The Impact of GPS Positioning Errors on the Hop Distance in Vehicular Adhoc Networks (VANETs)

Wen-Hsing Kuo and Shih-Hau Fang
Department of Electrical Engineering,
Yuan Ze University,
Taoyuan, Taiwan
Email: whkuo@saturn.yzu.edu.tw

Abstract—**In this paper, we study the impact of GPS positioning errors on the operation of Vehicular Adhoc Networks. To the best of our knowledge, this important issue has not been investigated before. First, we formulate a straight-road model. Then, to reduce the computational complexity of finding the expected degradation, we propose an approximate formula. The results of the simulations show that GPS errors do indeed degrade the hop-distance in VANETs. Moreover, the proposed approximation method yields good accuracy under sparse density conditions. Our study provides a good starting point for further research into this issue.**

Index Terms—**VANET, GPS error, position-based routing**

I. Introduction

The Vehicular Adhoc Network (VANET) [1] is an emerging mobile technology that provides wireless communications between vehicles within the network, as well as between the vehicles and fixed equipment in the coverage area. With this technology, many traffic-related applications, such as collision warning, congestion detection and road-event broadcasts, can be implemented to provide drivers with a more comfortable driving experience and improved ability to decide the most appropriate driving route. To communicate with each other, vehicles in a VANET must be equipped with a wireless interface, which typically operates under the IEEE 802.11 Wi-Fi standard [2]. In addition to the physical devices, a wireless ad hoc routing scheme that can decide each packet's transmission path efficiently and dynamically is also essential. Because of the dynamic topology and the positions of mobile nodes in a VANET, position-based routing schemes are especially suitable. The basic concept of a position-based routing algorithm is to repeatedly find the node whose position is the most feasible to relay a packet to its destination (typically the node closest to the destination). Under this approach, each relay node can relay the packets to the next-hop node independently without knowing the whole network topology or the node distribution in the network.

To implement the above routing protocol, each node must know the location of nodes nearby. Therefore, most routing protocols proposed for VANETs assume that each vehicle is equipped with a GPS (Global Position System) device. By receiving and comparing the signals from several satellites whose positions are known, the geometric location (i.e., the longitude and latitude) of the receiver can be pinpointed. GPS technology is very suitable for VANETs because the receiving devices are always outdoors and can therefore sense the satellite signals. However, a number of factors, such as satellite position, signal attenuation and clocking errors [3], cause typical GPS applications to experience positioning errors of up to 30 meters on average; while the advanced version, DGPS, still yields an average error of 5 to 10 meters [4]. As a result, the vehicle closest to the destination may not always be chosen as the forwarding node, which may reduce the hop distance (i.e., the distance a packet travels in each hop,) and increase the hop count of each packet transmission. This leads to degraded throughput in the VANET and increased packet delay. To analyze the performance and estimate the capability of a VANET accurately, the hop distance degradation caused by positioning errors must be taken into consideration.

Position-based routing and the GPS positioning error problem have been studied separately by a number of researchers. For example, in [5], the authors propose the well-known greedy perimeter stateless routing (GPSR) protocol, which combines greedy routing and face routing to provide an integral and practical position-based solution. Similar protocols, which may differ in terms of the MAC layer and operating details, are presented in [6], [7] and [8]. Meanwhile, several works consider the accuracy of GPS technology. The improved accuracy of new GPS technologies is evaluated in [9], while a field test of current GPS devices in different environments is reported in [10]. In contrast, comparatively few works consider the impact of positioning errors on position-based routing. Among them, [11] proposes an on-demand position-based routing protocol. The authors also believe that positioning errors can affect the performance of position-based routing. Based on their analysis, they conclude that positioning errors increase the hop-count and the failure probability of route discovery. However, since the positioning error issue is not the focus of their study, the analysis is very simple and only suitable for their proposed routing scheme. In [12], the authors claim that their approach is the first to measure the GPS error for VANETs. However, they only provide the results of field tests on positioning errors, and do not measure the impact of such errors on VANETS, or propose an analysis method.

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To the best of our knowledge, there are no studies on the impact of GPS errors on the performance of VANETs. Therefore, to address this research gap, we focus on the issue in this paper and propose an estimation approach. We conduct simulations and try to identify trends in the results, and also evaluate the accuracy of the proposed approximation method. We find that positioning errors result in significant degradation of the hop distance, while our approximation method can accurately estimate trends when the traffic density is sparse. Although we only consider the straight road model in this paper, we hope that our results will motivate the study of more complicated road topologies, such as cross or grid networks. For those who are also interested in this important issue, this research provides a good starting point.

The remainder of this paper is structured as follows. In Section 2, we formulate the problem and state our assumptions. Based on the model, we analyze the hop-distance degradation caused by positioning errors. We also propose a simplified formula that serves as a useful approximation method. In Section 3, we discuss the simulations conducted to observe hop-distance degradation in different environments. Finally, the conclusions are summarized in Section 4.

II. Analysis of The Hop-Distance

A. System Model and Problem Specification

We study the model of a straight road with a single lane. Assuming the distribution of vehicles is memory-less, the probability density function (PDF) of the distance *d* between two adjacent vehicles is exponentially distributed and can be expressed as $p_{inteval}(d) = \delta e^{-\delta d}$, where δ is the average density. Let *R* denote the transmission range of each node.

In this paper, we study the "distance degradation of each hop". This is because hop distance degradation is the most direct effect of positioning errors. Typical performance metrics (e.g., throughput, end-to-end delay, and hop count) can be calculated basing on the hop distance. However, they need the information of the physical network deployment and the utilized routing protocol, both of which are beyond the scope of this paper.

Consider a straight-road network topology in which a message can be relayed in either the forward or backward direction. In a position-based scheme, the vehicle that appears to have the largest hop distance (i.e., the one closest to the boundary of the transmission range,) in the direction that the packet is traveling is chosen as the forwarding node. Since the transmission range is relatively large compared to the width of the road, we can neglect the lane of each vehicle and only consider the positions of cars in the direction of the road. Therefore, we can index each vehicle in decreasing order of the hop distance, as shown in Fig. 1.

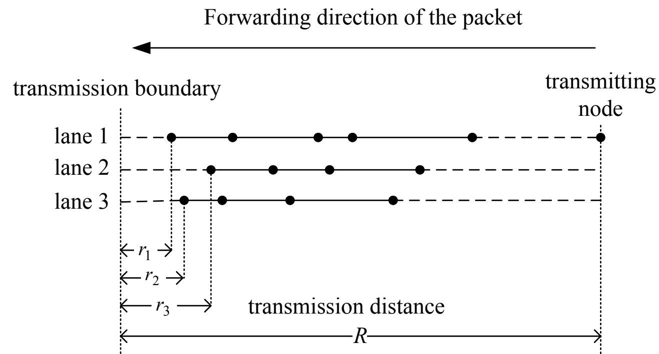


Figure 1. A multi-lane straight road.

Let r_i denote the position (i.e., distance to the boundary of the transmission range) of a vehicle i; and let r_i' denote the sensed position (i.e., the location the GPS detects) of vehicle r_i'. Because of GPS positioning errors, r_i' is not always equal to r_i. The next forwarding node is determined according to the position of r_i', which can be affected by random errors. Therefore, even vehicle *1* has the largest hop distance, it may not always be chosen. To ensure that the analysis is feasible, we assume that the distribution of the positioning error for each vehicle is independent and identical. Therefore, the PDF of r_i' can be presented as $e(r - r_i)$, where $e(r)$ is the PDF of the positioning error *r*. To estimate the probability that a vehicle will be selected as the forwarding node, we first consider the so-called *swap order*, which means that two nodes, say *i* and *j*, have different positioning orders when sorted in terms of the sensed positions and the actual positions of nodes (i.e., $(r_i' - r_j')(r_i - r_j) < 0$). Based on $e(\cdot)$, we can derive the function $P_{swap}(d)$, which refers to the possibility that the "swap order" involves two nodes whose actual distance is $|r_i - r_j| = d$. $P_{swap}(\cdot)$ can be derived easily from $e(\cdot)$ by letting $P_{swap}(d) = \int_{-\infty}^{\infty} e(r)\big(1 - E(d - r)\big)\,dr = e(d) * (1 - E(d))$, where $E(\cdot)$ is the CDF of $e(\cdot)$. This is because, when the distance between two nodes is *d*, for a node with a GPS error *r*, another node must have at least a (*d-r*) GPS error so that the "swap order" can occur. $P_{swap}(d)$ is a decreasing function because the larger the distance between two nodes, the lower will be the probability that they can be swapped.

Since vehicle i only becomes the forwarding node when it has the largest sensed hop distance among all vehicles (i.e., $r_i' < r_j', \forall j \neq i$), by combining the position distributions ($\delta e^{-\delta r}$) and P_{swap} of all vehicles, the probability that vehicle i will become the forwarding node can be expressed as follows:

$$
\begin{aligned}
&\mathrm{P}(r_i' < r_j', \forall j \neq i) \\
&= \prod_{\mathrm{j}\neq\mathrm{i}} \mathrm{P}(r_i' < r_j') \\
&= \prod_{\mathrm{j=1}}^{\mathrm{i-1}} \mathrm{P}(i \text{ and } j \text{ swap order}) \prod_{\mathrm{j=i+1}}^{\infty} \mathrm{P}(i \text{ and } j \text{ do not swap order}) \\
&= \lim_{\mathrm{k}\to\infty} \int_0^{\mathrm{R}} \delta e^{-\delta \mathrm{r_1}} \int_{\mathrm{r_1}}^{\mathrm{R}} \delta e^{-\delta(\mathrm{r_2}-\mathrm{r_1})} \ldots \int_{\mathrm{r_k}}^{\mathrm{R}} \delta e^{-\delta(\mathrm{r_k}-\mathrm{r_{k-1}})} \prod_{\mathrm{j=1}}^{\mathrm{i-1}} P_{swap}(r_i - r_j) \prod_{\mathrm{j=i+1}}^{\mathrm{k}} \big(1 - P_{swap}(r_j - r_i)\big)\, \mathrm{dr_k} \ldots \mathrm{dr_3 dr_2 dr_1} \qquad (1)
\end{aligned}
$$

The expected degradation of the hop distance can be derived in a similar manner. The degradation formula shown in

(2) combines the reduced hop distance of vehicle *i* (i.e., $r_i - r_1$,) and the probability that it will become the next hop node, for *i*=*1* to *k*, in all possible positions:

$$\lim_{k\to\infty} \int_0^R \delta e^{-\delta r_1} \int_{r_1}^R \delta e^{-\delta(r_2-r_1)} \dots \int_{r_k}^R \delta e^{-\delta(r_k-r_{k-1})} \prod_{j=1}^{i-1} P_{swap}(r_i - r_j) \prod_{j=i+1}^{k} \left(1 - P_{swap}(r_j - r_i)\right) (r_i - r_1) dr_k \dots dr_3 dr_2 dr_1 \quad (2)$$

Although (2) is accurate, it is not useful because the computational complexity is very high, and the P_{swap} values of all vehicles must be known in advance. In the next subsection, we reduce the number of factors considered in order to find an approximation approach that can calculate the degradation efficiently.

B. Approximation

To reduce the complexity of the analysis, we consider the sparse density scenario, in which the distance between vehicles is relatively large. Among all vehicles *i*>1, vehicle 2 is the most likely candidate to become the next hop node because it is closest to vehicle 1. In other words, it is the most likely candidate to swap its order with vehicle 1. Under the assumption of sparse density, compared with vehicle 2, the other vehicles (3, 4, etc.) are relatively far from vehicle 1. Thus, the probability that any of them will become the next hop can be ignored. By eliminating all such vehicles from (2), the expected hop degradation can be simplified as follows:

$$\int_0^R \delta e^{-\delta r_1} \int_{r_1}^R \delta e^{-\delta(r_2-r_1)} (r_2 - r_1) p_{swap}(r_2 - r_1) dr_2\, dr_1 \quad (3)$$

By denoting $(r_2 - r_1)$ and r_1 as r' and r respectively, (3) can be written as:

$$\delta^2 \int_0^R e^{-\delta r} \int_0^{R-r} e^{-\delta r'} r' \cdot p_{swap}(r') dr'\, dr. \quad (4)$$

To simplify (4), we let $u(r) = \int_0^{R-r} e^{-\delta r'} r' \cdot p_{swap}(r') dr'$, $dv(r) = e^{-\delta r} dr$, following $du(r) = -e^{-\delta(R-r)}(R-r) \cdot p_{swap}(R-r) dr$ and $v(r) = \frac{e^{-\delta r}}{-\delta}$. Considering the face that $\int u\, dv = uv - \int v\, du$, we have:

$$\begin{aligned}
&\delta^2 \int_0^R e^{-\delta r} \int_0^{R-r} e^{-\delta r'} r' \cdot p_{swap}(r') dr'\, dr \\
&= \delta^2 \left[\frac{e^{-\delta r}}{-\delta} \int_0^{R-r} e^{-\delta r'} r' \cdot p_{swap}(r') dr'\right]_{r=0}^{R} - \\
&\quad \delta^2 \int_0^R \frac{e^{-\delta r}}{-\delta} \left(-e^{-\delta(R-r)}(R-r) \cdot p_{swap}(R-r)\right) dr \\
&= \delta^2 [0 - \frac{1}{-\delta} \int_0^R e^{-\delta r'} r' \cdot p_{swap}(r') dr'] + \\
&\quad \delta \int_0^R e^{-\delta r} \left(-e^{-\delta(R-r)}(R-r) \cdot p_{swap}(R-r)\right) dr \\
&= \delta \int_0^R e^{-\delta r'} r' \cdot p_{swap}(r') dr' - \delta \int_0^R e^{-\delta R}(R-r) \cdot \\
&\quad p_{swap}(R-r) dr.
\end{aligned}$$

Then, by denoting $(R-r)$ as q, the formula can be rewritten as :

$$\delta \int_0^R e^{-\delta r'} r' \cdot p_{swap}(r') dr' - \delta e^{-\delta R} \int_0^R q \cdot p_{swap}(q) dq. \quad (5)$$

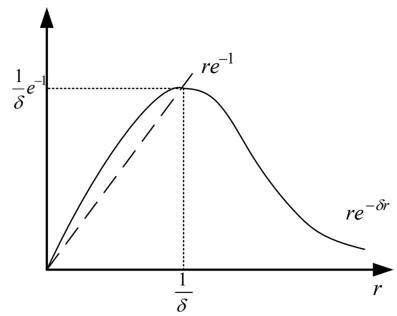


Figure 2. Approximating $re^{-\delta r}$ by re^{-1} when $r < \frac{1}{\delta}$.

To further simplify (5), we consider the term $e^{-\delta r'} r'$. Figure 2 shows that $e^{-\delta r} r$ has the maximal value $\frac{1}{\delta} e^{-1}$ when $r = \frac{1}{\delta}$. Therefore, when $r < \frac{1}{\delta}$, the function $e^{-\delta r} r$ can be approximated by $e^{-1} r$. This approximation is reasonable because it is assumed that δ is small so that $\frac{1}{\delta}$ tends to be large. By substituting $e^{-\delta r'} r'$ with $e^{-1} r'$, (5) can be rewritten as follows:

$$\delta e^{-1} \int_0^R r' \cdot p_{swap}(r') dr' - \delta e^{-\delta R} \int_0^R q \cdot p_{swap}(q) dq \quad (6)$$

By letting $E_{swap} = \int_0^R r \cdot p_{swap}(r) dr$, the approximation of the expected hop degradation is

$$\delta(e^{-1} - e^{-\delta R}) E_{swap} \quad (7)$$

Note that E_{swap} can be computed from $p_{swap}(r)$ and $e(r)$, or measured directly. When a vehicle has only one neighbor vehicle which is randomly distributed in its transmission range, (i.e., whose PDF is $\frac{1}{R}$ for $0 \le r \le R$), the expected distance between the two nodes when the "swap order" condition exists is $\int_0^R r \cdot p_{swap}(r) \frac{1}{R} dr = \frac{E_{swap}}{R}$. By measuring this distance, the value of E_{swap} can be found easily since *R* is known.

Therefore, unlike (2) which is hard to compute, (7) can estimate the degradation of the hop distance efficiently as long as δ, *R* and E_{swap} are known. Moreover, instead of describing the GPS positioning error by a function or a matrix, our approach only uses a single parameter, E_{swap}, which makes both the estimation process and the measurement beforehand easier and more efficient.

III. Numerical Results

A. Simulation Settings

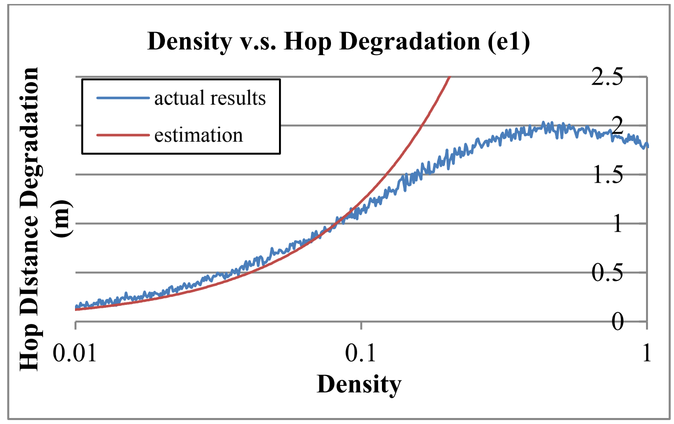
We conduct simulations to observe the trend of the positioning error and verify the accuracy of the approximation. To eliminate unrelated factors, such as the distance to the destination and the forwarding mechanism, we only simulate a forwarding scenario in which we have to find the most appropriate vehicle to relay a packet. The actual position r_i of each vehicle *i* is generated by an exponential random generator based on a given density δ. Then r_i' can be derived by adding a positioning error (i.e., $e(r)$,) to each r_i. To investigate the forwarding behavior of nodes under different distributions of positioning errors, we consider two positioning error patterns: $e_1(r) = \begin{cases} \frac{1}{20}, |r| < 10 \\ 0, \text{otherwise} \end{cases}$ and $e_2(r) = \begin{cases} \frac{1}{20} - \frac{|r|}{400}, |r| < 20 \\ 0, \text{otherwise} \end{cases}$. In other words, the sensed position r_i' for $e_1(r)$ is uniformly distributed within ($r_i \pm 10$) meters, while $e_2(r)$ causes r_i' to be distributed within ($r_i \pm 2\ 0$) meters, but it has a higher possibility to be more approximate to r_i. For the estimation, E_{swap}, we compute $e_1(r)$ and $e_2(r)$ in advance as approximately 33.3 meters and 25 meters respectively.

Among the sensed positions r_i' for all i, the vehicle that appears to have the largest hop distance (i.e., vehicle $i_{min} = \arg\min_i r_i'$) is chosen as the forwarding node, and the hop degradation caused by the positioning error (i.e., $r_{i_{min}} - r_1$) is recorded. For each setting, we average the results of 1,000 simulations. The corresponding approximation $\delta(e^{-1} - e^{-\delta R})E_{swap}$ is also computed for comparison with the actual results.

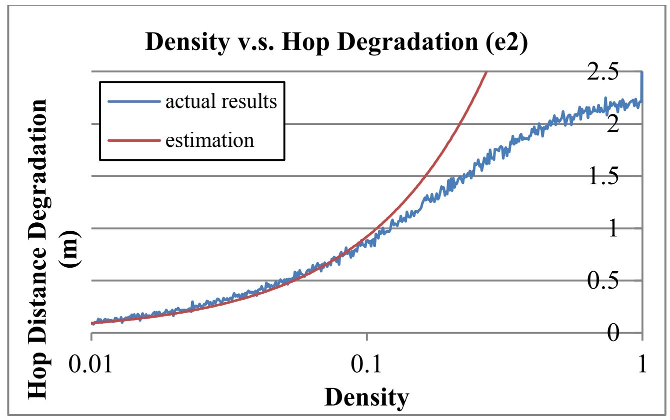
B. *Scenario I*

In the first simulation, we fix the transmission range at 100 *m* and adjust the vehicle density to observe how the density leads to hop degradation in VANETs. To assess the system's behavior under a wide range of vehicle densities, the value of δ is adjusted from 0.01 to 1(*vehicle/m*), and the *x* coordinate of the graph is set as logarithmic. The results of $e_1(r)$ and $e_2(r)$ are shown in Figs.3 (a) and 3(b) respectively. We observe that, in both figures, the degradation becomes more acute and then gradually saturates as δ increases. This is because the greater the vehicle density, the smaller will be the distance between vehicles, which increases the probability that one of the other vehicles (vehicle 2, 3 etc.) will wrongfully become the next hop node. However, under higher densities, an incorrectly chosen vehicle may also be closer to vehicle 1 and thus yield less hop degradation. When δ is larger and makes the latter effect dominates, the hop degradation gradually saturates.

It is also obvious that under sparse density conditions, the proposed estimation method can accurately approximate the actual results. In Figs. 3(a) and 3(b), when δ<0.1, the approximation is similar to the actual results. This is because our approach makes two assumptions: (i) vehicles after vehicles 1 and 2 can be neglected; and (ii) $e^{-\delta r} \cong e^{-1}r$. However, the assumptions only hold when is small. Despite this limitation, our approximation method is still practical because, in reality, the condition of low traffic density (δ<one vehicle per 10 meters) is very common (e.g., in country areas). Since the estimations in both figures are close to the actual results, it is clear that our approach can be applied to different positioning error patterns.



(a) $e_1(r)$



(b) $e_2(r)$

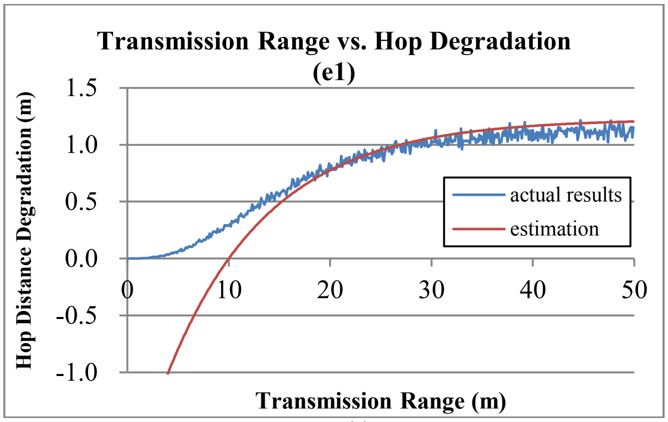
Figure 3. Vehicle Density vs. Hop Distance Degradation

C. *Scenario II*

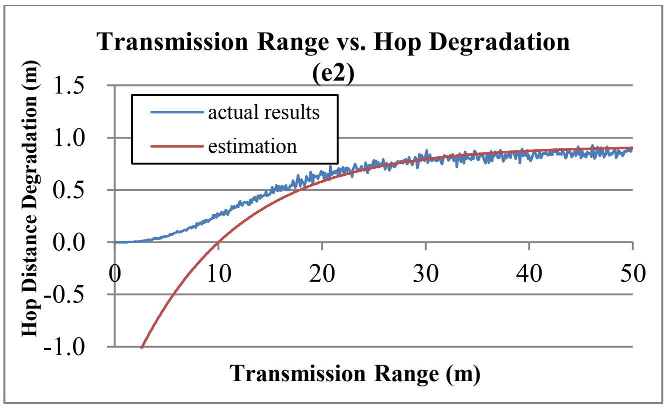
In the second simulation, we fix the vehicle density at 0.1 and study the impact of various transmission ranges. The transmission range is adjusted from 0 to 50 meters, and the trend of hop degradation is observed. The results of $e_1(r)$ and $e_2(r)$ are shown in Figs. 4(a) and 4(b), respectively.

First of all, we find that as *R* increases, the degradation of the hop distance increases, but gradually saturates. This is because there are fewer vehicles in the transmission range when *R* is small, thus limiting the probability that the "swap order" condition will occur. However, when *R* is larger, the presence of more distant vehicles does not make much difference because the vehicle closest to vehicle 1 has the highest probability of becoming the forwarding node.

The results also demonstrate the accuracy of our approximation method. When *R* is large enough, the method can reflect the saturation trend of both $e_1(r)$ and $e_2(r)$ accurately. Although the method fails to correctly estimate the results under small transmission ranges (i.e., *R*<10*m*), this range is too small to be realized. Therefore, the results still imply that our approach can accurately estimate the degradation in the hop distance under most transmission ranges and different positioning error patterns.



(a) $e_1(r)$



(b) $e_2(r)$

Figure 4. Transmission Range vs. Hop Distance Degradation

IV. CONCLUSION AND FUTURE WORK

In this paper, we study how GPS positioning errors affect the operation of VANETs. To the best of our knowledge, this important issue has not been investigated before. Based on our proposed model, we show that analyzing the performance degradation caused by GPS positioning errors is complicated, even under a straight-road model. To solve this problem, we propose an approximation approach that can efficiently estimate the degradation of the hop distance, basing on the given the vehicle density, the transmission range of the radio device, and the factors of a GPS positioning error.

We conducted simulations to verify our analysis and to observe the trend of the hop distance degradation. The results show that positioning errors do produce observable hop distance degradation, and thus affect the performance of a VANET. The degradation increases and then saturates gradually as the vehicle density and the transmission range increase. Our approximation approach provides a very close estimate under most conditions.

Because of space limitations, we cannot provide a more sophisticated analysis here. In the future, based on the observations and results reported in this paper, we will consider different road topologies, and also study the impact of positioning errors on other performance metrics, such as the transmission rate and packet delay.

ACKNOWLEDGMENT

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