Robust analysis and design of load frequency controller for power systems

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ABSTRACT

Robust load frequency control for power systems is discussed. A detailed robustness analysis of the existing control laws shows that parameter variation is not a critical issue but more attention should be paid to the unmodeled dynamics in robust load frequency controller design. A new robust load frequency control method is then proposed considering the unmodeled dynamics of power systems. Finally, a new configuration is proposed to overcome the effects of generation rate constraints (GRC). Simulation results show that the design method and the anti-GRC configuration are effective.

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1. Introduction

In power systems, changes in the load affect the frequency and bus voltages in the systems. For small changes in the load the frequency deviation problem can be separated or decoupled from the voltage deviation. The problem of controlling the real power output of generating units in response to changes in system frequency and tie-line power interchange within specified limits, is known as load frequency control (LFC) [1]. It is generally regarded as a part of automatic generation control (AGC) and is very important in the operation of power systems.

Conventional LFC uses an integral controller. It is well-known that a high integral gain may deteriorate the system performance causing large oscillations and instability. Thus, the integral gain must be set to a level to provide a compromise between a desirable transient recovery and low overshoot in the dynamic response of the overall system. A lot of approaches have been reported in the literature to tune the gain of the integral controller [2].

To improve the transient performance, extension of the conventional integral controller has been considered. An extended integral control was proposed in [3] to obtain zero steady-state error as well as having a controlled overshoot in system performance. Fuzzy PI controllers were suggested in [4,5] for load frequency control of power systems. Moon [6] observed that a differential feedback in LFC can indeed improve system damping, and [7] proposed a derivative structure that can achieve better noise-reduction than a conventional practical differentiator. In [8–10] the design and tuning of PID load frequency controllers were reported.

With the increase in size and complexity of modern power systems, the system oscillation might propagate into wide area resulting in a wide-area blackout. So advanced control methods were applied in LFC, e.g., optimal control [11–13]; variable structure control [14,15]; adaptive and self-tuning control [16,17]; and intelligent control [18,19]. Recently, LFC under new deregulation market [20,21], LFC with communication delay [22], and LFC with new energy systems [23,24] received much attention. See [25] for a complete review of recent philosophies in AGC.

Usually a linear model around a nominal operating point is used in the load frequency controller design. However, power system components are inherently nonlinear, the real plant usually differs from the model. Therefore robustness becomes a main issue in the attempt to design a controller to satisfy the basic requirements for zero steady state and acceptable transient frequency deviations. Many robust control methods have been applied to load frequency control problem, for example, Riccati equation approach [26], $H_{\infty}$ control [27], $\mu$-synthesis approach [28], robust pole assignment approach [29]. These papers use the same model parameters in the design thus a comparison among them is possible.

In this paper, the robust stability and robust performance of the above methods against parameter variation and unmodeled dynamics are compared. It is found that for load frequency control problem the parameter variation is not a critical issue while unmodeled dynamics may pose severe performance degradation problem. A robust load frequency control method is then proposed considering the unmodeled dynamics of power systems. Finally, a new configuration is proposed to overcome the effects of generation rate constraints (GRC). Simulation results show that the design method and the anti-GRC configuration are effective.

For simplicity, we will concentrate on the case of a single generator supplying power to a single service area. Since for the
load-frequency control problem the power system under consideration is expressed only to relatively small changes in load, it can be adequately represented by the linear model shown in Fig. 1 (obtained by linearizing the plant around the operating point).

The symbols are explained in Table 1.

2. General framework for robustness analysis

Structured singular value is a powerful tool for analyzing the robustness of a system subject to parameter variation and/or unmodeled dynamics [30]. Given a matrix $M \in \mathbb{C}^{r \times n}$ and a block structure $\Delta \subset \mathbb{C}^{m \times n}$

$$\Delta = \{ \text{diag} [\delta_1 K_1, \ldots, \delta_5 K_5, \Delta_1, \ldots, \Delta_F] : \delta_i \in \mathbb{C}, \Delta_k \in \mathbb{C}^{m_k \times n_k} \}$$

(1) where the number of repeated scalar blocks is denoted by $S$, and the number of full blocks is denoted by $F$. $C_i$ $(i = 1, \ldots, S)$ denotes the dimension of the $i$th repeated scalar block, and $m_k$ $(k = 1, \ldots, F)$ denotes the dimension of the $k$th full block. The (complex) structured singular value (SSV) of $M$ with respect to the block structure $\Delta$ is defined as

$$\mu_{\Delta}(M) = \min_{\Delta \in \Delta} \max_{\delta \in \mathbb{R}} |\delta| \cdot \det(I - \Delta M) = 0$$

(2) unless no $\Delta \in \Delta$ makes $I - \Delta M$ singular, in which case $\mu_{\Delta}(M) = 0$.

The basis for robust stability analysis is the small $\mu$ theorem [31].

Theorem 1. Consider the interconnected system in Fig. 2(a). Let $\beta > 0$. The system is well-pose and internally stable for all $\Delta$ such that $||\Delta||_{\infty} < (1/\beta)$ if and only if

$$\mu_{\Delta}(M) \leq \beta$$

(3)

Robust stability is not the only issue in robust controller design. Instead, robust performance is the ultimate goal. Consider the robust performance problem in a general framework shown in Fig. 2(b). Let

$$M_\Delta := F_{\Delta}(P, K), T_{zw} := F_{\Delta}(M, \Delta_s)$$

(4)

where $F_{\Delta}(\cdot, \cdot)$ ($F_{\Delta}(\cdot, \cdot)$) is the lower (upper) linear fractional transformation [31], and $T_{zw}$ represents the transfer function from $w$ to $z$. The robust performance problem is to find a stabilizing controller $K$ such that $T_{zw} \|_{\infty} \leq \beta$ for all model uncertainty $\Delta_s$ with $||\Delta_s||_{\infty} < (1/\beta)$.

The following result shows that the robust performance problem can be transformed to an augmented robust stability problem [31].

Table 1 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>$\Delta P$</td>
<td>load disturbance (p.u.MW)</td>
<td></td>
</tr>
<tr>
<td>$K_p$</td>
<td>electric system gain</td>
<td></td>
</tr>
<tr>
<td>$T_p$</td>
<td>electric system time constant (s)</td>
<td></td>
</tr>
<tr>
<td>$T_r$</td>
<td>turbine time constant (s)</td>
<td></td>
</tr>
<tr>
<td>$T_c$</td>
<td>governor time constant (s)</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>speed regulation due to governor action (Hz/p.u.MW)</td>
<td></td>
</tr>
<tr>
<td>$\Delta f(t)$</td>
<td>incremental frequency deviation (Hz)</td>
<td></td>
</tr>
<tr>
<td>$\Delta P_d(t)$</td>
<td>incremental change in generator output (p.u.MW)</td>
<td></td>
</tr>
<tr>
<td>$\Delta X_g(t)$</td>
<td>incremental change in governor valve position</td>
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In such a case, the plant can be represented by:

$$\dot{x}(t) = Ax(t) + Bu(t) + F\Delta P_d$$

(8)

where

$$A = \begin{bmatrix} -1/T_p & K_p/T_p & 0 & 0 \\ 0 & -1/T_r & 1/T_r & -1/T_C \\ 1/T_C & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 1/T_C & 0 \end{bmatrix}^T, F = [-K_p/T_p \ 0 \ 0 \ 0]^T$$

(9)

Typical values of the system parameters are

$$K_p = 120, \quad T_p = 20, \quad T_r = 0.3, \quad T_C = 0.08, \quad R = 2.4$$

(10)

and the state feedback gains designed in [26–29] for the typical parameters (10) are given by:

$$K_1 = [-1.893 \ 4.762 \ 1.516 \ 1.658]$$

$$K_2 = [5.03 \ 8.725 \ 2.1324 \ 1.6252]$$

$$K_3 = [-2.7321 \ 4.0167 \ 0.8506 \ 0.4318]$$

$$K_4 = [-13.738 \ 16.077 \ 2.837 \ 19.118]$$

(11)

where $K_3$ has been transformed from the transfer function form to the state feedback form.
3.1. Parameter variation

Robustness are tested for the following parameter variation \([26–29]\):

\[
\begin{align*}
\frac{1}{T_p} &\in [2.564, 4.762], \\
\frac{1}{T_c} &\in [9.615, 17.857], \\
\frac{1}{T_p} &\in [0.033, 0.1], \\
\frac{K_p}{T_p} &\in [4, 12], \\
\frac{1}{R T_c} &\in [3.081, 10.639]
\end{align*}
\]

(12)

To show the robust stability of the state feedback system under the parameter variation, we define the uncertain parameters as

\[
\delta_1 := \frac{1}{T_p}, \quad \delta_2 := \frac{K_p}{T_p}, \quad \delta_3 := \frac{1}{T_T}, \quad \delta_4 := \frac{1}{R T_c}, \quad \delta_5 := \frac{1}{T_G}
\]

(13)

In fact, we can directly use \(T_T, T_c, \text{etc}.\) as uncertain parameters; however, it is more convenient to use the parameters defined above since they appear explicitly in the state-space model (9). The model with uncertain parameters is shown in Fig. 3.

To analyze the robust stability of the uncertain system, we need to pull out the uncertain parameters and put it in the \(M\)-\(A\) structure as shown in Fig. 2(a). Define the uncertainty structure ‘\(\Delta\)’ as

\[
\Delta = \begin{bmatrix} \Delta_s \\ \Delta_p \end{bmatrix}
\]

(19)

then the robust performance problem for the power system under parameter variation can be put in the structure shown in Fig. 2(a), with ‘\(M\)’ given by

\[
M_p := \begin{bmatrix} A + BK & E_0 & F \\ w_s (F_0 + F_2 K) & 0 & w_s F_1 \\ \gamma w_p C & 0 & 0 \end{bmatrix}
\]

(20)

Here \(\gamma\) is the nominal performance, \(C = [1 \ 0 \ 0 \ 0]\), and \(w_p\) is a factor to make sure that \(\mu_A(M_p) < 1\). It reflects the percentage of the nominal performance that a state-feedback control law \(u = Kx\) can retain, thus reflects the robust performance.

3.2. Unmodeled dynamics

It is obvious that the plant for load frequency control consists of three parts:

- Governor with dynamics: \(G_p(s) = 1/(T_c s + 1)\).
- Turbine generator with dynamics: \(G_t(s) = 1/(T_T s + 1)\).
- Power systems with dynamics: \(G_p(s) = K_p/(T_p s + 1)\).

Consider the uncertain system shown in Fig. 4, where all the three parts of the system are subject to multiplicative perturbations, i.e.,

\[
\tilde{C}_p = G_p (1 + \Delta_1), \quad \tilde{C}_t = G_t (1 + \Delta_2), \quad \tilde{C}_g = G_g (1 + \Delta_3)
\]

(21)

For the uncertain system in Fig. 4, if we define the uncertain structure as

\[
\Delta_u = \text{diag} \{\Delta_1, \Delta_2, \Delta_3\}
\]

(22)

then the transfer function ‘\(M\)’ in Fig. 2(a) after pulling out the \(\Delta_u\) is

\[
M_u = \tilde{N} \begin{bmatrix} G_p G_t G_k k_1 + G_p G_t G_k k_2 + G_p G_t G_k k_3 + G_p G_t \\
G_p G_t k_1 + G_p G_t k_2 + G_p G_t k_3 + G_t \\
G_p G_t k_1 \\ G_p G_t k_2 \\ G_p G_t k_3 \end{bmatrix}
\]

(23)
where the state feedback gain is $K = [k_1 \ k_2 \ k_3 \ k_4]$ and

\begin{align*}
\tilde{k}_1 &:= k_1 + \frac{(k_4 - 1)k_E}{s} - \frac{1}{R} \\
\tilde{k}_2 &:= k_1 G_p + k_2 \\
\tilde{k}_3 &:= k_1 G_p G_t + k_2 G_t + k_3 \\
\tilde{N} &:= \frac{1}{1 - k_1 G_p G_t G_g - k_2 G_t G_g - k_3 G_g}
\end{align*}

If only $G_p$ contains unmodeled dynamics, while the other two models do not contain unmodeled dynamics ($\Delta_2 = 0$, $\Delta_3 = 0$), then we just need to look at the singular values of the (1,1) block of $M_u$ in (23), which is

$$M_{11} = \frac{G_p G_t G_g \tilde{k}_1}{1 - k_1 G_p G_t G_g - k_2 G_t G_g - k_3 G_g}. \quad (24)$$

4. Discussion

For the parameter variations in (12), the weighting matrix $w_s$ is given by

$$w_s = \text{diag}(0.5038, 0.5, 0.3, 0.5509, 0.3) \quad (25)$$

The robust stability charts ($\mu_{\Delta s}(M_u)$) are shown in Fig. 5 for the controllers $K_i$ in (11). Each peak value is less than 0.6, meaning that the closed-loop systems remain stable for each parameter given in (12), and in fact at least $1/0.6 \times 100\%$ larger than the specified range, so all the methods are very robust against parameter variation.

The nominal performance $\gamma_i$ for each controller $K_i$ is

$$\gamma_1 = 3.2452, \quad \gamma_2 = 2.1682, \quad \gamma_3 = 1.8193, \quad \gamma_4 = 1.4719$$

respectively, and the weight $w_{pl}$ for each controller such that $\mu_{\Delta s}(M_{pl}) < 1$ is

$$w_{p1} = 0.71, \quad w_{p2} = 0.81, \quad w_{p3} = 0.53, \quad w_{p4} = 0.76 \quad (27)$$

respectively, which means that $K_1$ can retain at least 71% of its nominal performance, $K_2$ can retain at least 81% of its nominal performance, $K_3$ can retain at least 53% of its nominal performance, and $K_4$ can retain at least 76% of its nominal performance. So the best nominal performance is achieved by $K_4$, but the best robust performance for the parameters specified in (12) is achieved by $K_2$.

It seems that uncertainty is not a critical issue in load frequency control. But it is not true. For example, if the real dynamics in the power systems is

$$\tilde{G}_p(s) = G_p(s) \frac{1}{s + 1} \quad (28)$$

then the closed-loop system by $K_4$ is not stable, while other three are stable, but oscillatory (as shown in Fig. 6). The best performance in this case is achieved by $K_2$. So the robust control methods discussed above are not quite suitable for the unmodeled dynamics in the power system model.

The robust stability charts ($\mu_{\Delta s}(M_u)$) for the state-space controllers given in (11) are shown in Fig. 7(a), together with the inverse
of the magnitude of the uncertainty
\[
\Delta_1 = \frac{S}{s + 1}, \quad i = 1, \ldots, 3
\] (29)
i.e., each part is perturbed to
\[
\tilde{G}_g = G_g \frac{1}{s + 1}, \quad \tilde{G}_t = G_t \frac{1}{s + 1}, \quad \tilde{G}_p = G_p \frac{1}{s + 1}
\] (30)
The condition \(\mu_\Delta(M_u) < \frac{1}{\|\Delta_u\|_\infty}\) is violated, so none of the controllers can guarantee robust stability.
If only \(G_p\) contains unmodeled dynamics, the robust stability charts \(\|M_{11}\|_\infty\) for the state-space controllers given in (11) are shown in Fig. 7(b). \(K_4\) violates the robust stability condition thus cannot guarantee stability; while \(K_2\) is the farthest from the uncertainty thus has the best robustness. These results go well with the simulation in Fig. 6.

5. Robust load frequency controller design
From the previous analysis, robust load frequency control should pay more attention to unmodeled dynamics instead of parameter variation. A compromise between optimal load rejection and stability robustness against unmodeled dynamics should be con-
Fig. 9. Anti-GRC configuration

is considered in LFC design. For simplicity, we consider the case that only $G_P$ contains unmodeled dynamics and a dynamic output feedback $u(s) = K(s) \Delta f(s)$ is used to control the system shown in Fig. 1. By similar arguments as in the previous section for state-feedback case, robustness against multiplicative uncertainty in $G_P$ requires that

$$P_2 := \frac{G_P G_T G_G (1 - 1/R)}{1 - G_P G_T G_G (1 - 1/R)}$$

(31)

is small. So a compromise is achieved with the following $H_\infty$ optimization problem:

$$\left\| \begin{bmatrix} W_P P_1 \\ W_S P_2 \end{bmatrix} \right\|_\infty < 1$$

(33)

where $W_P$ is a weight reflecting the load rejection ability, and $W_S$ is a weight reflecting the multiplicative uncertainty in $G_P$.

The weights $W_P$ and $W_S$ can be chosen according to the methods explained in [31]. For the power system considered above, since the unmodeled dynamics in $G_P$ is $(s/s + 1)$, so we choose $W_S = (s + 9.9/10)$, the magnitude of which is right below the inverse of magnitude of the uncertainty. $W_P$ is chosen to be a PI so that the final controller contains integrator. The parameters are chosen by trial and error, and finally we choose $W_P = (0.2s + 0.1/s)$ to

Fig. 10. Responses of system with GRC (solid: proposed; dashed: $K_2$; dashdotted: $K_4$). (a) $\Delta f(t)$, (b) $\Delta P_d(t)$, (c) $\Delta X_G(t)$. 
have an acceptable performance. A load frequency controller is then obtained by solving (33):

\[ K = - \frac{1.902 \times 10^5 s^3 + 3.049 \times 10^7 s^2 + 6.638 \times 10^7 s + 2.009 \times 10^7}{s^4 + 595.8s^3 + 2.674 \times 10^5 s^2 + 4.924 \times 10^7 s} \]  

(34)

The responses of the closed-loop systems for the proposed controller, \( K_2 \) and \( K_4 \) for the nominal case are shown in Fig. 8(a). The case that the uncertainties occur in parameters (12) are shown in Fig. 8(b), and the case that the uncertainties occur in parameters (12) and unmodeled dynamics (28) simultaneously are shown in Fig. 8(c) and (d). It is clear that the proposed controller has good disturbance rejection performance and good robust stability against parameter variation and unmodeled dynamics.

Due to physical limitations of governor and turbine, there are constraints for the generation rate, which are demonstrated to cause instability [14]. Note that the proposed method uses the output feedback, and it is of high order, thus the methods proposed in the existing literature [3,26] are not applicable here. To deal with this problem, we propose a new anti-GRC structure. The structure is shown in Fig. 9, where \( K(s) \) is the designed controller and it is decomposed as

\[ K(s) = \frac{k_i}{s} + K_m(s) \]  

(35)

where \( k_i \) is the integral gain and \( K_m(s) \) is the part that does not contain any integral action.

Fig. 9 shows that when the signal that passes through a dynamics that is identical to the turbine is not the same as \( \Delta P_C(t) \), then it will be compensated with a gain \( k_i \) till they are identical. So the additional attention is on reset configuration for PI controllers, except (1) the controller is not limited to PI; (2) a turbine model is needed since the constraint cannot be measured directly.

It is shown that the larger the anti-GRC gain \( k_i \), the faster the accumulated error will tend to zero. However, too large a value might slow down the process of \( \Delta P_C(t) \) to zero. A trial and error method is needed to find a good choice of \( k_i \).

The time responses of the closed-loop systems for the proposed controller \( K \), \( K_2 \) and \( K_4 \) with GRC of 0.01 r.u./min, or 0.0017 r.u./s are shown in Fig. 10(a). Corresponding responses of governor valve and turbine generated power are shown in Fig. 10(b) and (c). Here we choose \( k_c = 5 \). The responses of the proposed method return to their nominal values quickly and smoothly.

6. Conclusion

Robustness is critical in load frequency controller design for power systems since the operating conditions are continuously changing. By performing a detailed robustness analysis of the existing control laws it was concluded that parameter variation is not a critical issue in robust load frequency design. More attention should be paid to the unmodeled dynamics. A new robust load frequency control method and a new anti-GRC configuration were proposed to overcome the effects of unmodeled dynamics and generation rate constraints. The robust load frequency control for multi-area power systems is under investigation.

Acknowledgment

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Appendix A. Notation and symbols

**C** \( \times n \) complex matrix space with dimension \( n \times n 

\( \mu_\Delta(M) \) structured singular value of a matrix \( M \) with respect to a block structure \( \Delta \nolinebreak[4]

\( \mathcal{F}_2(P, K) \) lower linear fractional transformation, defined by \( \mathcal{F}_2(P, K) = P_{11} + P_{12}K(1 - P_{22}K)^{-1}P_{21} \) assuming that \( P \) is decomposed as \( P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \) and \( P_{22} \) has compatible dimensions with \( K \nolinebreak[4]

\( A \begin{bmatrix} B \\ C \end{bmatrix} D \) shorthand for state-space realization \( C(sl - A)^{-1}B + D \nolinebreak[4]

\( RH_\infty \) space of all real rational stable transfer functions

\( \|M\|_\infty = \sup_{j \in \mathbb{R}} \|M(j\omega)\| \)

\( \text{diag}(\delta_1, \delta_2) \) diagonal matrix with \( \delta_1 \) and \( \delta_2 \) as diagonal elements

References


