W_1$ and $W_2$, and then apply a generating function, $g(W_1, W_2)$, to create a transmit signal $X_r$ for the BCSI channel. As a result, the received signal at $i$-th destination can be written as $Y_i = \sqrt{P_i}h_iX_i + Z_i$, $i = 1, 2$. For the sake of simplicity, we also impose an average transmit power constraint on $X_i$, i.e., $\mathbb{E}\{|X_i|^2\} = 1$, for $i = 1, 2, r$. Since the messages $W_1$ and $W_2$ originate from $S_1$ and $S_2$, respectively, each destination is able to cancel out the interference caused by the its own message.

B. Asymmetric relay strategies

Throughout this paper, we assume that $|h_2| \geq |h_1|$ without loss of generality. Let $n_1$ and $n_2$, $n_1 \neq n_2$, denote the number of information bits from the two sources, respectively. As mentioned earlier, the relay node transmits the re-encoded signals by applying $g(W_1, W_2)$ onto the outputs of MAC. Depending on the actual function $g(W_1, W_2)$, two different approaches are analyzed.

1) Joint modulation [7]: By combining the MAC outputs, $W_1$ and $W_2$, into a new sequence, $W_r = [W_1, W_2]$, the relay node can encode and modulate the resulting sequence jointly, regardless of the sizes of the original messages. Such a DF scheme with joint modulation is hereby defined DF-JM, i.e.,

$$X_r = g(W_1, W_2) = \mu_r(W_r)$$

where $\mu_r$ denotes the constellation used by the relay node.

In one form of DF-JM, optimal constellation labeling maps are used to maximize the intra-subset Euclidean distance at each destination by exploiting the redundant bits or side information [7]. Such a strategy allows the DF-JM to achieve interference-free reception at both destinations. Accordingly, we can describe the achievable rate region of DF-JM for asymmetric relaying as:

$$R_{JM} = \{(R_1, R_2) : R_1 \leq \min\{R_{1r}, R_{r2}\}, \quad R_2 \leq \min\{R_{2r}, R_{1r}\}, \quad R_1 + R_2 \leq R_r\}$$

where we define $C(x) \triangleq \log(1 + x)$ and have

$$R_{1r} = t_1C(P_1 |h_1|^2), \quad R_{2r} = t_1C(P_2 |h_2|^2), \quad R_{1c} = (1 - t_1)C(P_r |h_1|^2), \quad R_{2c} = (1 - t_1)C(P_r |h_2|^2), \quad R_r = t_1C(P_1 |h_1|^2 + P_2 |h_2|^2).$$

2) Network and Superposition coding [8][9]: For BCSI, the longer sequence (e.g., $W_1$) can be split into two subsequences ([$W_{11}W_{12}$]), with $W_{11}$ having the same length as the short sequence (e.g., $W_2$). Hence, $X_r$ can be expressed as $X_r = g(W_1, W_2) = \sqrt{\lambda_1}\mu_r(W_{11} \oplus W_2) + \sqrt{\lambda_2}\mu_r(W_{12})$, where $\oplus$ denotes the bit-wise XOR operation and $\lambda_i$, $i = 1, 2$, denotes the power allocation factor that satisfies the total power constraint at the relay node, i.e., $\lambda_1 + \lambda_2 = 1$. For decoding at the destination, the self-interference cancellation can be performed at $S_1$ due to the side information $W_1$. However, for the decoding at $S_2$, the XORed signal must be decoded first with the superposed signal (i.e., $W_{12}$) as interference. Only after this step can the XORed signal be removed from the received signal, leading to interference-free decoding of $W_{12}$. The achievable rate of DF-NSC (assuming $n_1 > n_2$) in BCSI is given by

$$R_{r1} = t_2 \min\{C(\lambda_1P_r |h_2|^2), C(\lambda_2P_r |h_1|^2)\}, \quad R_{r2} = R_{r1} + (1 - t_1)C(\lambda_2P_r |h_2|^2).$$
By substituting (4) and (5) into (2), the rate region of DFNSC can be obtained. The achievable rate for the case with \( n_1 < n_2 \) can be similarly derived:

\[
R_{t_1} = (1-t_1)C(P_r|h_1^2), \quad \tag{6}
\]

\[
R_{t_2} = (1-t_2)C\left(\frac{\lambda_1 P_r |h_1|^2}{1 + \lambda_2 P_r |h_1|^2}\right). \quad \tag{7}
\]

**Remark 1:** Since the achievable rates of MAC and BCSI are function of their allocated time, it is essential to adjust \( t_1 \) and \( t_2 \) to optimize the end-to-end relay rates. In other words, the rates corresponding to MAC and BCSI must match under given performance measures. In the following analyses, we assume that both sources have sufficient transmit power, i.e., rates of MAC \( \geq \) rates of BCSI, and focus on the maximization of the BCSI rates subject to the relay power and QoS constraint.

### III. OPTIMAL RESOURCE ALLOCATION AND ANALYSIS

In this section, we formulate and solve the resource allocation problem for the two DF schemes. Their optimized achievable rates are then analyzed and compared. The goal here is to derive the optimal time duration and the optimum power allocation at the relay node to maximize the total rates subject to the QoS constraint. Specifically, one of the links is constrained to be greater than a pre-defined threshold \( R_t \) (i.e., we assume \( R_1 \geq R_t \) without loss of generality). The optimization problem is formulated as follows:

\[
\begin{align*}
\max_{t_1, t_2} & \quad \{ R_{t_2} + R_{t_1} \} \\
\text{s.t.} & \quad R_t \leq R_{t_2} \leq R_{t_1}, R_{t_1} \leq R_{t_2}, \\
& \quad R_{t_2} + R_{t_1} \leq R, t_1 \in (0, 1), \sum_{i=1}^{2} \lambda_i = 1
\end{align*} \tag{8}
\]

where \( \sum_{i=1}^{2} \lambda_i = 1 \) is not necessary for the DF-JM scheme.

#### A. DF-JM

For this strategy, the constraints in (8) can be rewritten as

\[
\frac{C(P_r|h_2^2)}{C(P_1|h_1^2) + C(P_r|h_2^2)} \leq t_1 \leq 1 - \frac{R_t}{C(P_r|h_2^2)}; \\
t_1 \geq \frac{R_t}{C(P_1|h_1^2)}; \quad t_1 \geq \frac{C(P_r|h_1^2)}{C(P_1|h_1^2) + C(P_r|h_2^2)}; \quad \tag{9}
\]

\[
t_1 \geq \frac{C(P_1|h_1^2) + C(P_r|h_2^2)}{C(P_1|h_1^2) + P_2|h_2^2) + C(P_r|h_1^2) + C(P_r|h_2^2)}. \]

Let \( V = \max\{\frac{C(P_1|h_1^2) + C(P_r|h_2^2)}{C(P_1|h_1^2) + C(P_r|h_2^2)}, \frac{R_t}{C(P_1|h_1^2)}\} \) and \( \Psi \) be a set that satisfies \( \Psi = \{P_1, P_2, R_t\} : P_t \geq (2^{\frac{R_t}{C(P_{r_2}|h_2^2)}} - 1)/2^{\frac{R_t}{C(P_{r_2}|h_2^2)}}\). From the resulting expression in (2), we note that \( R_{t_2} \) and \( R_{t_1} \) are continuous and strictly decreasing in \( t_1 \). This means the optimal time allocation of (8) is reached at

\[
t_1^* = V \quad \text{when} \quad t_1^* \leq 1 - \frac{R_t}{C(P_r|h_2^2)}, \quad \tag{10}
\]

We observe that, the optimal time allocation \( t_1^* \) depends on \( P_r, P_1, P_2 \) and constraints on \( R_t \). Thus, for any given \( (P_r, P_1, P_2, R_t) \in \Psi \), (10) is fulfilled, which means the optimal time allocation is at \( t_1^* = V \). If (10) does not hold (i.e., \( (P_r, P_1, P_2, R_t) \notin \Psi \)), this means the two-way relay systems is unable to support the desired minimum rate.

**Remark 2:** When \( R_t = 0 \), i.e., without a minimum rate constraint, the two-way asymmetric relaying system reaches the maximum sum rate with \( t_1^* = V \).

#### B. DF-NSC

Rate maximization using the DF-NSC scheme is more involved due to the fact that both the time duration and the power allocation need to be optimized. Depending on the length of the original messages, we study two cases separately.

**Case 1** \((n_1 < n_2)\): Recalling the achievable rate region for this strategy in (6) and (7), it can be shown that the object of (8) is continuous and strictly increasing in \( \lambda_1 \) and decreasing in \( t_1 \) under the constraints. Let \( f_1(\lambda_1) = \max\{C(P_r|h_2^2) + C(P_1|h_1^2), (1 + \lambda_2 P_r |h_1|^2) C(P_r|h_1^2)\} \)

\[
C(P_r|h_2^2) + C(P_1|h_1^2) + C(P_r|h_1^2) + C(P_r|h_2^2) + C(P_1|h_1^2) + C(P_r|h_2^2) \}
\]

Further define \( D_1 = \{\lambda_1 : 0 \leq f_1(\lambda_1) \leq 1 - R_t/C(1 + \lambda_2 P_r |h_1|^2)\} \) and \( D_2 = \{\lambda_1 : f_1(\lambda_1) > 1 - R_t/C(1 + \lambda_2 P_r |h_1|^2) \geq 0\} \). Note that the optimal power allocation \( \lambda_1^* \) is not in the set \( D_2 \) based on the constraints in (8). We introduce the following lemma to facilitate the optimization.

**Lemma 3:** For any \( \lambda_1 \in D_1 \), the optimal time allocation is given by \( t_1^* = f_1(\lambda_1) \).

**Proof:** This can be proved by contradiction. It is easy to see that under the constraints in (8), \( t_1^* > f_1(\lambda_1) \) cannot be less than \( f_1(\lambda_1) \) or greater than \( 1 - R_t/C(1 + \lambda_2 P_r |h_1|^2) \). Hence, \( 1 - R_t/C(1 + \lambda_2 P_r |h_1|^2) \geq t_1^* > f_1(\lambda_1) \) given any \( \lambda_1 \in D_1 \). Assuming we can find a \( \tilde{t}_1 \) such that \( f_1(\lambda_1) \leq \tilde{t}_1 < t_1^* \leq 1 - R_t/C(1 + \lambda_2 P_r |h_1|^2) \) while meeting all constraints in (8), then \( R_{t_2}(\tilde{t}_1 + R_{t_1}(\tilde{t}_1)) > R_{t_2}(t_1^*) + R_{t_1}(t_1^*) \). This contradicts the assumption that \( t_1^* \) is optimal time allocation for a given \( \lambda_1 \). Thus, for any \( \lambda_1 \in D_1 \), the Lemma results.

Using the above Lemma, it is clear that the optimal power allocation and the optimal time allocation can be rewritten as

\[
\max\{\lambda_1 \in D_1 \} \{C(P_r|h_2^2) + C(P_1|h_1^2) + C(P_r|h_2^2)\}
\]

\[
\text{s.t.} \quad 0 \leq \lambda_1 \leq 1, \quad \lambda_2 = 1 - \lambda_1, \quad \tag{11}
\]

\[
0 \leq f_1(\lambda_1) \leq 1 - \frac{R_t}{C(1 + \lambda_2 P_r |h_1|^2)}.
\]

Substituting \( f_1(\lambda_1) \) back into (11), we observe that increasing \( \lambda_1 \) will increase the objective function in (11). Note that for any \( \lambda_1 \), \( f_1(\lambda_1) \leq f_1(1) \) holds. Depending on the power at the relay node, the solution is given under three scenarios.
Scenario 1: $P_r \geq \frac{R_t}{|h_1|^2 - |h_2|^2}$. Here, $\lambda_1 = 1 \in D_1$. The optimal solution set of the above problem is $\lambda^*_1 = 0, \lambda^*_2 = 0$, and $t_i^* = f_1(t_i)$.  

Scenario 2: $(\frac{R_t}{|h_1|^2 - |h_2|^2}) < P_r \geq \frac{2R_t}{|h_1|^2}$. This case corresponds to $\lambda_1 = 1 \in D_2$. It can be easily verified that the optimal problem also is convex and the optimum solution can be determined through an iterative search.  

Scenario 3: $P_r < \frac{2R_t}{|h_1|^2}$. In this case, the constraint in (11) can be rewritten as $f_1(\lambda_1) \leq f_1(1) < 1 - R_t/C(P_r| |h_1|^2) < 0$. However, since $t_1$ must be larger than 0, the DF-NSC scheme cannot achieve the requirement of $R_1 \geq R_t$.  

Case II $(n_1 > n_2)$: Let $f_2(\lambda_1) = \max\{C(P_r| |h_2|^2) + C(P_r| |h_1|^2)/|h_1|^2 + \lambda_2 P_r + \lambda_2^2 P_r^2), C(P_r| |h_1|^2)/|h_1|^2 + C(P_r| |h_1|^2)/|h_1|^2 + \lambda_2 P_r + \lambda_2^2 P_r^2), C(P_r| |h_2|^2)\}$.

Similarly, for any $\lambda_1$ that meets all the constraints in (8), it is straightforward to prove that Lemma 1 also holds for Case II. The optimization problem becomes  

$$\max_{\lambda_1} (1 - f_2(\lambda_1))\{C\left(\frac{\lambda_1 P_r |h_2|^2}{1 + \lambda_2 P_r |h_2|^2} + C(P_r |h_2|^2)\right), (12)$$

$$s.t. \lambda_2 \leq 1, \lambda_2 = 1 - \lambda_1,$$

$$0 \leq f_2(\lambda_1) = 1 - \frac{R_t}{C(P_r| |h_2|^2)}.$$

Since $t_1$ must be less than 1 and $\lambda_1$ must be larger than 0, the transmit power of the relay node must satisfy: $P_r \geq \max\{\frac{2R_t}{|h_2|^2 - |h_1|^2}, \frac{|h_2|^2 - |h_1|^2}{|h_2|^2 - |h_1|^2}\}$. Again, three scenarios need to be considered.  

Scenario 1: $P_r \geq \max\{\frac{2R_t}{|h_2|^2 - |h_1|^2}, \frac{|h_2|^2 - |h_1|^2}{|h_2|^2 - |h_1|^2}\}$. Let us define $G_1 = \{(P_r, P_1, P_2, R_t) | f_2(1 - \frac{R_t}{C(P_r| |h_2|^2)}) \leq 1 - \frac{R_t}{C(P_r| |h_2|^2)}\}$ and $G_2 = \{(P_r, P_1, P_2, R_t) | f_2(1 - \frac{R_t}{C(P_r| |h_2|^2)}) > 1 - \frac{R_t}{C(P_r| |h_2|^2)}\}$.

Note that, $f_2(\lambda_1)$ and the objective function (12) are increasing with $\lambda_1$. As a result, the optimal power allocation of the relay node is given by  

$$\lambda^*_1 = \begin{cases} 1 - \frac{|h_2|^2 - |h_1|^2}{P_r |h_2|^2 - |h_1|^2} & \text{if } (P_r, P_1, P_2, R_t) \in G_1, \\ \lambda_1 & \text{if } (P_r, P_1, P_2, R_t) \in G_2 \end{cases}$$

where $\lambda_1$ satisfies $f_2(\lambda_1) = 1 - R_t/C(P_r| |h_2|^2)$. Further, we obtain $\lambda_2^* = 1 - \lambda_1^*$ and $t_i^* = f_2(t_i)$.  

Scenario 2: $(\frac{R_t}{|h_2|^2 - |h_1|^2}) > P_r \geq \frac{2R_t}{|h_2|^2}$. Since $(|h_2|^2 - |h_1|^2)/(|h_2|^2 - |h_1|^2) > P_r$, the objective function in (8) becomes $(1 - t_i)\left(2C(\lambda_1 P_r |h_1|^2) + C((1 - \lambda_1) P_r |h_2|^2)\right)$. (8) is not longer a convex problem. As a matter of fact, it corresponds to an extreme case where the relay node has very limited transmission power: $P_r \leq 1/|h_1|^2 - 1/|h_2|^2$.  

Scenario 3: $P_r < \min\{\frac{2R_t}{|h_2|^2 - |h_1|^2}, \frac{|h_2|^2 - |h_1|^2}{|h_2|^2 - |h_1|^2}\}$. In this case, the DF-NSC scheme cannot achieve the rate requirement ($R_1 \geq R_t$).

C. Achievable rates analysis

Having optimized the time and power allocation, we next compare the two DF schemes and show that the DF-JM protocol performs better than the DF-NSC in terms of the end-to-end relay rates.

Theorem 4: Under the minimum rate constraint, the DF-JM scheme provides a higher sum rate than the DF-NSC scheme.

Proof: For both DF-JM and DF-NSC to meet the minimum rate constraint, it holds that $P_r \geq \max\{\frac{2R_t}{|h_2|^2}, \frac{R_t}{|h_1|^2} - 1\}/|h_2|^2, \frac{2R_t}{|h_2|^2} - 1\}/|h_1|^2, \frac{(|h_2|^2 - |h_1|^2)}{|h_2|^2 - |h_1|^2}\}$. Under the optimal resource allocation, the achievable rate for DF-JM is given by  

$R_{JM} = \min\{\frac{2C(\lambda P_r |h_1|^2)}{1 + \frac{2C(\lambda P_r |h_1|^2) + C_1(\lambda P_r |h_2|^2)}{C(P_r |h_2|^2)}}, \frac{\sum_{i=1}^{\lambda} C(\lambda P_r |h_i|^2)}{1 + \frac{\sum_{i=1}^{\lambda} C(\lambda P_r |h_i|^2)}{C(P_r |h_2|^2)}}, \frac{C(\lambda P_r |h_1|^2)}{1 + \frac{\sum_{i=1}^{\lambda} C(\lambda P_r |h_i|^2)}{C(P_r |h_2|^2)}}\}$.

For Case I, we obtain the following sum rate for DF-NSC:  

$R_{NSC}^{\lambda} = \min\{\frac{2C(\lambda P_r |h_1|^2)}{1 + \frac{2C(\lambda P_r |h_1|^2)}{C(P_r |h_2|^2)}}, \frac{2C(\lambda P_r |h_1|^2)}{1 + \frac{\sum_{i=1}^{\lambda} C(\lambda P_r |h_i|^2)}{C(P_r |h_2|^2)}}, \frac{\sum_{i=1}^{\lambda} C(\lambda P_r |h_i|^2)}{1 + \frac{\sum_{i=1}^{\lambda} C(\lambda P_r |h_i|^2)}{C(P_r |h_2|^2)}}, \frac{C(\lambda P_r |h_1|^2)}{1 + \frac{\sum_{i=1}^{\lambda} C(\lambda P_r |h_i|^2)}{C(P_r |h_2|^2)}}\}$.

Note that $|h_2|^2 \geq |h_1|^2$ and $P_r < P_1$, the following inequalities result: $\alpha_1 \geq \beta_1$, for $i = 1, \ldots, 4$. Let $R_{JM} = \alpha_1$ and $R_{NSC}^{\lambda} = \beta_4$, we have $R_{JM} = \alpha_1 > \beta_1 \geq \beta_4 = R_{NSC}^{\lambda}$. Therefore $R_{JM} > R_{NSC}^{\lambda}$ for any $\alpha_1$ and $\beta_4$.  

Similarly results can be derived for Case II. Details are skipped in this paper due to space limitation.

IV. NUMERICAL RESULTS

In this section, some numerical results are presented to demonstrate the performance of the optimal resource allocation for the two-way asymmetric relaying using different DF protocols. To compare the sum rate, we consider a network where both sources are separated by a normalized distance equal to one. Let $d_1 = (1 - d_1)$ be the distance between the source $S_1$ ($S_2$) and the relay node. The channel gains from the relay node to the sources are computed as $|h_1|^2 = 1/d_1^3$ and $|h_2|^2 = 1/(1 - d_1)^3$.

In Fig.2 we show the performance gain of the optimal resource allocation over the equal resource allocation in terms of the total achievable rate. For the equal resource allocation, we fix $t_1 = t_2 = 1/2$ and $\lambda_1 = \lambda_2 = 1/2$. As expected, the optimal resource allocation achieves a significant
gain over the equal resource allocation. We observe that the best performance is achieved by the optimized DF-JM strategy, while the optimized DF-NSC protocol in Case II performs similar to the optimized DF-JM strategy in high $P_r$ region. The reason for this behavior is that $t_1^*$ is very close for the DF-JM and DF-NSC schemes in high $P_r$. The difference between DF-JM and DF-NSC is dominated by the term $\log((1 + P_r |h_1|^2)/(|h_2|^2 + P_r |h_1|^2))$. It is obvious that $\log((1 + P_r |h_1|^2)/(|h_2|^2 + P_r |h_1|^2)) \to 0$ when $P_r$ is high.

In Fig. 3, we illustrate the optimal achievable rate of MAC and BCSI for the two-way asymmetric relaying. It is noteworthy that no matter which DF protocol is used, the achievable rate of MAC is equal to that of BCSI after optimizing the time and power allocation. We can also see that $R_1 > R_2 = 1$ for all the DF protocols. For comparison, we present the region of the AF scheme. It can be seen that the AF relaying has the worst performance and cannot satisfy the QoS constraint.

The achievable rates corresponding to different relay node positions are presented in Fig. 4. It is observed that no matter where the relay is, the DF-JM yields the best achievable rate, while the worst performance is achieved by the AF. We observe that, as $d_1$ increases, i.e., when the relay moves towards the source $S_2$, the achievable rate of DF-NSC with Case I decreases. This behavior can be explained by (6) and (7). If the relay node is placed at the middle position, all DF strategies yields the same achievable rate. Interestingly, when the relay is placed closer to one source, the DF-JM and DF-NSC under Case II yield the best performance.

V. CONCLUSIONS

In this paper, we have studied the optimization problem for two DF strategies over two-way asymmetric relaying channels. Our analyses were based on the maximization of the end-to-end sum rate under a QoS constraint (i.e., $R_1 \geq R_2$). We have formulated and derived the optimal resource allocation for both DF protocols. Several important observations have been drawn from the analytical results. First of all, the optimal time and power allocation does not always exist under the minimum rate constraint, regardless of the relay strategies. Secondly, the optimized DF-JM outperforms the optimized DF-NSC in terms of the total rate. Thirdly, through optimizing the time and power allocation, the achievable rate of MAC can be exactly matched up with that of BCSI, leading to the optimal end-to-end relay performance. Furthermore, it has been shown that the optimal resource allocation achieves a significant gain over the equal resource allocation in terms of the achievable rate.

ACKNOWLEDGMENT

This work was supported in part by the national science foundation grant number 0801997. This work was supported in part by National Natural Science Foundation of China (Grant No. 61072058), and the State Major Science and Technology Special Projects (Grant No. 2010ZX03003-003-01).

REFERENCES