Tolerance analysis and synthesis of cam-modulated linkages

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ABSTRACT

Cam-modulated linkages that can achieve superior kinematic performance as compared with pure linkages have been widely applied in a variety of automation machinery. The tolerance analysis and synthesis are thus quite essential in the design and manufacture of precision cam-modulated linkages. This paper presents comprehensive and systematic mathematical tools for dealing with the tolerance analysis and synthesis of cam-modulated linkages. Based on the concept of equivalent linkage and the derived correlation between the radial-dimension errors and the normal-direction errors of the cam profile, the sensitivity analysis method for equivalent linkages is introduced for analyzing the tolerances in cam-modulated linkages, whose output error equations can be derived through an analytical means. Then, by incorporating the sensitivity analysis method and the concept of design for manufacturing and assembly (DFMA), an optimization model for synthesizing the tolerances in cam-modulated linkages is developed. The objective of this optimization model is to maximize the manufacturability and assembly of a cam-modulated linkage while maintaining acceptable kinematic accuracy of its output motion. A practical case study of analyzing and synthesizing the tolerances in a cam-modulated linkage type pick-and-place device is then performed to illustrate the presented methods. It shows that the mathematical tools presented can be helpful to the design and manufacture of precision cam-modulated linkages.

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1. Introduction

The cam-modulated linkage, also known as the cam-integrated linkage or the combined cam-linkage mechanism, is a composite mechanism consisting of at least one cam-follower pair in combination with a linkage [1–6]. Such cam-linkage composite mechanisms have been widely applied in a variety of automation machinery. Their capabilities of generating prescribed functions, paths and motions are superior to those of pure linkages according to the viewpoint of kinematic design. Nevertheless, cams are irregular-shaped mechanical components, their profiles cannot be accurately machined with relative ease as compared with dimensions of linkages. In high-speed machinery, even slight deviations in cam contour may still produce excessive noise, wear, and vibrations [7–9]. The required kinematic accuracy of the output motion, as well as the dynamic performance, of the cam-modulated linkage may not be retained as expected. To achieve reasonably acceptable kinematic accuracy and dynamic performance, the manufacturing and assembly errors in a cam-modulated linkage cannot exceed their specified tolerances. Therefore, the tolerance analysis and synthesis [10–12], also known as the analysis and synthesis of mechanical errors [13–16], are quite essential in the design and manufacture of precision cam-modulated

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The tolerance analysis aims at evaluating the output motion errors of a mechanism caused by known deviations (or specified tolerances) in the design parameters of the mechanism. The tolerance synthesis, on the contrary, aims at deciding the optimal combination of tolerances in the design parameters of a mechanism according to the output motion errors that are limited to acceptable extents and also the controlled production cost of the mechanism. From the viewpoint of design for manufacturing and assembly (DFMA) [17,18], the tolerance of the cam profile and that of each design parameter should be specified at the largest (or optimal) values to make the components in a cam-modulated linkage be fabricated easily. At the same time, the optimal combination of tolerances should also meet the operating or functional requirements of the output motion. Therefore, mathematical tools for dealing with both the tolerance analysis and synthesis must be established in order to aid the production of precision cam-modulated linkages.

A variety of analytical or numerical approaches have been presented for the tolerance analysis of various types of cam mechanisms [12–14,19–25] or for that of linkages [15,16,26–29]. Some of these approaches are further extended for the tolerance synthesis tasks with optimization models being established [12–14,23–29]. However, up to the present, less literature addresses systematic approaches for dealing with the tolerance analysis and synthesis of cam-modulated linkages. In addition, some representative optimization models are suggested by considering the minimum production cost [10–14,24–29] with some defined cost–tolerance models based on the reciprocal, reciprocal power or exponential functions. Most of these cost–tolerance models can give a reasonably qualitative estimation, but not an exact quantitative prediction. If the used cost–tolerance models cannot reasonably reflect the actual situations in the manufacturing and assembly of mechanisms, the sequential optimization results may be inapplicable. Therefore, establishing a more appropriate optimization model for the tolerance synthesis of cam-modulated linkages is still a matter of concern until now. The present paper, employing the concepts of equivalent linkage [15,19,20] and DFMA, demonstrates comprehensive and systematic mathematical tools for analyzing and synthesizing the tolerances in cam–modulated linkages. A practical case study of analyzing and synthesizing the tolerances in a cam-modulated linkage type pick-and-place device is then performed to illustrate the presented approaches.

2. Fundamentals of the tolerance analysis

The kinematic analysis of a cam-modulated linkage can be simplified by replacing the mechanism by an equivalent linkage with lower pairs [15,30,31]. Such a replacement can provide a convenient means for analyzing the tolerance in the cam-modulated linkage. Fig. 1(a) shows a three-link direct–contact mechanism, which may be part of a cam-modulated linkage. In the figure, points O2 and O3 are the fixed pivots of the cam (link 2) and the follower (link 3) related to the frame (link 1), respectively, and points K2 and K3 are the centers of curvature of the cam and the follower in contact at point A, respectively. The common normal at the contact point A must always pass through points K2, K3, and the instant velocity center I3. The equivalent linkage of this direct-contact mechanism is the four-bar linkage O2K2K3O3 shown in Fig. 1(b), in which the coupler (link 4) of the linkage connects the centers of curvature of the cam, K2, and of the follower, K3. The instantaneous kinematic characteristics of links O2K2 and O2K3 are identical to those of the cam and the follower, respectively.

The actual profile of a machined cam may slightly deviate from the theoretical contour, and a deviation in the follower motion will thus occur. Since the instantaneous kinematic characteristics of a cam-modulated linkage are identical to those of its equivalent linkage with lower pairs, the tolerance analysis of a cam-modulated linkage can be performed through the aid of its equivalent linkage with lower pairs. In other words, if the profile error in the normal direction of a machined cam, \( \Delta n \), equals the coupler-length error of the equivalent linkage, \( \Delta n = \Delta n \), their output links will have the same motion deviations. Similarly, the dimensional errors of other theoretical design parameters of a cam-modulated linkage arising from manufacturing and assembly errors can also be transformed into their corresponding link-length errors of the equivalent linkage with lower pairs. For a cam-modulated linkage including more than one cam-follower pair, all cam-follower pairs in the cam-modulated linkage can thus be simultaneously transformed into their equivalent four–bar chains for performing the tolerance analysis. As a result, the procedure developed by Hartenberg and Denavit [15], based on the concept of sensitivity analysis [10,11,32], can be applied to calculate the output motion deviation of the equivalent linkage with lower pairs. A general form of the sensitivity analysis method for equivalent linkages is derived below.

2.1. Sensitivity analysis method for equivalent linkages

For an N-degree-of-freedom closed-loop linkage with m specified functional output variables, its implicit constraint or displacement equations, relating the constant link-length parameters \( r_1, r_2, \ldots, r_n \) to the input variables \( \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_N \) and the functional output variables \( \psi_1, \psi_2, \ldots, \psi_m \) may be written as

\[
F(r, \theta, \psi) = \begin{bmatrix} F_1(r, \theta, \psi) \\ F_2(r, \theta, \psi) \\ \vdots \\ F_m(r, \theta, \psi) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\]

(1)
Fig. 1. Three-link direct-contact mechanism and its equivalent four-bar linkage.

where $F$ is an $m$-dimensional implicit function of vectors $\mathbf{r}$, $\theta$, and $\psi$; in which, $\mathbf{r} = [r_1 \ r_2 \ \cdots \ r_n]^T$, $\theta = [\hat{\theta}_1 \ \hat{\theta}_2 \ \cdots \ \hat{\theta}_N]^T$, and $\psi = [\psi_1 \ \psi_2 \ \cdots \ \psi_m]^T$. Through kinematic analysis, the $m$-by-1 unknown vector $\psi$ would be solved analytically or numerically if vectors $\mathbf{r}$ and $\theta$ are given. If the link-length parameters and the variables have slight deviations $1\mathbf{r}$, $1\theta$, and $1\psi$, the Taylor expansion of Eq. (1) can be written as

$$F(\mathbf{r} + 1\mathbf{r}, \theta + 1\theta, \psi + 1\psi) \approx F(\mathbf{r}, \theta, \psi) + \frac{\partial F}{\partial \mathbf{r}} 1\mathbf{r} + \frac{\partial F}{\partial \theta} 1\theta + \frac{\partial F}{\partial \psi} 1\psi = 0.$$ (2)

The differential of the function $F$ may be written in terms of the Jacobian matrices of its partial derivatives as

$$dF \approx \Delta F$$

$$= F(\mathbf{r} + \Delta \mathbf{r}, \theta + \Delta \theta, \psi + \Delta \psi) - F(\mathbf{r}, \theta, \psi) = \frac{\partial F}{\partial \mathbf{r}} \Delta \mathbf{r} + \frac{\partial F}{\partial \theta} \Delta \theta + \frac{\partial F}{\partial \psi} \Delta \psi = 0.$$ (3)

For a given vector of the input variables $\theta$ and letting $\Delta \theta = 0$, the overall deviations of the functional output variables will be

$$\Delta \psi = - \left( \frac{\partial F}{\partial \psi} \right)^{-1} \left( \frac{\partial F}{\partial \mathbf{r}} \right) \Delta \mathbf{r} = J \Delta \mathbf{r}$$ (4)

where $J$ is called the sensitivity Jacobian matrix [32]. (Note that, in general, $\hat{\theta}_1$, $\hat{\theta}_2$, . . . , $\hat{\theta}_N$ are inputs that are not subject to deviations or tolerances unless they are derived from the outputs of previous mechanisms or other devices [16]. Thus, the assumption of $\Delta \theta = 0$ could be given when considering that $\theta$ is an independent input vector.) Furthermore, the deviations of the functional output variables caused by each link-length error can be expressed as

$$\Delta \psi_i = \begin{bmatrix} \Delta \psi_{i1} \\ \Delta \psi_{i2} \\ \vdots \\ \Delta \psi_{im} \end{bmatrix} = - \left( \frac{\partial F}{\partial \psi} \right)^{-1} \left( \frac{\partial F}{\partial \mathbf{r}} \right)_i \Delta \mathbf{r}_i = J_i \Delta \mathbf{r}_i \quad \text{for } i = 1, 2, \ldots, n$$ (5)

where

$$\left( \frac{\partial F}{\partial \mathbf{r}} \right)_i = \begin{bmatrix} \frac{\partial F_1}{\partial r_i} \\ \frac{\partial F_2}{\partial r_i} \\ \vdots \\ \frac{\partial F_m}{\partial r_i} \end{bmatrix} \quad \text{for } i = 1, 2, \ldots, n$$ (6)
and \( \mathbf{j} \) is the sensitivity vector relating the functional output variables and the \( i \)-th link-length parameter. In Eq. (5), the scalar \( \Delta \psi_{i(0)} \) represents the deviation of the \( j \)-th functional output variable caused by the dimensional error of the \( i \)-th link-length parameter. From the stochastic viewpoint, the worst-case deviations of the functional outputs will be [11,15,16]

\[
\Delta \psi_{wor} = \left\{ \begin{array}{l}
\Delta \psi_{1,wor} \\
\Delta \psi_{2,wor} \\
\vdots \\
\Delta \psi_{m,wor}
\end{array} \right\} = \left\{ \begin{array}{l}
\sum_{i=1}^{n} |\Delta \psi_{1(i)}| \\
\sum_{i=1}^{n} |\Delta \psi_{2(i)}| \\
\vdots \\
\sum_{i=1}^{n} |\Delta \psi_{m(i)}|
\end{array} \right\}
\]

(7)

in which the scalars represent the worst combinations of the output deviation caused by each link-length error. The maximum expected deviations of the functional outputs will be [11,15,16,33]

\[
\Delta \psi_{rss} = \left\{ \begin{array}{l}
\Delta \psi_{1,ss} \\
\Delta \psi_{2,ss} \\
\vdots \\
\Delta \psi_{m,ss}
\end{array} \right\} = \left\{ \begin{array}{l}
\sqrt{\sum_{i=1}^{n} (\Delta \psi_{1(i)})^2} \\
\sqrt{\sum_{i=1}^{n} (\Delta \psi_{2(i)})^2} \\
\vdots \\
\sqrt{\sum_{i=1}^{n} (\Delta \psi_{m(i)})^2}
\end{array} \right\}
\]

(8)

in which the scalars are obtained by the widely used root sum of squares (RSS) approach [11].

In practice, the accuracy of a machined cam can be controlled through examining whether the radial dimension of the actual cam profile meets its specified tolerance. That is, the radial dimension of the actual cam profile with respect to each cam angle must lie within a specified zone along the theoretical profile. To evaluate how the radial-dimension errors of the actual cam profile affect the output error of the mechanism, the analytical approach proposed by Wu and Chang [19,20] for correlating the radial-dimension error and the normal-direction error of the cam profile can be applied.

**2.2. Correlation between radial profile error and normal-direction error**

In practice, the accuracy of a machined cam can be controlled through examining whether the radial dimension of the actual cam profile meets its specified tolerance. That is, the radial dimension of the actual cam profile with respect to each cam angle must lie within a specified zone along the theoretical profile. To evaluate how the radial-dimension errors of the actual cam profile affect the output error of the mechanism, the analytical approach proposed by Wu and Chang [19,20] for correlating the radial-dimension error and the normal-direction error of the cam profile can be applied.

**Fig. 2** shows a cam in contact with its follower, in which the theoretical cam profile is shown in solid line and the actual cam profile in dashed line. The theoretical contact point is designated by \( A \), and its normal to the cam profile intersects the actual cam profile at point \( A_2 \); line \( OA_2 \) intersects the actual cam profile at point \( A_1 \). For a sufficiently small value of normal-direction profile error \( AA_0 \), since \( A_1A_0 \) will be tangent to the actual cam profile and \( A_1A_0 \) will be normal to the profile, triangle \( AA_0A_1 \) can be approximated as a right-angled triangle, and thus

\[
\Delta n \approx \Delta r \cos \lambda
\]

(9)

where \( \Delta n = AA_0, \Delta r = AA_1 \), and \( \lambda = \angle AA_0A_1 \). (The quantity \( \Delta n \) and \( \Delta r \) are negative if the actual cam profile is smaller than the theoretical one; in the figure shown they are negative.) The subtending angle between lines \( AA_0 \) and \( AA_1 \), \( \lambda \), is called the shift angle [19,20]. In addition, the common normal at the contact point \( A \) must always pass through point \( Q \), which is also the instant center \( I_{23} \), and so

\[
\lambda = \angle AA_0 = \angle O_2AQ = \sin^{-1}\left( \frac{QA \times O_2A}{||QA|| \cdot ||O_2A||} \right)
\]

(10)
where $Q_A = QA(\theta)$ and $O_2A = O_2A(\theta)$ are derived parametric vector equations with respect to the input variables $\theta$ and related to a fixed Cartesian coordinate system $O_2-XY$ fixed on the cam. Equation (9) shows how to transform the radial profile error of a cam into its corresponding normal-direction error, i.e. the coupler-length error of the equivalent linkage, by applying the shift angle. If the parametric vector equations and the radial dimensions of a cam profile are obtained, the output deviations arising from the cam profile error (or tolerance) can be evaluated by combining the analytical transformation of cam profile errors with the sensitivity analysis method introduced in Section 2.1.

3. Fundamentals of the tolerance synthesis

One of the practical considerations in designing a cam-modulated linkage is the optimal tolerance synthesis of each design parameter to achieve the desired kinematic accuracy of the output motion. The optimal objective can either be the minimum production cost or the maximum manufacturability and assemblyability. The latter is based on the consideration of DFMA [17,18] and aims at achieving the lowest level of difficulty of manufacturing and assembling cam-modulated linkages at the design phase. By incorporating the equivalent linkage method presented in Section 2 and the concept of DFMA, the procedure for optimal tolerance synthesis with maximization of the manufacturability and assemblyability of cam-modulated linkages as the objective is developed in this section.

3.1. The manufacturability and assemblyability indices

Every manufacturing and assembly error may affect the kinematic accuracy of the output motion, and the specified tolerance of each design parameter has its own respective cost. Lower costs imply a lower level of difficulty in the manufacture and assembly of mechanisms. In order to optimally synthesize the tolerances of the design parameters of a cam-modulated linkage, the cost of each manufacturing and assembly tolerance must be evaluated. Therefore, the deviations of the design parameters of a cam-modulated linkage are classified as the errors arising from manufacturing and the errors arising from assembly. Their respective costs are indirectly evaluated by means of the manufacturability and assemblyability of cam-modulated linkages, and the quantitative correlation model between manufacturability versus tolerance and assemblyability versus tolerance must be established first.

Considering that there are $n$ theoretical design parameters $(r_1, r_2, \ldots, r_n)$ in a cam-modulated linkage, for all the $n$ dimensional tolerances $(\Delta r_1, \Delta r_2, \ldots, \Delta r_n)$, they can be divided into two groups: $h$ manufacturing tolerances $(\Delta r_{M1}, \Delta r_{M2}, \ldots, \Delta r_{Mh})$ and $k$ assembly tolerances $(\Delta r_{A1}, \Delta r_{A2}, \ldots, \Delta r_{Ak})$ where $n = h + k$. Here, the manufacturing tolerances $(\Delta r_{M1}, \Delta r_{M2}, \ldots, \Delta r_{Mh})$ refer to the allowable deviations of the design parameters assigned for manufacturing inaccuracy, such as the radial-dimension tolerance of the cam profile. On the other hand, the assembly tolerances $(\Delta r_{A1}, \Delta r_{A2}, \ldots, \Delta r_{Ak})$ refer to the allowable deviations of the design parameters assigned for assembly misalignment, such as the tolerance amount for the center distance between cam and follower pivots. In other words, a manufacturing tolerance is assigned to the dimension of a design parameter that can quantitate the geometric characteristic of one rigid member, while an assembly tolerance is assigned to the dimension of a design parameter that can quantitate the geometric correlation between assembled rigid members. (It must be emphasized that fits for assembled members are not considered in the optimal procedure because they have already been standardized. In fact, the effects of fits for assembled members on influencing assembly tolerances can also be included if necessary.) Accordingly, a manufacturability index, MI, quantitatively
modeling the level of difficulty of manufacturing members in a cam-modulated linkage is defined as
\[
MI = \sum_{i=1}^{h} w_{Mi} |\Delta r_{Mi}| \quad \text{for} \quad \sum_{i=1}^{h} w_{Mi} = 1 \quad \text{and} \quad 0 < w_{Mi} < 1.
\]  
(11)

Additionally, an assembly index, AI, quantitatively modeling the level of difficulty of assembling members in a cam-modulated linkage is defined as
\[
AI = \sum_{i=1}^{k} w_{Ai} |\Delta r_{Ai}| \quad \text{for} \quad \sum_{i=1}^{k} w_{Ai} = 1 \quad \text{and} \quad 0 < w_{Ai} < 1.
\]  
(12)

The coefficients \((w_{M1}, w_{M2}, \ldots, w_{Mh})\) and \((w_{A1}, w_{A2}, \ldots, w_{Ak})\) are the specified weighting factors that reveal the extents of manufacturability and assembly influenced by their respective tolerances. The indices MI and AI that can be respectively expressed in an exchanged consistent unit are employed here to quantitatively indicate the subtotal extent of the manufacturability and assembly when a cam-modulated linkage is fabricated; the larger the specified tolerances, the more obtainable the manufacturability and assembly. Similarly, the magnitude of each weighting factor \((w_{M1}, w_{M2}, \ldots, w_{Mh})\) and \((w_{A1}, w_{A2}, \ldots, w_{Ak})\) quantitatively indicate that the larger the weighting factor specified to the tolerance amount of a design parameter, the more obtainable the manufacturability or assembly of this dimension. In other words, the assignment of weighting factors \((w_{M1}, w_{M2}, \ldots, w_{Mh})\) and \((w_{A1}, w_{A2}, \ldots, w_{Ak})\) represents a recognized fact that not each tolerance amount can provide equal contribution to the overall manufacturability or assembly. For instance, considering that a disk cam mechanism with an oscillating roller follower is fabricated, the machining of the disk cam usually requires more complicated manufacturing processes (resulting in much production cost and time) than those required by the machining of the oscillating arm. The tolerance grade of the cam profile is thus preferred larger than that of the arm-length of the oscillating arm. That is, the enlargement of the tolerances of design parameters in a mechanism that involve more complicated manufacturing or assembly processes can efficiently and effectively contribute to better manufacturability or assembly. Hence, the weighting factors \((w_{M1}, w_{M2}, \ldots, w_{Mh})\) and \((w_{A1}, w_{A2}, \ldots, w_{Ak})\) actually involve the information of manufacturing and assembly processes and their required effort and time. Thus, a larger weighting factor should be specified to the tolerance amount of a design parameter that needs more complicated and expensive processes to achieve. At the design phase, the values of weighting factors \((w_{M1}, w_{M2}, \ldots, w_{Mh})\) and \((w_{A1}, w_{A2}, \ldots, w_{Ak})\) can be evaluated and decided through comprehensive process analysis and cost evaluation, which can be undertaken via the use of some computer aided design/computer aided manufacture (CAD/CAM) software incorporating with the designer’s judgement and experience based on the practical considerations of required manufacturing machines, tools, materials, workmanship, working time, and so on.

To simultaneously maximize the manufacturability \((MI)\) and the assembly \((AI)\), a comprehensive index, CI, for quantitatively modeling the level of difficulty of manufacturing and assembling members in a cam-modulated linkage, as the objective function of the optimization can be defined as
\[
CI = CI(\Delta r_1, \Delta r_2, \ldots, \Delta r_n)
\]
\[
≡ w_{C(M)}MI + w_{C(A)}AI = \sum_{i=1}^{h} (w_{C(M)} w_{Mi}) |\Delta r_{Mi}| + \sum_{i=1}^{k} (w_{C(A)} w_{Ai}) |\Delta r_{Ai}|
\]  
(13)

where
\[
w_{C(M)} + w_{C(A)} = 1, \quad 0 < w_{C(M)} < 1, \quad \text{and} \quad 0 < w_{C(A)} < 1.
\]  
(14)

In these equations, the weighting factors \(w_{C(M)}\) and \(w_{C(A)}\) are respectively assigned to MI and AI to combine them into a single index for the multiobjective problem [34]. The comprehensive index, CI, can also be expressed in an exchanged consistent unit. The weighting factors \(w_{C(M)}\) and \(w_{C(A)}\) involve information on the relative complexity between the overall manufacturing and assembly processes, as well as their relative importance on influencing the expected overall production cost or time. The values of weighting factors \(w_{C(M)}\) and \(w_{C(A)}\) can also be evaluated and decided at the design phase through comprehensive process analysis and cost evaluation, which can be undertaken via the use of some CAD/CAM software incorporating with the designer’s judgement and experience. The value of \(w_{C(M)}\) is usually much greater than that of \(w_{C(A)}\) since the manufacturing processes may be much complicated than the assembly processes.

3.2. The optimization model

As mentioned earlier, the optimal tolerance synthesis for a cam-modulated linkage is the maintenance of acceptable kinematic accuracy of its output motion with maximum manufacturability and assembly. In practice, considering the \(j\)-th maximum expected deviation function \(\Delta \psi_{j,\text{rss}}(\theta)\) of the functional output variables shown in Eq. (8), its maximum value \((\Delta \psi_{j,\text{rss}})_{\text{max}}\) cannot exceed a specified upper bound, \(\Delta \psi_{j}^{(u)}\). Also, the difference between the maximum and minimum values of \(\Delta \psi_{j,\text{rss}}(\theta)\), \((\Delta \psi_{j,\text{rss}})_{\text{max}}\) and \((\Delta \psi_{j,\text{rss}})_{\text{min}}\), must be smaller than another specified upper bound, \(\delta_{\psi_{j}}^{(u)}\), in order to maintain...
the relative accuracy of the expected positions of the output link(s), especially the relative accuracy of the positions between the low and high dwells of a follower. According to the comprehensive index, CI, defined in Eq. (13), the optimization model can be formulated as follows:

\[
\text{find } X = \begin{bmatrix}
\Delta r_1 \\
\Delta r_2 \\
\vdots \\
\Delta r_n
\end{bmatrix}
\]

which maximize

\[
f(X) = Cl(\Delta r_1, \Delta r_2, \ldots, \Delta r_n)
\]

subject to

\[
(\Delta \psi_j)_{\text{max}} \leq (\Delta \psi_j)_{\text{U}} \quad \text{for } j = 1, 2, \ldots, m
\]

\[
\delta_{\psi_j} = (\Delta \psi_j)_{\text{max}} - (\Delta \psi_j)_{\text{min}} \leq (\Delta \psi_j)_{\text{U}} \quad \text{for } j = 1, 2, \ldots, m
\]

\[
\Delta r_i^{(L)} \leq \Delta r_i \leq \Delta r_i^{(U)} \quad \text{for } i = 1, 2, \ldots, n
\]

in which, \(\Delta r_i^{(L)}\) and \(\Delta r_i^{(U)}\) respectively denote the lower and upper bounds on the \(i\)-th dimensional tolerance, \(\Delta r_i\), in a cam-modulated linkage. The evaluation of the extreme values \((\Delta \psi_j)_{\text{max}}\) and \((\Delta \psi_j)_{\text{min}}\) shown in Eqs. (16) and (17) is based on the sensitivity analysis method shown in Section 2.1, hence the sensitivity of each dimensional tolerance on influencing the output deviations of the cam-modulated linkage can also be considered in the proposed optimization model. The optimization model shows a typical constrained optimization problem that must be solved numerically [34].

4. A practical case study

The tolerance analysis and synthesis methods presented above will be illustrated by the following practical case study. Fig. 3(a) shows the kinematic diagram of the MEG X6061 mechanism [35,36], which is an existing pick-and-place device of planar cam-modulated linkage type, and Fig. 3(b) shows the equivalent ten-bar linkage of this mechanism. Return springs (not shown in the figure) are used in the device to retain the intended contact of every higher pair. This device utilizes dual cams and dual primary followers to generate the end effector motion of the follower train. The dual cams A and B are fixed on a common shaft which rotates with respect to the frame and serves as the driving link of the mechanism. The frame is commonly numbered as 1, the driving link as 2, and other links are successively numbered. Cam A drives an oscillating follower (link 3) which in turn drives the end effector (link 5), a floating secondary follower, to generate the desired horizontal motion of the end effector. Similarly, cam B drives the other oscillating follower (link 4) which in turn drives a secondary translating follower (link 6) to generate the desired vertical motion of the end effector. Finally, a clamping device (not shown in the figure) is attached at the left end of the end effector to effectively pick and place target objects. In order to perform safely and reliably its intended function, the kinematic accuracy of the end effector must be evaluated through the tolerance analysis and synthesis of the mechanism.

As shown in Fig. 3(a), dual cams A and B are both pivoted at O2, and dual oscillating follower arms (links 3 and 4) are both pivoted at O3. The center distance O2O3 can be expressed as \(f = O2O3 = (f_h^2 + f_v^2)^{1/2}\) where \(f_h(=O2F)\) and \(f_v(=O3F)\) are the horizontal and vertical components of the distance, respectively. Then the subtending angle \(\alpha_{f} = \tan^{-1}(f_h/f_v)\). The base circle of cam A is \(r_{fA}\) and that of cam B is \(r_{fB}\). Rollers C and G, which are mounted on link 3, and rollers D and H, which are mounted on link 4, are all have the same radius of \(r_f\). The dimensions of link 3 are \(l_c = O2C, l_c = O3G,\) and \(\eta_A = \angle CO3G;\) the dimensions of link 4 are \(l_0 = O3D, l_0 = O3H,\) and \(\eta_B = \angle DO3H.\) The distance from the right-end plate of link 5 to its reference point \(P = l_2;\) the distance from the top-end plate of link 6 to its reference point \(M = l_6.\) The central axis of link 5, line PM, remains in the horizontal direction, the central axis of link 6, line EM, remains in the vertical direction, and thus \(\alpha_E = 90^\circ.\) The central axis of link 6 has an offset of \(e (=O2E)\) from the cam center. The cams rotate counterclockwise with a constant angular velocity of \(\omega_{c} \text{ rad/s}.\) By setting up a Cartesian coordinate system X–Y fixed on the driving link and with its origin at the fixed pivot O2, after the desired motions of the end effector have been specified, the profile coordinates of both cams can be expressed in terms of the cam rotation angle \(\theta,\) which is measured counterclockwise from the horizontal line to the X-axis. The cam rotation angle \(\theta\) is also the input variable of the mechanism in this case. The kinematic design of this mechanism has been studied by Chang et al. [36] in order to analytically determine the cam profiles, the paths of cutters, the pressure angles and the radii of curvature of the dual cams. The necessary equations and information based on the previous work [36] for undertaking the tolerance analysis and synthesis of the mechanism are provided below.

By assigning point P as the reference point of the end effector, after the horizontal motion program \(S_h(\theta)\) and the vertical motion program \(S_v(\theta)\) of point P have been specified, the position functions of point P relative to the cam center O2 can consequently expressed as the following two functions [36]:

\[
L_h(\theta) = l_5 + r_f - f_h + l_c \cos \left( \alpha_f + \eta_A - \cos^{-1} \left( \frac{f_h^2 + f_v^2 - (r_{fA} + r_f)^2}{2l_cf} \right) \right) + S_h(\theta)
\]
Fig. 3. A cam-modulated linkage type pick-and-place device (the MEG X6061 pick-and-place device [35,36]) and its equivalent ten-bar linkage.

and

\[
L_v(\theta) = l_6 - r_f - f_h + l_H \sin \left( \alpha_f + \eta_B - \cos^{-1} \left[ \frac{\eta_B + f_h - (r_{AB} + r_f)^2}{2l_H f_h} \right] \right) + S_v(\theta)
\]  

(20)

where \(L_h(\theta)\) is the horizontal displacement function and \(L_v(\theta)\) is the vertical displacement function of point \(P\). Here, functions \(S_h(\theta)\) and \(L_h(\theta)\) increase positively when point \(P\) moves leftward, and functions \(S_v(\theta)\) and \(L_v(\theta)\) increase positively when point \(P\) move downward; all of them are positive in Fig. 3(a). Then, the angular displacement functions of dual primary oscillating followers, \(O_3C\) and \(O_3D\), can be respectively expressed as [36]

\[
\xi_A(\theta) = \alpha_f + \eta_A - \gamma_A(\theta) - 90^\circ = \alpha_f + \eta_A - \cos^{-1} \left[ \frac{L_h(\theta) - l_5 - r_f + f_h}{l_c} \right]
\]  

(21)
and

$$\xi_B(\theta) = \alpha_f + \eta_B - \gamma_B(\theta) = \alpha_f + \eta_B - \sin^{-1}\left[\frac{l_c(\theta) - l_6 + r_f + f_c}{l_{II}}\right]$$

(22)

where \(\gamma_A(\theta)(=\alpha_f + \eta_A - \xi_A(\theta) - 90^\circ)\) and \(\gamma_B(\theta)(=\alpha_f + \eta_B - \xi_B(\theta))\) are the angular displacement functions of oscillating arms \(O_3G\) and \(O_3H\), respectively. The profile coordinates of cams \(A\) and \(B\) can be respectively expressed as \(36\)

$$O_2A(\theta) = \left\{\begin{array}{l}
q_A \cos(\alpha_f - \theta) + (Q_A - r_f) \cos(\alpha_f + \alpha_A - \theta) \\
q_A \sin(\alpha_f - \theta) + (Q_A - r_f) \sin(\alpha_f + \alpha_A - \theta)
\end{array}\right\}$$

(23)

and

$$O_2B(\theta) = \left\{\begin{array}{l}
q_B \cos(\alpha_f - \theta) + (Q_B - r_f) \cos(\alpha_f + \alpha_B - \theta) \\
q_B \sin(\alpha_f - \theta) + (Q_B - r_f) \sin(\alpha_f + \alpha_B - \theta)
\end{array}\right\}$$

(24)

where, by labeling instant centers \(I_{23}\) and \(I_{24}\) as \(Q_A\) and \(Q_B\), respectively,

$$q_A = O_2Q_A = O_2I_{23} = \frac{f \frac{d\xi_A(\theta)}{d\theta}}{1 + \frac{d^2\xi_A(\theta)}{d\theta}} \sqrt{V_B(\theta) + \left[\frac{l_c^2}{\xi_A(\theta)} - l_5 - r_f + f_c\right]^2}$$

(25)

$$q_B = O_2Q_B = O_2I_{24} = \frac{f \frac{d\xi_B(\theta)}{d\theta}}{1 + \frac{d^2\xi_B(\theta)}{d\theta}} \sqrt{V_B(\theta) - \left[\frac{l_c^2}{\xi_B(\theta)} - l_5 - r_f + f_c\right]^2}$$

(26)

in which, \(v_B(\theta) = dl_B(\theta)/d\theta = ds_B(\theta)/d\theta\) and \(v_A(\theta) = dl_A(\theta)/d\theta = ds_A(\theta)/d\theta\); and

$$Q_A = \sqrt{l_c^2 + (f - q_A)^2 - 2l_c(f - q_A) \cos \xi_A(\theta)}$$

(27)

$$Q_B = \sqrt{l_d^2 + (f - q_B)^2 - 2l_d(f - q_B) \cos \xi_B(\theta)}$$

(28)

$$\alpha_A = \angle CO_AO_3 = \sin^{-1}\left[\frac{l_c \sin \xi_A(\theta)}{Q_A C}\right]$$

(29)

$$\alpha_B = \angle DQ_BO_3 = \sin^{-1}\left[\frac{l_d \sin \xi_B(\theta)}{Q_B D}\right]$$

(30)

Also, the pressure angles can be expressed as \(36\)

$$\phi_A = 90^\circ - \alpha_A - \xi_A(\theta)$$

(31)

$$\phi_B = 90^\circ - \alpha_B - \xi_B(\theta)$$

(32)

and the shift angles can be expressed as

$$\lambda_A = \angle O_2AQ_A = \sin^{-1}\left[\frac{q_A \sin \alpha_A}{\|O_2A\|}\right]$$

(33)

$$\lambda_B = \angle O_2BQ_B = \sin^{-1}\left[\frac{q_B \sin \alpha_B}{\|O_2B\|}\right]$$

(34)

In this case, the specified motion programs, \(S_B(\theta)\) and \(S_A(\theta)\), are shown in \(4(a)\), while their analytical expressions can be found in the Appendix. Such motion programs can lead to the resulting path of point \(P\) as shown in \(4(b)\); that is, point \(P\) will successively occupy positions \(P_0, P_1, P_2, P_3, \ldots\) with \(P_0\) being the initial position. In addition, the path positions of point \(P\) are correlated with cam rotation angles. In other words, point \(P\) stays at its initial position \(P_0\) while the cams rotate counterclockwise from 0° to 13°, then moves downward via \(P_1\) to its first working position \(P_2\) for 40° cam rotation and then dwells for the next 30°. When the cams rotate from 83° to 123°, point \(P\) moves upward from position \(P_2\) via \(P_3\) to position \(P_4\); during this period, when \(\theta = 106^\circ\), point \(P\) also starts its leftward motion at position \(P_2\), simultaneously. When the cams rotate from 123° to 178°, point \(P\) moves leftward from position \(P_4\) to position \(P_5\). From 178° to 218°, point \(P\) moves downward again from position \(P_5\) via \(P_6\) to position \(P_7\), its second working position, and then dwells for the next 30°; during this period, when \(\theta = 195^\circ\), point \(P\) completes its leftward motion at \(P_5\). From 248° to 288°, tracing the former path, point \(P\) moves upward from position \(P_7\) via \(P_8\) to position \(P_9\); then from position \(P_9\), it moves rightward to position \(P_2\) for the next 72°. During this period, when \(\theta = 271^\circ\), it must occupy position \(P_3\). The horizontal and vertical travel lengths of point \(P\) are 100 and 30 mm, respectively, both with cycloidal motions \(8\). Other dimensions of the design parameters are: \(f_b = 60\) mm, \(f_c = 75\) mm, \(l_c = l_0 = 60\) mm, \(l_c = 190\) mm, \(l_{II} = 160\) mm, \(\eta_A = 80^\circ\), \(\eta_B = 15^\circ\), \(r_f = 8\) mm, \(r_B = 38\) mm, \(r_{Bb} = 72\) mm, \(l_5 = 230\) mm, \(l_6 = 200\) mm, \(e = 100\) mm, and \(\alpha_E = 90^\circ\). Fig. 4(c) shows the angular displacements...
of the oscillating followers. The determined cam profiles and their corresponding pressure angles ($\phi_A$ and $\phi_B$), shift angles ($\lambda_A$ and $\lambda_B$) and auxiliary angles ($\alpha_A$ and $\alpha_B$) are shown in Fig. 5(a)–(d), respectively. The cams $A$ and $B$ have their maximum radial dimensions of 68.008 and 72 mm, respectively. As can be observed in Fig. 5(a), for cam $A$, the radius of base circle, $r_{bA}$, is its minimum radial dimension. Also, for cam $B$, the radius of base circle, $r_{bB}$, is its maximum radial dimension.

4.1. Tolerance analysis results

For the equivalent ten-bar linkage of the MEG X6061 mechanism shown in Fig. 3(b), the couplers are the added links 7 and 8 of the linkage and they respectively connect the centers of curvature of cams $A$ and $B$ ($K_A$ and $K_B$) and the roller centers ($C$ and $D$). Also, the half joint $J_c$ is replaced by slider 9 which connects links 3 and 5, and the half joint $J_h$ is replaced by slider 10 which connects links 4 and 6. The instantaneous kinematic characteristics of the rockers (links 3 and 4) with respect to the driving link (link 2) shown in Fig. 3(b) will be identical to those of the dual primary follower with respect to the dual cams shown in Fig. 3(a).

For the equivalent linkage shown in Fig. 3(b), the input variable $\theta_2$ is the angular displacement of the driving link; it is measured counterclockwise from the horizontal line to $O_2K_A$. After points $K_A$ and $K_B$, the curvature centers of cams $A$ and $B$, have been located, the magnitude of angle $\theta_2$ together with the subtending angle $\beta = \angle K_AO_2K_B$ can be found. That is, the input variable $\theta_2$ is dependent to the cam rotation angle $\theta$, or $\theta_2 = \theta_2(\theta)$. Also, the variables $\theta_3$ and $\theta_4$ can be directly determined by $\theta_3 = 180^\circ + \alpha_f - \xi_A$ and $\theta_4 = 180^\circ + \alpha_f - \xi_B$. From the four-bar loops $O_2K_AC_3$ and $O_2K_BD_3$ of the equivalent linkage shown in Fig. 3(b), the implicit constraint equations that relate design parameters $r_{1h}$, $r_{1v}$, $r_{2A}$, $r_{2B}$, $r_{3C}$, $r_{4D}$, $r_7$, $r_8$, and
Fig. 5. Profiles and geometric angles of the dual cams A and B [36].
\[ \beta \text{ to the variables } \theta_2, \theta_3, \text{ and } \theta_4 \text{ may be respectively written as [15]} \]
\[
\begin{align*}
G_A &= G_A(r_{1h}, r_{1v}, r_{2A}, r_{3C}, r_7, \theta_2, \theta_3) \\
&= 2[r_{1h}(r_{3C} \cos \theta_3 - r_{2A} \cos \theta_1) + r_{1v}(r_{3C} \sin \theta_3 - r_{2A} \sin \theta_2) - r_{2A} r_{3C} \cos(\theta_2 - \theta_3)] \\
&\quad + r_{1h}^2 + r_{1v}^2 + r_{2A}^2 + r_{3C}^2 - r_7^2 = 0
\end{align*}
\] (35)

and
\[
\begin{align*}
G_B &= G_B(r_{1h}, r_{1v}, r_{2B}, r_{4D}, R_6, \beta, \theta_2, \theta_4) \\
&= 2[r_{1h}[r_{4D} \cos \theta_4 - \cos(\theta_2 + \beta)] + r_{1v}[r_{4D} \sin \theta_4 - \sin(\theta_2 + \beta)] \\
&\quad - r_{2A} r_{4D} \cos(\theta_2 + \theta_4)] + r_{1h}^2 + r_{1v}^2 + r_{2A}^2 + r_{4D}^2 - R_8^2 = 0
\end{align*}
\] (36)

where \( r_{1h} = OxF = f_{1h}, r_{1v} = OyF = f_{1v}, r_{2A} = OyK_A, r_{2B} = OyK_B, r_{3C} = OyC = l_c, r_{4D} = OyD = l_p, r_7 = K_A C, r_8 = K_B D, \) and \( \beta = \angle K_A OyK_B. \) From Eqs. (35) and (36), functions \( \theta_3 = \theta_3(r_{1h}, r_{1v}, r_{2A}, r_{3C}, r_7, \theta_2) \) and \( \theta_4 = \theta_4(r_{1h}, r_{1v}, r_{2B}, r_{4D}, r_8, \beta, \theta_2) \) exist implicitly according to \( G_A = 0 \) and \( G_B = 0, \) respectively. Further, by dealing with the loop closure equation of \( O_x O_y + O_y P - O_x P = 0, \) the constraint equations relating parameters \( r_{1h}, r_{1v}, r_{3C}, r_{4D}, r_5, r_6, e, \eta_A, \eta_B, \) and \( \alpha_E \) to the dependent and output variables \( \theta_3, \theta_4, l_h, \text{ and } l_v \) can be derived and expressed as
\[
F = F(r_{1h}, r_{1v}, r_{3C}, r_{4D}, r_5, r_6, e, \eta_A, \eta_B, \alpha_E, \theta_3(r_{1h}, r_{1v}, r_{2A}, r_{3C}, r_7, \theta_2(\theta))), \]
\[
\theta_4(r_{1h}, r_{1v}, r_{2B}, r_{4D}, \beta, \theta_2(\theta)), L_h(\theta), L_v(\theta))
\] (37)

where \( r_{3C} = OyG = l_c, r_{4D} = OyH = l_p, r_5 = l_5 + r_7, \) and \( R_6 = l_6 - r_7. \) In this case, \( r = [r_{1h} \ r_{1v} \ r_{2A} \ r_{3C} \ r_{4D} \ r_{3C} \ r_{4D} \ r_5 \ r_6 \ r_7 \ r_8 \ e \ \eta_A \ \eta_B \ \alpha_E] \)\(^T\), \( \theta = [\theta_2(\theta)], \) and the displacement functions of point \( P \) are designated as \( \Psi \) and defined by
\[
\Psi = \begin{bmatrix}
L_h(\theta) \\
L_v(\theta)
\end{bmatrix}
\] (38)

The 14 design parameters \( r_{1h}, r_{1v}, r_{3C}, r_{4D}, r_{3C}, r_{4D}, r_5, r_6, r_7, e, \eta_A, \eta_B, \) and \( \alpha_E \) in the equivalent ten-bar linkage as shown in Fig. 3(b) are considered to have deviations. Then by applying Eqs. (5), (35)-(38) and recalling that \( \alpha_E = 90^\circ, \) the output errors of the functional output variables, \( \Delta L_h \) and \( \Delta L_v, \) caused by the deviation of each design parameter can be derived and expressed as
\[
\Delta \Psi(1) = \begin{bmatrix}
\Delta L_h(1) \\
\Delta L_v(1)
\end{bmatrix} = -\begin{bmatrix}
\frac{\partial F}{\partial \Psi}
\end{bmatrix}^{-1} \begin{bmatrix}
\frac{\partial F}{\partial r_{1h}} \\
\frac{\partial F}{\partial r_{1v}}
\end{bmatrix} \Delta r_{1h} = \begin{bmatrix}
-l_c \cos \gamma_A \cos (\alpha_A + \alpha_f) \\
l_c \cos \gamma_A \cos (\alpha_A + \alpha_f)
\end{bmatrix} \frac{\partial \Delta L_h}{\partial \Psi}
\]
\[= \begin{bmatrix}
-l_c \cos \gamma_A \cos (\alpha_A + \alpha_f) \\
l_c \cos \gamma_A \cos (\alpha_A + \alpha_f)
\end{bmatrix} \frac{\partial \Delta L_h}{\partial \Psi}
\]
(39)

\[
\Delta \Psi(2) = \begin{bmatrix}
\Delta L_h(2) \\
\Delta L_v(2)
\end{bmatrix} = -\begin{bmatrix}
\frac{\partial F}{\partial \Psi}
\end{bmatrix}^{-1} \begin{bmatrix}
\frac{\partial F}{\partial r_{1v}} \\
\frac{\partial F}{\partial r_{1v}}
\end{bmatrix} \Delta r_{1v} = \begin{bmatrix}
-l_c \cos \gamma_A \sin (\alpha_A + \alpha_f) \\
l_c \cos \gamma_A \sin (\alpha_A + \alpha_f)
\end{bmatrix} \frac{\partial \Delta L_v}{\partial \Psi}
\]
\[= \begin{bmatrix}
-l_c \cos \gamma_A \sin (\alpha_A + \alpha_f) \\
l_c \cos \gamma_A \sin (\alpha_A + \alpha_f)
\end{bmatrix} \frac{\partial \Delta L_v}{\partial \Psi}
\]
(40)

\[
\Delta \Psi(3) = \begin{bmatrix}
\Delta L_h(3) \\
\Delta L_v(3)
\end{bmatrix} = -\begin{bmatrix}
\frac{\partial F}{\partial \Psi}
\end{bmatrix}^{-1} \begin{bmatrix}
\frac{\partial F}{\partial r_{3C}} \\
\frac{\partial F}{\partial r_{3C}}
\end{bmatrix} \Delta r_{3C} = \begin{bmatrix}
-l_c \cos \gamma_A \tan \phi_A / l_c \\
0
\end{bmatrix} \Delta l_c
\]
(41)
\[ \Delta \Psi(4) = \begin{bmatrix} 0 \\ \frac{\partial F}{\partial \psi} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F}{\partial \psi} \\ \frac{\partial F}{\partial \psi} \end{bmatrix} \Delta r_{4d} = \begin{bmatrix} 0 \\ r_{4l} \cos(\theta_4 + \eta_B) \frac{\partial G_B}{\partial \theta_4} \end{bmatrix} \Delta r_{4d} \]

\[ = \begin{bmatrix} 0 \\ -l_{lt} \cos \gamma_B \tan \phi_B \end{bmatrix} \Delta l_b \]  

(42)

\[ \Delta \Psi(5) = \begin{bmatrix} 0 \\ \frac{\partial F}{\partial \psi} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F}{\partial \psi} \\ \frac{\partial F}{\partial \psi} \end{bmatrix} \Delta r_{5c} = \begin{bmatrix} -\cos(\theta_5 + \eta_A) \\ 0 \end{bmatrix} \Delta r_{5c} = \begin{bmatrix} -\sin \gamma_A \\ 0 \end{bmatrix} \Delta l_c \]  

(43)

\[ \Delta \Psi(6) = \begin{bmatrix} 0 \\ \frac{\partial F}{\partial \psi} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F}{\partial \psi} \\ \frac{\partial F}{\partial \psi} \end{bmatrix} \Delta r_{6l} = \begin{bmatrix} 0 \\ -\sin(\theta_4 + \eta_B) \end{bmatrix} \Delta r_{6l} = \begin{bmatrix} 0 \\ \sin \gamma_B \end{bmatrix} \Delta l_c \]  

(44)

\[ \Delta \Psi(7) = \begin{bmatrix} 0 \\ \frac{\partial F}{\partial \psi} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F}{\partial \psi} \\ \frac{\partial F}{\partial \psi} \end{bmatrix} \Delta r_{7s} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta r_{7s} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta l_s \]  

(45)

\[ \Delta \Psi(8) = \begin{bmatrix} 0 \\ \frac{\partial F}{\partial \psi} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F}{\partial \psi} \\ \frac{\partial F}{\partial \psi} \end{bmatrix} \Delta r_{8l} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta r_{8l} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta l_s \]  

(46)

\[ \Delta \Psi(9) = \begin{bmatrix} 0 \\ \frac{\partial F}{\partial \psi} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F}{\partial \psi} \\ \frac{\partial F}{\partial \psi} \end{bmatrix} \Delta r_{9g} = \begin{bmatrix} -r_{3g} \sin(\theta_3 + \eta_A) \frac{\partial G_A}{\partial \theta_5} \\ 0 \end{bmatrix} \Delta r_{9g} = \begin{bmatrix} \frac{\partial G_A}{\partial \theta_5} \end{bmatrix} \Delta r_{9g} \]  

(47)

\[ \Delta \Psi(10) = \begin{bmatrix} 0 \\ \frac{\partial F}{\partial \psi} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F}{\partial \psi} \\ \frac{\partial F}{\partial \psi} \end{bmatrix} \Delta r_{10h} = \begin{bmatrix} 0 \\ r_{4h} \cos(\theta_4 + \eta_B) \frac{\partial G_B}{\partial \theta_4} \end{bmatrix} \Delta r_{10h} \]  

(48)

The radius errors of the follower rollers may be ignored since they generally have a small tolerance grade if precision rollers are used. As can be seen from Eq. (49), the variation of the offset (\( \Delta e \)) does not appear in the equation and thus will not affect the output errors. By substituting Eqs. (9), (33) and (34) into Eqs. (47) and (48) and ignoring the radius errors of the follower rollers, the respective output errors caused by the radial profile errors of cams A and B are

\[ \Delta \Psi(9) = \begin{bmatrix} \Delta L_{9h} \\ \Delta L_{9v} \end{bmatrix} = \begin{bmatrix} \frac{l_c \cos \gamma_A}{l_c \cos \phi_A} \\ 0 \end{bmatrix} \Delta r_A \cos \lambda_A \]  

(53)

\[ \Delta \Psi(10) = \begin{bmatrix} \Delta L_{10h} \\ \Delta L_{10v} \end{bmatrix} = \begin{bmatrix} 0 \\ -l_{lt} \cos \gamma_B/(l_d \cos \phi_B) \end{bmatrix} \Delta r_B \cos \lambda_B. \]  

(54)
Note that in the final expressions of Eqs. (39)–(54), parameters $r_{2A}$, $r_{2B}$, $r_7$, $r_8$, and $\beta$ are not actually involved. Thus, locating the curvature centers of the cam profiles is unnecessary for the tolerance analysis. Then, by using Eqs. (39)–(46) and (49)–(54), the worst case deviations of the functional outputs will be

$$\Delta \Psi_{wor} = \left\{ \frac{\Delta L_{b,wor}}{\Delta L_{v,wor}} \right\} = \left\{ \sum_{i=1}^{14} |\Delta L_{b(i)}| \right\} \quad (55)$$

and the maximum expected deviations of the functional outputs will be

$$\Delta \Psi_{rss} = \left\{ \frac{\Delta L_{b,rss}}{\Delta L_{v,rss}} \right\} = \left\{ \sum_{i=1}^{14} (\Delta L_{b(i)})^2 \right\} \quad (56)$$

For a tolerance grade of IT6 being specified to the mechanism, the 14 design parameters may have their extreme deviations of $\Delta f_x = \Delta f_y = \Delta c = \Delta f = \Delta \alpha = 19 \, \mu$m, $\Delta \alpha_c = \Delta H = 29 \, \mu$m, $\Delta c_5 = 25 \, \mu$m, $\Delta c_6 = 22 \, \mu$m, $\Delta \gamma_8 = 0.019^\circ = 3.32 \times 10^{-4}$ rad, $\Delta \gamma_9 = 0.011^\circ = 1.92 \times 10^{-4}$ rad, and $\Delta \epsilon_8 = 0.022^\circ = 3.84 \times 10^{-4}$ rad. Based on the derived output error equations, only $\Delta f_x$, $\Delta f_y$ and $\Delta \epsilon_8$ will lead to both horizontal and vertical output errors simultaneously. The deviations $\Delta \alpha_c$, $\Delta L_c$, $\Delta L_5$, and $\Delta \gamma_8$ will only lead to the horizontal output error, and the deviations $\Delta \gamma_8$, $\Delta \alpha_c$, $\Delta H$, and $\Delta \gamma_9$ will only lead to the vertical output error. The deviation of the offset ($\Delta \epsilon$) has no effect on the output errors.

All functions of the tolerance analysis results that might be of interest are shown in Fig. 6. Fig. 6(a) shows the resultant output errors of the horizontal displacement of point $P$ arising from each dimensional deviation. Therefore $\Delta L_{b(4)} = \Delta H_{b(6)} = \Delta L_{b(8)} = \Delta L_{b(10)} = \Delta L_{b(11)} = \Delta L_{b(13)} = 0$ and $\Delta L_{b(7)} = \Delta L = 29 \, \mu$m are invariant, they are not shown in Fig. 6(a). The seven error functions shown in Fig. 6(a) can be further divided into six types by their trends. Firstly, $\Delta L_{b(12)}$ (caused by $\Delta \eta_8$) apparently has only slight variation. Since from Eq. (50), the trend of $\Delta L_{b(12)}$ is dominated by $\cos \gamma_8$, which also has only slight variation while $\gamma_8$ varies only from about $-10^\circ$ to $20^\circ$ as shown in Fig. 4(c). That is, other error equations involving $\cos \gamma_8$ are also slightly influenced by this factor. Secondly, $\Delta L_{b(9)}$ (caused by $\Delta f_x$) and $\Delta L_{b(9)}$ (caused by $\Delta f_y$) have almost the same trend and extreme position when their signs are ignored. As shown in Fig. 5(c) and (d), $\alpha_c$ and $\lambda_\alpha$ have quite similar trends. Thus, from Eqs. (40) and (53), $\sin(\alpha_A + \alpha_f)/\cos \phi_A$ and $\cos \lambda_\alpha/\cos \phi_A$ are proportional to each other. Thirdly, the trend of $\Delta L_{b(1)}$ (caused by $\Delta f_x$) mainly dominated by $\cos(\alpha_A + \alpha_f)/\cos \phi_A$ is similar to that of $\alpha_A$ as shown in Fig. 5(d). Fourthly, $\Delta L_{b(3)}$ (caused by $\Delta L_c$) mainly dominated by $\tan \phi_A$ is proportional to pressure angle $\phi_A$. Fifthly, the trend of $\Delta L_{b(5)}$ (caused by $\Delta L_c$) is proportional to that of $\sin \gamma_8$. Finally, the trend of $\Delta L_{b(14)}$ (caused by $\Delta \epsilon_8$) mainly dominated by the vertical displacement function $L_v$ is similar to that of $S_A$ as shown in Fig. 4(a) when their signs are ignored. An important effect of $\Delta L_{b(14)}$ is that it appears variant motion error during horizontal dwells, while the others appear invariant ones. As can be seen, $\Delta \alpha_c$, $\Delta f_x$, $\Delta f_y$, and $\Delta \eta_8$ have the most effects on the output error $\Delta L_{bh}$, while $\Delta H$, $\Delta \gamma_8$ and $\Delta L_{b(13)}$ have the secondary effects.

Similarly, Fig. 6(b) shows the resultant output errors of the vertical displacement of point $P$ arising from each dimensional deviation. Therefore $\Delta L_{v(3)} = \Delta L_{v(5)} = \Delta L_{v(7)} = \Delta L_{v(9)} = \Delta L_{v(11)} = \Delta L_{v(12)} = 0$ and $\Delta L_{v(8)} = \Delta L = 29 \, \mu$m are invariant, they are not shown in Fig. 6(b). The seven error functions shown in Fig. 6(b) can also be further divided into six types by their trends, just like those shown in Fig. 6(a). Firstly, $\Delta L_{v(13)}$ (caused by $\Delta \eta_8$) obviously has only slight variation. Since from Eq. (51), the trend of $\Delta L_{v(13)}$ is dominated by $\cos \gamma_8$, which also has only slight variation while $\gamma_8$ varies only from about $10^\circ$ to $20^\circ$ as shown in Fig. 4(c). Hence, other error equations involving $\cos \gamma_8$ are also slightly influenced by this factor. Secondly, $\Delta L_{v(2)}$ (caused by $\Delta f_x$) and $\Delta L_{v(10)}$ (caused by $\Delta f_y$) have almost the same trend and extreme position when their signs are ignored. As shown in Fig. 5(c) and (d), $\alpha_c$ and $\lambda_\beta$ have quite similar trends. Thus, from Eqs. (39) and (54), $\sin(\alpha_B + \alpha_f)/\cos \phi_B$ and $\cos \lambda_\beta/\cos \phi_B$ are proportional to each other. Also, because the variations of $\sin(\alpha_B + \alpha_f)$ and $\cos \lambda_\beta$ are both flatted by $1/\cos \phi_B$, the magnitudes of $\Delta L_{v(2)}$ and $\Delta L_{v(10)}$ have only slight variations. Thirdly, the trend of $\Delta L_{v(14)}$ (caused by $\Delta f_x$) mainly dominated by $\cos(\alpha_B + \alpha_f)/\cos \phi_B$ is similar to that of $\alpha_B$ as shown in Fig. 5(d). Fourthly, $\Delta L_{v(4)}$ (caused by $\Delta L_c$) mainly dominated by $\tan \phi_B$ is proportional to pressure angle $\phi_B$. Fifthly, $\Delta L_{v(6)}$ (caused by $\Delta H$) is proportional to that of $\sin \gamma_8$. Finally, the trend of $\Delta L_{v(14)}$ (caused by $\Delta \epsilon_8$) mainly dominated by the horizontal displacement function $L_h$ is similar to that of $S_A$ as shown in Fig. 4(a) when their signs are ignored. An important effect of $\Delta L_{v(14)}$ is that it appears variant motion error during vertical dwells, while the others appear invariant ones. As can be seen, $\Delta \gamma_8$ has the most effect on the output error $\Delta L_{lv}$, while $\Delta H$, $\Delta \gamma_8$, and $\Delta \epsilon_8$ have the secondary effects.

As shown in Fig. 6(c), the extreme value of $\Delta L_{b,wor}$ occurring at $\theta = 331.68^\circ$ is $414.47 \, \mu$m, the extreme value of $\Delta L_{v,rss}$ occurring at $\theta = 331.42^\circ$ is $167.81 \, \mu$m; both are very close to $\theta = 331.91^\circ$, where the extreme pressure angle $\phi_A$ occurs. Also, the extreme value of $\Delta L_{v,wor}$ occurring at $\theta = 200.42^\circ$ is $235.69 \, \mu$m, and the extreme value of $\Delta L_{v,rss}$ occurring at $\theta = 200.72^\circ$ is $90.4 \, \mu$m; both are very close to $\theta = 200.63^\circ$, where the extreme pressure angle $\phi_B$ occurs.
Fig. 6. Tolerance analysis results of a cam-modulated linkage type pick-and-place device.

viewpoint of the position accuracy of the functional outputs, for total end effector travels of 100 mm in horizontal direction and 30 mm in vertical direction, position deviations of $\Delta L_{h,\text{max}} = 414.47 \mu m$ and $\Delta L_{v,\text{max}} = 235.69 \mu m$ imply a quite low accuracy. That is, if the worst situation occurs, although the design parameters are specified to have a small tolerance grade of IT6, the horizontal functional output will have a degraded accuracy ranging from IT12 (350 \mu m) to IT13 (540 \mu m), and the vertical functional output will also have a degraded accuracy ranging from IT12 (210 \mu m) to IT13 (330 \mu m). Even for better, the maximum expected deviations of the horizontal and vertical functional outputs will still lead to a degraded accuracy ranging from IT10 (140 and 84 \mu m for each direction) to IT11 (220 and 130 \mu m for each direction).

In addition, the relative accuracy of the expected positions between the low and high dwells of the end effector, i.e. the operating accuracy of the mechanism, dominates the operating performance of the pick-and-place device. To maintain and improve operating accuracy of the pick-and-place device should be of concern for designers. When considering the maximum expected deviations, the relative deviations between the low and high dwells of the horizontal and vertical functional outputs are merely 25.41 and 14.13 \mu m, respectively. Such slight deviations imply an acceptable operating accuracy of the mechanism ranging from IT6 (22 and 13 \mu m for each direction) to IT7 (35 and 21 \mu m for each direction). The
operating performance of the pick-and-place device can thus be maintained when its all design parameters are specified to have a tolerance grade of IT6.

4.2. Tolerance synthesis results

According to the optimization model presented in Section 3.2, the 14 dimensional tolerances \( \Delta f_h, \Delta f_e, \Delta l_c, \Delta l_d, \Delta l_c, \Delta l_l, \Delta l_b, \Delta l_e, \Delta r_A, \Delta r_B, \Delta e, \Delta \eta_A, \Delta \eta_B, \) and \( \Delta \alpha_E \) in the MEG X6061 pick-and-place device desired to be optimally synthesized are divided into manufacturing and assembly tolerances. Their corresponding weighting factors and lower and upper bounds are given in Table 1. In this case, according to the cause of these tolerance amounts, the four tolerances \( \Delta f_h, \Delta f_e, \Delta e, \) and \( \Delta \alpha_E \) are assigned for assembly, while the other 10 tolerances are assigned for manufacturing. After roughly estimating the overall manufacturing and assembly processes and their required costs and times, the weighting factors \( w^{(M)} \) and \( w^{(A)} \) for this case are reasonably given as 0.9 and 0.1, respectively. The weighting factors of the radial cam profile tolerances are undoubtedly much greater than those of other tolerances since the cam profiles must be accurately machined and inspected with relatively complicated procedures. In fact, for different manufacturers, the weighting factors listed in Table 1 may be modified via their estimation. The lower and upper bounds on each tolerance are respectively based on the tolerance grades of IT1 and IT12 for their corresponding dimensions of design parameters. Note that the lower and upper bounds on the radial profile tolerances of cams A and B (\( \Delta r_A \) and \( \Delta r_B \)) are based on the tolerances of their corresponding maximum radial dimensions.

Recall from Fig. 4(b) that the pick-and-place device is used to convey objects between the two working positions \( P_2 \) and \( P_7 \); thus, the relative accuracy between the two positions dominates the operating performance of this device. Hence, the relative variation of the horizontal output motion error \( \delta_{th} = (\Delta L_{h, rss})_{max} - (\Delta L_{h, rss})_{min} \) and that of the vertical output motion error \( \delta_{tv} = (\Delta L_{v, rss})_{max} - (\Delta L_{v, rss})_{min} \) must be controlled in small amounts. Moreover, the maximum output motion errors \( (\Delta L_{h, rss})_{max} \) and \( (\Delta L_{v, rss})_{max} \) can be larger since their effects may be reduced by slightly adjusting the fixed location of the clamping device attached on the end effector of the pick-and-place device. For this case, the upper bounds on the maximum output motion errors are respectively given as follows: \( \Delta L_{h}^{(0)} = 0.22 \) mm and \( \Delta L_{v}^{(0)} = 0.13 \) mm, based on a large tolerance grade of IT11 for the corresponding horizontal and vertical strokes of point \( P \), which are 100 and 30 mm, respectively. Also, for a tolerance grade of IT7 of the horizontal and vertical strokes, the upper bounds on the relative accuracy of the output motions are respectively given as \( \delta_{th} = 0.035 \) mm and \( \delta_{tv} = 0.021 \) mm. Accordingly, the optimization model can be formulated as follows:

\[
\text{find } \mathbf{X} = \{ \Delta f_h, \Delta f_e, \Delta l_c, \Delta l_d, \Delta l_c, \Delta l_l, \Delta l_b, \Delta l_e, \Delta r_A, \Delta r_B, \Delta e, \Delta \eta_A, \Delta \eta_B, \Delta \alpha_E \}^T
\]

which maximize

\[
f(\mathbf{X}) = \text{Cl}(\Delta f_h, \Delta f_e, \Delta l_c, \Delta l_d, \Delta l_c, \Delta l_l, \Delta l_b, \Delta l_e, \Delta r_A, \Delta r_B, \Delta e, \Delta \eta_A, \Delta \eta_B, \Delta \alpha_E) \]

subject to

\[
(\Delta L_{h, rss})_{max} \leq \Delta L_{h}^{(0)}
\]

\[
(\Delta L_{v, rss})_{max} \leq \Delta L_{v}^{(0)}
\]

Table 1

Weighting factors and lower and upper bounds of tolerances in a cam-modulated linkage type pick-and-place device.

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Weighting factors</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta f_h )</td>
<td>0.12</td>
<td>0.002 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta f_e )</td>
<td>0.12</td>
<td>0.002 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta l_c )</td>
<td>0.013</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta l_d )</td>
<td>0.0117</td>
<td>0.0035 mm</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>( \Delta l_l )</td>
<td>0.06</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta l_b )</td>
<td>0.06</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta l_e )</td>
<td>0.06</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta r_A )</td>
<td>0.4</td>
<td>0.02 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta r_B )</td>
<td>0.4</td>
<td>0.02 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta \eta_A )</td>
<td>0.015</td>
<td>0.0225 mm</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>( \Delta \eta_B )</td>
<td>0.015</td>
<td>0.0225 mm</td>
<td>0.35 mm</td>
</tr>
</tbody>
</table>

Assignment for manufacturing tolerances (with \( w^{(M)} = 0.9 \))

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta f_h )</td>
<td>0.002 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta f_e )</td>
<td>0.002 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta l_c )</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta l_d )</td>
<td>0.0035 mm</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>( \Delta l_l )</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta l_b )</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta l_e )</td>
<td>0.0045 mm</td>
<td>0.46 mm</td>
</tr>
<tr>
<td>( \Delta r_A )</td>
<td>0.4</td>
<td>0.02 mm</td>
</tr>
<tr>
<td>( \Delta r_B )</td>
<td>0.4</td>
<td>0.02 mm</td>
</tr>
<tr>
<td>( \Delta \eta_A )</td>
<td>0.0012*</td>
<td>0.35*</td>
</tr>
<tr>
<td>( \Delta \eta_B )</td>
<td>0.0012*</td>
<td>0.35*</td>
</tr>
</tbody>
</table>

Assignment for assembly tolerances (with \( w^{(A)} = 0.1 \))

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta f_h )</td>
<td>0.002 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta f_e )</td>
<td>0.002 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta l_c )</td>
<td>0.002 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>( \Delta l_d )</td>
<td>0.00025 mm</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>( \Delta \eta_A )</td>
<td>0.00025 mm</td>
<td>0.35 mm</td>
</tr>
<tr>
<td>( \Delta \eta_B )</td>
<td>0.00025 mm</td>
<td>0.35 mm</td>
</tr>
</tbody>
</table>
where the maximum expected deviations $\Delta L_{h,\text{rss}}$ and $\Delta L_{v,\text{rss}}$ shown in Eqs. (58)–(61) can be referred to in Eq. (56).

The MATLAB Optimization Toolbox [37] is utilized to solve the constrained optimization problem. The sequential quadratic programming (SQP) method is used to search the optimal solution, in which the Hessian matrix is estimated using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) formula, and a quadratic programming (QP) subproblem is solved at each iteration. The convergence tolerances for the numerical evaluation of the design variables, the constraint violation and the objective function are given as $1 \times 10^{-8}$, $1 \times 10^{-8}$, and $1 \times 10^{-10}$, respectively. The initial values of the 14 dimensional
tolerances, namely the initial design variables, are listed in Table 2, which are assigned based on a tolerance grade of IT6 for their corresponding dimensions of design parameters. The initial value of the objective function, CI, and those of the four extremes, $(\Delta L^L_{h, rss} \max), (\Delta L^C_{h, rss} \max), \delta_h$, and $\delta_l$, for evaluating the constraints are then calculated by adopting the initial design. It can be seen that the initial value of $\delta_h$ is 0.07075 mm, which is more than twice its upper bound $\delta_h^{(0)} (=0.035 \text{ mm})$. That is, the constraint of $\delta_h \leq \delta_h^{(0)} = 0.035 \text{ mm}$ is violated, and the initial design is infeasible. After the iteration process, the optimization results are also listed in Table 2. The maximum expected deviations $\Delta L^L_{h, rss}$ and $\Delta L^C_{h, rss}$ caused by the optimally synthesized tolerances are computed and shown in Fig. 7. As can be seen in Table 2, there are quite obvious differences between the initial values and the optimization results. The improved rates of CI, MI, and AI are 224.01%, 227.62%, and 191.43%, respectively. Both the manufacturability and assembly of the pick-and-place device can be increased twice with no constraint being violated if the optimally synthesized tolerances are adopted. That is, the SQP method is effective in searching the optimal design within the feasible region for the constrained optimization problem even the initial design is infeasible.

The optimally synthesized tolerances for $l_C, l_h, l_B, f_h, f_v$, and $\alpha_C$ are referred to small tolerance grades ranging from IT1 to IT3, especially $\Delta l_C$ which is at its lower bound. In the contrast, the optimally synthesized tolerances for $l_C, l_h, l_B$, and $e$ are referred to large tolerance grades ranging from IT9 to IT12, especially $\Delta e$ which is at its upper bound. The optimal tolerance grades for the radial dimensions of the cam profiles ($r_h$ and $r_b$) range from IT6 to IT8, which are larger than those for most of other parameters. This may be a benefit to help lower the production costs of the cams. In particular, the optimal tolerance grades of parameters $l_C, l_h, l_B$, and $e$ are larger than those of the cam profiles. It simply means that their deviations are more non-sensitive than the cam profile errors to contribute to the output errors, $\Delta L^L_{h, rss}$ and $\Delta L^C_{h, rss}$, or their relative variations, $\delta_h$ and $\delta_l$. Hence, not every tolerance with a smaller weighting factor needs to have a smaller tolerance grade. The limitations of the kinematic accuracy of the functional output are all satisfied, especially $\delta_l$, which is much smaller than its upper bound. As shown in Fig. 7, the relative deviations of the positions between the low and high dwells of $L_h$ and $L_v$ will not exceed 17.07 and 1.51 $\mu$m, respectively. The operating performance of the pick-and-place device can thus be ensured when the obtained optimal tolerances are specified.

5. Conclusions

This paper has presented comprehensive and systematic mathematical tools for dealing with the tolerance analysis and synthesis of cam-modulated linkages. By employing the concept of equivalent linkage and the derived correlation between the radial-dimension errors and the normal-direction errors of the cam profile, the sensitivity analysis method for equivalent linkages can be applied to analytically determine the output motion errors caused by the deviations (or tolerances) in the design parameters of a cam-modulated linkage. Then, by incorporating the sensitivity analysis method and the concept of DFMA, an optimization model for synthesizing the tolerances in cam-modulated linkages has been developed. The objective of this optimization model is to maximize the manufacturability and assembly of a cam-modulated linkage while maintaining acceptable kinematic accuracy of its output motion. A practical case study of analyzing and synthesizing the tolerances in the MEG X6061 cam-modulated linkage type pick-and-place device has been performed. In the worst case, owing to the joined effects of various design parameters, the accuracy of the functional outputs of the pick-and-place device may degrade considerably when the design parameters are specified to have a tolerance grade of IT6. But, the relative accuracy of the expected positions between the low and high dwells of the end effector, i.e. the operating accuracy of the mechanism, can maintain at an acceptable level. The tolerance synthesis results of the pick-and-place device show that the optimal tolerance grades for the cam profiles can be larger than those for most of other design parameters. This may be a significant benefit to help lower the production costs of the cams. In conclusion, the mathematical tools presented can be helpful to the design and manufacture of precision cam-modulated linkages.
Acknowledgment

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Appendix

The analytical expressions of the specified motion programs, $S_h(\theta)$ and $S_v(\theta)$, adopted in the practical case study are given by

$$S_h(\theta) = \begin{cases} 
0 \text{ mm when } 0^\circ \leq \theta < 106^\circ \\
100 \left\{ \frac{\theta - 106^\circ}{89^\circ} - \frac{1}{2\pi} \sin \left[ 2\pi \left( \frac{\theta - 106^\circ}{89^\circ} \right) \right] \right\} \text{ mm when } 106^\circ \leq \theta < 195^\circ \\
100 \left\{ 1 - \frac{\theta - 271^\circ}{89^\circ} + \frac{1}{2\pi} \sin \left[ 2\pi \left( \frac{\theta - 271^\circ}{89^\circ} \right) \right] \right\} \text{ mm when } 271^\circ \leq \theta \leq 360^\circ 
\end{cases}$$

(76)

$$S_v(\theta) = \begin{cases} 
0 \text{ mm when } 0^\circ \leq \theta < 13^\circ \\
35 \left\{ \frac{\theta - 13^\circ}{40^\circ} - \frac{1}{2\pi} \sin \left[ 2\pi \left( \frac{\theta - 13^\circ}{40^\circ} \right) \right] \right\} \text{ mm when } 13^\circ \leq \theta < 53^\circ \\
35 \left\{ 1 - \frac{\theta - 83^\circ}{40^\circ} + \frac{1}{2\pi} \sin \left[ 2\pi \left( \frac{\theta - 83^\circ}{40^\circ} \right) \right] \right\} \text{ mm when } 83^\circ \leq \theta < 123^\circ \\
35 \left\{ \frac{\theta - 178^\circ}{40^\circ} - \frac{1}{2\pi} \sin \left[ 2\pi \left( \frac{\theta - 178^\circ}{40^\circ} \right) \right] \right\} \text{ mm when } 178^\circ \leq \theta < 218^\circ \\
35 \left\{ 1 - \frac{\theta - 248^\circ}{40^\circ} + \frac{1}{2\pi} \sin \left[ 2\pi \left( \frac{\theta - 248^\circ}{40^\circ} \right) \right] \right\} \text{ mm when } 248^\circ \leq \theta < 288^\circ \\
0 \text{ mm when } 288^\circ \leq \theta \leq 360^\circ 
\end{cases}$$

(77)

In both specified motion programs, cycloidal motions (also known as sinusoidal acceleration functions) [8] are adopted for their non-dwell portions.

References